

Comparative study between Sample Entropy and Detrended Fluctuation Analysis performance on EEG records under data loss

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Abstract— This study compares two signal entropy measures, Sample Entropy (SampEn) and Detrended Fluctuation Analysis (DFA) over real EEG signals after a randomized sample removal. Both measures have demonstrated their ability to discern between, among others: control and pathologic EEG signals, seizure free or not, control and opened eyes EEG, and side of brain signals.

Results show that SampEn behaves better when analyzing control signals, while DFA provides better segmentation results between epileptic signals, in the context of sample loss, particularly when discerning between seizure and seizure free signal intervals.

I. INTRODUCTION

Biomedical signals are usually highly nonlinear and non-stationary, their statistical characteristics often change with time. Classical linear or non linear dynamic analysis methods are thus not suitable for their analysis as they are based on an implicit assumption of stationarity, which is rarely the case in this type of signals [1].

Entropy measures are a family of statistics that provide information about the chaotic or deterministic nature of a signal by quantifying the time–series regularity, namely, they measure the likelihood that runs of patterns that are close, remain close in the next incremental comparison [1].

This work compares Sample Entropy (SampEn) and Detrended Fluctuation Analysis (DFA) to characterize sample loss on EEG signals. A previous work showed that SampEn was found to be the regularity measure which provides better segmentation results for the analyzed EEG signals, using SampEn, Approximate Entropy (ApEn) and Multi Scale Entropy (MSE), considered in [2].

As a novelty, in this study we compare SampEn against Detrended Fluctuation Analysis (DFA), a less used entropy estimator, but that has provided excellent results where other entropy metrics failed. The comparative analysis is carried out in the context of sample loss, situation often found

in signal compression or transmission schemes, such as telemedicine applications.

Ambulatory monitoring or home monitoring are common practice nowadays. The acquired data need to be remotely sent to a medical facility for its analysis. This data transmission is performed using radio links, which can undergo connection interruptions, packet loss, high noise or interference [3], [4]. These techniques present limitations, such as energy saving or hardware design, which requires data compression to be performed [5]. All the previous mentioned can also introduce a randomized sample loss into the original signal, which needs to be characterized.

II. METHODS AND MATERIALS

We computed SampEn and DFA over different types of EEG records after a randomized data removal. Results were evaluated in terms of the cross correlation coefficient (CC), the confidence intervals (CI), and statistical tests.

A. SampEn algorithm

SampEn is an entropy measure proposed as an improvement of Approximate Entropy (ApEn) derived by Richmann *et. al.* [1] in order to reduce ApEn bias due to self-matches and record length. It is the negative logarithm of the conditional probability that two sequences which are similar for m points within a tolerance r , remain similar at the next point, where self-matches are not included in the computations of the conditional probabilities [1].

Let's consider a time–series $\{u(j) : 1 \leq j \leq N\}$ of length N , and define the runs of length m according to: $x_i = x_m(i) = \{u(i+k) : 0 \leq k \leq m-1\}$. A dissimilarity measure needs to be defined in order to determine which runs are similar within a tolerance r . The dissimilarity measure is defined as $d_m(x_i, x_j) = \max\{|x_m(i) - x_m(j)| : 0 \leq j \leq N-m\}$, then the conditional probability and thus SampEn are obtained by:

$$B_i^m(r) = \frac{1}{N-m+1} \sum_{\substack{j=1 \\ j \neq i}}^{N-m} (d_m(x_i, x_j) \leq r) \quad (1)$$

$$A_i^m(r) = \frac{1}{N-m+1} \sum_{\substack{j=1 \\ j \neq i}}^{N-m} (d_{m+1}(x_i, x_j) \leq r) \quad (2)$$

$$B^m(r) = \frac{1}{N-m} \sum_{i=1}^{N-m} B_i^m(r) \quad (3)$$

$$A^m(r) = \frac{1}{N-m} \sum_{i=1}^{N-m} A_i^m(r) \quad (4)$$

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$$\text{SampEn}(m, r, N) = -\log \frac{A^m(r)}{B^m(r)} \quad (5)$$

B. Detrended Fluctuation Analysis (DFA)

DFA is defined as the modified root mean square of a random walk. It enables the detection of long-range correlations embedded in a seemingly nonstationary time-series, while avoiding spurious detection of apparent long correlations that are an artifact of nonstationarity. DFA explores the potential utility of scaling alterations in the detection of pathologic states [6].

Let's define a windowing sequence with different window lengths $\{L_i(k) : 1 \leq i \leq I, 1 \leq k \leq K\}$, where I is the maximum number of windows in the sequence, and K the length of the window, and compute an integration of the series defined:

$$y(n) = \sum_{n=1}^n u(n) - \bar{u} \quad (6)$$

where \bar{u} is the signal mean. For each windowing length L_i of K samples, a least-square fit with a 1st order polynomial function, $y_k(n)$, is computed for each segment inside the window and calculate the root-mean square error as:

$$F_i^2(K) = \sum_{n=(i-1)K+1}^{iK} y(n) - y_k(n) \quad (7)$$

$$F(K) = \sqrt{\frac{1}{I} \sum_{i=1}^I F_i^2(K)} \quad (8)$$

Finally, a scaling coefficient, α is obtained from the relation between the window lengths and (8) in the logarithmic scale, this is: $F(K) \propto K^\alpha$ [6].

