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Medical Image Restoration with Different Types of Noise*

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Abstract— **The images obtained by X-Ray or computed tomography (CT) in adverse conditions may be contaminated with noise that can affect the detection of diseases. A large number of image processing techniques (filters) have been proposed to remove noise. These techniques depend on the type of noise present in the image. In this work, we propose a method designed to reduce the Gaussian, the impulsive and speckle noise and combined noise. This filter, called PGNDF, combines a non-linear diffusive filter with a peer group with fuzzy metric technique. The proposed filter is able to reduce efficiently the image noise without any information about what kind of noise might be present. To evaluate the filter performance, we use mammographic images from the mini-MIAS database which we have damaged by adding Gaussian, impulsive and speckle noises of different magnitudes. As a result, the proposed method obtains a good performance in most of the different types of noise.**

I. Introduction

The denoising techniques to restore noisy images are an important subject nowadays, for example, medical images obtained by X-Ray or computed tomography CT in adverse conditions, or a mammographic image which may be contaminated with noise that can affect the detection of microcalcifications. The aim of this work is to design a filter system to remove the noise efficiently, without having the initial information about what kind of noise might be present. For this task, we used a mammogram image from the Database of mini-MIAS [1]. This mammogram has been added with Gaussian and/or impulsive (fixed) and speckle noise respectively.

Different methods have been proposed for image restoration depending on the type of noise in the image, for example, for Gaussian noise, the methods based on filtering the image in the space or in the frequency domain (see [2] for a review), methods based on solving regularized least-squares problems [3] and methods based on the use of total variation and non-linear diffusion equations [4], [5], [6], [7], [8], [9], [10], [11], [12]. In the case of impulsive noise, we can use recent techniques based on the concept of peer group with fuzzy metric which have provided good results in RGB images [13], [14], [15]. A total Variation model to remove speckle noise in images is proposed in [16] and [17]. The Sylvester–Lyapunov Equation is used to achieve medical image noise reduction [18].

The basic model with Gaussian noise assumes that the Gaussian (normal) distribution has zero mean and is not correlated with the image; this case is known as Gaussian white noise of zero mean.

Impulsive noise or "salt & pepper" have dark pixels and bright pixels in the image. This type of noise can be caused by analog-to-digital converter errors or bit errors in transmission. The "salt & pepper" noise is characterized by its density d, ratio between the number of corrupted pixels and the image size.

Speckle noise is a granular noise that inherently exists in and degrades the quality of the images. Speckle is a multiplicative noise. The equation used is $J=I+n*I$, where n is uniformly distributed random noise with zero mean.

In this work, we propose a method which combines a peer group with fuzzy metric method with non-linear diffusion method.

The paper is organized as follows: Section II explains the algorithm to remove impulsive, Gaussian and speckle noise. The results of the experimental study are shown in Section III and, the conclusions are presented finally, in Section IV.

II. Methods to remove noise

A. Peer Group and Fuzzy Metric (PGFM)

A class of denoising methods is based on the technique of peer group and fuzzy metric [13], [14], [15]. The process is divided into two steps. The aim of the first step is to detect erroneous pixels and the second, to correct them. For the detection stage, the fuzzy metric between pixel x_i and x_j is used as described in [13], this metric is defined by:

$$M(x_i,x_j) = \frac{min\{x_i,x_j\}+k}{max\{x_i,x_j\}+k}, \quad (1)$$

where $k > 0$.

Fuzzy metric is employed in peer group $P(x_i,d)$, where x_i is the central pixel in a window W with size n x n (in our work, n = 3 was considered) and $d \in [0,1]$. The $P(x_i,d)$ group is defined by [14]:

$$P(x_i,d) = \{x_j \in W : M(x_i,x_j) \geq d\} \quad (2)$$

The detection of corrupted pixels is performed in two phases. The first phase calculates the peer group of x_i in W and all pixels that belong to the peer group. It is declared as non-corrupted if the cardinality of the $P(x_i,d)$ is greater than

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(m+1), where m is a threshold. Otherwise they are labeled as undiagnosed. In the second phase, the pixels labeled as undiagnosed are analyzed. All pixels that belong to the peer group are labeled as non-corrupt if the cardinality of the $P(x_i,d)$ is greater than (m+1), otherwise the central pixel is marked as corrupted.

The three parameters (k, d and m), which are determined heuristically in the process described take values in a certain range depending on the input image. The image is executed with different d and k values and the quality of the result determines the parameter selection. The value of d depends on the amount and type of noise introduced.

In the correction step, given a x_i previously marked as corrupted, we replace it by the Arithmetic Mean Filter (AMF) of its neighboring pixel values (labeled as non-corrupted) in its window W.

B. Non-linear diffusive filter (NDF)

As mentioned in the introduction, a class of image restoration methods is based on the use of non-linear diffusion equations [4], [5], [6], [7], which appear associated to a variational problem and may be obtained from the minimization of the appropriate functional. The choice of a particular functional depends upon the specific goal of interest. For example, several diffusive filters, suitable for medical imaging [9], have been obtained from the minimization of the appropriate functional.

Let us consider the functional [10],

$$J(u,\beta,\mu,\varepsilon) = \int_\Omega \left(\sqrt{\beta^2 + \|\vec{\nabla} u\|^2} + \frac{\mu}{2}\left(u - I_0\right)^2 + \frac{\varepsilon}{2}(\vec{\nabla} u)^2 \right) d\vec{x}, \quad (3)$$

where I_0 is the observed image (with noise), u is filtered image, μ and ε are constant and Ω is a convex region of R^2 constituting the support space of the surface $u(x,y)$, representing the image. The first term in the functional for $\beta = 1$ represents the area of the surface representing the image [6], the second term gives an account of the distance between the observed image and the desired solution u, and the third term controls the regularity of the solution.

The denoising image process corresponds to the minimization problem [5], [6] (total variation):

$$\min_u J(u,\beta,\mu,\varepsilon) \quad subject \ to \quad \frac{\int_\Omega (u - I_0)^2 d\vec{x}}{\int_\Omega d\vec{x}} = \sigma^2. \quad (4)$$

The solution of this problem is the image u that minimizes the functional $J(u,\beta,\mu,\varepsilon)$ satisfying the above restriction. The condition means that the “error” between the original and the denoised images must be equal to σ, where σ is the standard deviation of the noise present in the image. It’s important a good estimation of σ to minimize equation (3).

In our work, we estimate the noise level present in the image by using a robust estimation proposed by Donoho in [12] based in the discrete wavelet transformed. According to [12], the estandard deviation of the noise present in the image can be estimated by $\sigma = \frac{median(|D_{ij}|)}{0.6745}$, where D_{ij} are the diagonal wavelet transformed coefficient. In this work, the wavelet used was the Daubechies wavelet of order 25. The estimation of the image noise level is the key stone of the non-linear diffusive filter.

The solution of the minimization problem leads to a time discretization and, therefore, to a iterative solution of the problem. For the time discretization, we use a semi-implicit scheme, and for solving the equations we use the alternative additive operator splitting (AOS) [7],[10]. The stopping time selection in the Diffusion equation was proposed by Mrázek and Navara, based on the decorrelation criterium[11].

C. Peer Group-Fuzzy Non-linear diffusion filter (PGFND)

This technique is the combination of PGFM and NDF method. The sequence of application of the methods is as follows: first PGFM and then NDF. The peer group with fuzzy metric approach removes the impulsive noise and the Gaussian noise is eliminated by NDF and both methods to eliminate speckle noise

III. RESULTS

In order to evaluate the performance of the proposed filter, we took images from the mini-MIAS database, we assumed that these images were free of noise, and we corrupted them by adding Gaussian, impulsive and speckle noise. Noisy images were filtered by using PGFM, NDF and PGFND methods and we compared the image quality obtained in each case. The results are shown in this section.

To quantify the amount of noise removed, we used the peak signal to noise ratio (PSNR), the mean squared error (MSE) and the mean absolute error (MAE). In particular, PSNR is used to measure noise reduction and MAE is used for the preservation of the signal.

The mean square error of two monochrome images u and I of size M x N is defined as:

$$MSE = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \left\| I(i,j) - u(i,j) \right\|^2. \quad (5)$$

The PSNR is defined as:

$$PSNR = 10 \log_{10} \left(\frac{MAX_I^2}{MSE} \right). \quad (6)$$

where MAX_I is the maximum possible pixel value of the image.

The mean absolute error is given by,

$$MAE = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \left\| I(i,j) - u(i,j) \right\|. \quad (7)$$

We have made the analysis experiment for several images and for different noise levels for each type of the noise, although we only show here the results for one image (figure 1) and one noise level for each type of noise.

Original images were in grayscale, the intensities ranged from 0 to 255. The test images were generated by adding gaussian, impulsive and speckle noise to the image with the function of MATLAB *imnoise*: gaussian white noise of zero

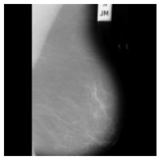


Figure 1. Original Image. 1024x1024

mean and variance 0.01, salt & pepper noise of density 0.10 and multiplicative noise of variance 0.04. We added these noises by separated or combined way.

As we mentioned in Section II, the fuzzy method requires the heuristically selection of several parameters, therefore, we previously selected the best performance parameter values for each type of noise. Table I shows the best results for the parameters *d* and *m* for each kind of noise. Values *d* and *m* depend on the type and amount of noise introduced. In the cases which involve the variance, *m* has the same value (8, all neighbors), otherwise the value is 4. With a variance of 0.01, the value of *d* is 0.92.

Through a similar process to that used in article [13] the value of *k* was obtained, whose optimal value is usually 1024 for this type of images.

Once the heuristic parameters *k*, *d* and *m* were determined, we filtered the noisy images with the proposed filter PGFND and with PGFM and NDF filters, described in Section II.

Applying the filters to the image with 10% fixed impulsive noise, we obtained the quality of the filtered image from the original shown in table II. As we can see, when the image contains only impulsive noise, the best method is PGFM. We can also use the PGFND method with a little quality difference below the PGFM method. The NDF method does not provide good image quality. Figure 2 shows the results of applying the filters. Figure 2*a* is the image with this type of noise. The 2*b* is the image filtered with PGFM, 2*c* and 2*d* is filtered with PGFM and NDF respectively. We can see that 2*b* and 2*c* are the best.

In the case of images only with Gaussian noise, the performance of Diffusion methods NDF and PGFND have similar results and are better than PGFM method. Table III shows the results.

TABLE I. BEST VALUE OF THE PARAMETERS.

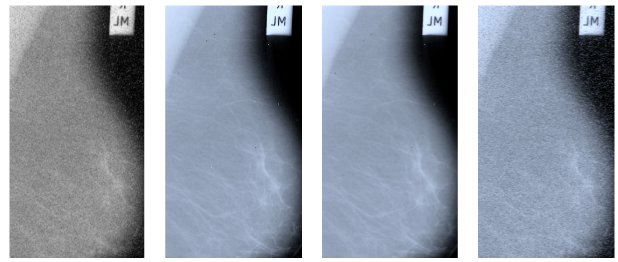
	m	*d*
D=0.10 for fixed impulsive noise (*D*=density)	5	0.85
σ^2= 0.01 for Gaussian noise	8	0.92
D=0.10 for fixed impulsive noise with σ^2=0.01 for Gaussian	8	0.92
σ^2 =0.04 for speckle noise	8	0.88

TABLE II. QUALITY MEASURES FOR THE IMAGE WITH DENSITY (D) = 0.10 (FIXED IMPULSIVE NOISE).

	MSE	PSNR	MAE
Filtered image with PGFM	5.5365	40.6985	0.1793
Filtered image with NDF	36.4523	24.3934	9.4899
Filtered image with PGFND	7.1813	39.5688	0.759
Noisy image	1.90E+02	15.3459	12.821

TABLE III. QUALITY MEASURES FOR IMAGE WITH VARIANCE=0.01 (GAUSSIAN NOISE).

	MSE	PSNR	MAE
Filtered image with PGFM	305.8954	23.2751	13.266
Filtered image with NDF	88.5022	28.6613	7.3277
Filtered image with PGFND	99.7893	28.14	6.9826
Noisy image	621.5723	20.1959	19.672



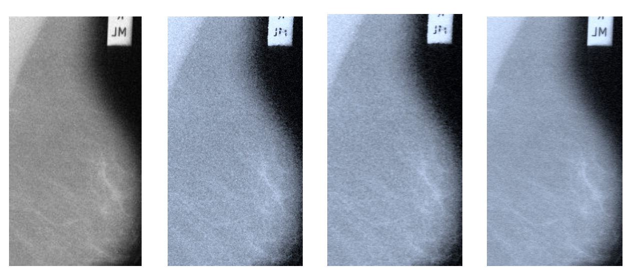
a b c d

Figure 2. Results for image size 512x960: a) Density (*D)* =0.01 for fixed impulsive noise, b) Filtered image with PGFM, c)Filtered image with PGFND, d) Filtered image with NDF.

NDF and PGFND are about 8 units PSNR with respect to the noisy image. Figure 3 shows the resulting images. Figure 3*a* is the image with this type of noise. 3*b* is the image filtered with PGFM, 3*c* with PGFND and 3*d* filtered with NDF. We can see that figures 3*c* and 3*d* are the best, PGFND and NDF methods.

For images contaminated with two types of noise (table IV), shows that the PGFND method is about 4 units PSNR better than the other methods and with respect to the original noisy image is approximately 13 units PSNR. Figure 4 shows the resulting images. Figure 4*a* is the image with 0.01 of Impulsive noise and 0.010 of Gaussian noise. 4*b* is the image filtered with PGFM, 4*c* with PGFND and 4*d* filtered with NDF. We can see that the 4*c* is the best filter.

Table V show the results obtained for speckle noise. We can see that PGFND and NDF have a similar behavior. The PGFM method with this kind of noise does not eliminate the noise present in the image. Figure 5 shows the resulting images. Figure 5*a* is the image with this type of noise. 5*b* is the image filtered with PGFM, 5*c* with PGFND and 5*d* filtered with NDF. We can see that figures 5*c* and 5*d* are the best filters.

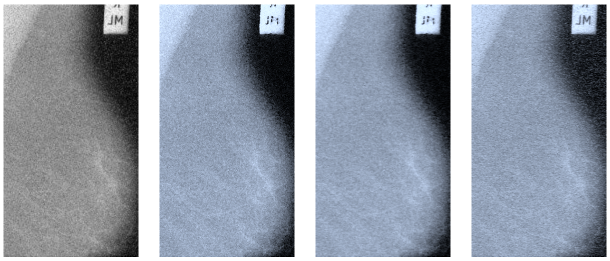


a b c d

Figure 3. Results for image size 512x960: a) σ^2 =0.01 for Gaussian noise, b) Filtered image with PGFM, c)Filtered image with PGFND, d) Filtered image with NDF.

TABLE IV. QUALITY MEASURES FOR IMAGE WITH (D) = 0.10 AND 0.01 OF GAUSSIAN (FIXED IMPULSIVE AND GAUSSIAN NOISE).

	MSE	PSNR	MAE
Filtered image with PGFM	322.9214	23.0398	13.6645
Filtered image with NDF	299.4993	23.3668	12.5675
Filtered image with PGFND	103.1628	27.9956	7.3411
Noisy image	2.23E+03	14.6383	29.6629

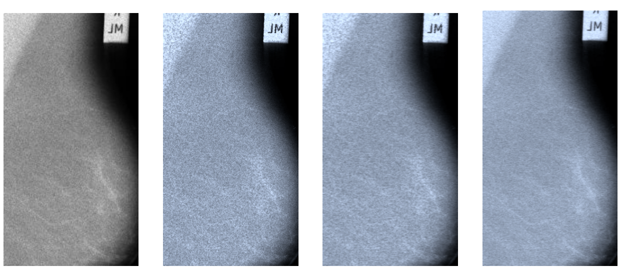


a b c d

Figure 4. Results for image size 512x960: a) D=0.10 for fixed impulsive and σ^2 =0.01 for Gaussian noise, b) Filtered image with PGFM, c)Filtered image with PGFND, d) Filtered image with NDF.

TABLE V. QUALITY MEASURES FOR IMAGE WITH SPECKLE NOISE FOR IMAGE SIZE 512X960.

	MSE	PSNR	MAE
Filtered image with PGFM	1.25E+02	20.3004	19.0856
Filtered image with NDF	1.02E+02	28.0572	7.412
Filtered image with PGFND	7.37E+03	27.1649	8.1496
Noisy image	8.47E+02	18.8536	23.3195



a b c d

Figure 5. Results for image size 512x960: a) Speckle noise, b) Filtered image PGFM, c)Filtered image PGFND, d) Filtered image NDF.

IV. CONCLUSION

In this paper we present the results obtained by applying the PGFND method and compare them with PGFM and NDF methods to remove the impulsive noise (fixed), Gaussian and speckle on a mammogram obtained from the database mini-MIAS. If the image contains only impulsive noise (fixed), the best technique is PGFM, although the method PGFND provides similar results. If the image contains only Gaussian or speckle noise, the best technique for removing noise has been the NDF method, followed closely by the PGFND method. When the image contains some combination of the noise discussed, PGFND method is the best with respect to other methods. We conclude that the PGFND is the best method when there is no information about the nature of the noise.

In future works, due to the high computational cost of the process, we will introduce high performance computing (GPUs, Multicore, libraries). The parameters *d, m* and *k* will be studied to find the general value for different images.

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