Power-law correlations can be divided into 3 main categories in terms of α . If $0 < \alpha < 0.5$ power-law correlations are said to be of different type, anti-correlated data, if $0.5 < \alpha \leq 1$ power-law correlations are persistent and of long-range and finally if $\alpha \geq 1$ correlations exist but cease to be of a power-law form [6].

C. Evaluation

Evaluation of the sample loss influence on entropy calculations is performed in terms of the correlation coefficient (CC), the confidence intervals (CI) and a statistical discrimination test.

The CC is computed according to Eq.(9), between the original series entropy value and the entropy value of the series after the random sample loss removal:

$$CC = \left| \frac{\sum_{i=1}^{N_s-1} (RM - \mu_{RM})(RM_r - \mu_{RM_r})}{\sqrt{\sum_{i=0}^{N_s-1} (RM - \mu_{RM})^2} \sqrt{\sum_{i=0}^{N_s-1} (RM_r - \mu_{RM_r})^2}} \right| \quad (9)$$

where RM refers to the regularity measure (SampEn or α) of the original time-series, RM_r is the RM of the reduced sample time-series, N_s is the number of signals

being analyzed and μ_{RM} is the mean value of RM . The correlation coefficient is used to assess the clinical validity of the results ($CC \geq 0.8-0.9$) [3].

The CI are computed in terms of the mean (μ) and the mean standardized error (σ), according to:

$$CI = [\mu - 2\sigma_\mu, \mu + 2\sigma_\mu] \quad (10)$$

$$\mu = \frac{1}{N_s} \sum_{i=1}^{N_s} RM \quad (11)$$

$$\sigma_\mu = \frac{\sigma}{\sqrt{N_s}} \quad (12)$$

where σ denotes the standard deviation of the data.

Finally, a Student T-Test is performed to obtain statistical values for the discrimination between data.

D. Experimental set

The experimental EEG database consists of 5 data sets, each containing 100 single-channel EEG segments of 23.6 sec. duration. These segments were selected and cut out form continuous multichannel EEG recording after visual inspection for artifacts. They also had to fulfill a stationarity criterion.

The five sets, denoted A-E contain different signals. Data sets A and B are healthy volunteer surface EEG, in a relaxed open eyes state (A) or relaxed close eyes state (B). Sets C, D and E contain epileptic signals. Set D was recorded from within the epileptogenic hippocampal formation and set C from the hippocampal formation of the opposite side of the brain. set E only contains seizure activity while C and D contain seizure free intervals [7].

SampEn was computed with traditional parameters, $m=2$ and $r=0.15$. DFA windowing lengths were define according to [8], where only integers divisors of N were considered, so as to consider all the information in the signal by not discarding any samples. Windowing lengths had to satisfy: $L_i : \text{mod}(N, L_i) = 0$.

E. Sample loss generation

For each signal in the database, an auxiliary time-series of length N in terms of a random distribution was generated. The data in the auxiliary signal were sorted, and the first R values, which corresponded to the data removal percentage, were removed from $u(j)$. This process was repeated 100 times for each removal percentage. The signals were characterized by the mean value of the 100 realizations.

DFA main drawback is the number of windows, as the sequence needs to have at least 3 to provide reliable results. Therefore, not all removal percentages were suitable for analysis. The removal percentages analyzed were $R = \{0, 5, 10, 15, 20, 30, 45, 60, 65, 70, 75, 80, 85\}$ which covered a wide range of values.

III. RESULTS

Both DFA and SampEn exhibit a $CC \geq 0.8$ for all sample loss ratios considered, enabling the clinical validity of the signals even when more than half of the samples are lost.

Fig.1–Fig.4 show the CI of the entropy measures for the different types of signals considered. Fig.1 shows SampEn (Fig.1.a) and DFA (Fig.1.b) in terms of the sample loss ratio between healthy and epileptic EEG. A better segmentation and a clearer and simpler segmentation boundary can be set for SampEn, although DFA only loses segmentation capability for 4/13 ratios considered (p Value = $\{0.274, 0.663, 0.053, 0.203\}$ for $R=\{20, 75, 80, 85\}$), respectively. There is not a clear trend in α for each group, as for SampEn, control (CT) signals present higher entropy values for any R value, indicating lower regularity, α is higher for some R values and lower for others.

Fig.2 presents the CI for open vs. closed eyes CT EEG. Both regularity measures, SampEn and DFA, allow a clear separation between both types, but as in Fig1 the separation boundary is again easier to compute in SampEn than in DFA. For the first one, a lower order polynomial function would be required. The p -value for Student T-test is 0 for any R value in both measures. CI are nonoverlapping.

Fig.3, presents the segmentation between seizure free signal intervals and seizure signal intervals on epileptic (EP) EEG signals, for lower R than 60% (p Value = 0.243, $CI=[1.114, 1.182]$ for seizure free and $CI=[1.333, 1.245]$ for seizure signals). SampEn allows the differentiation between both types of signals, while DFA enables it for the whole range of R considered, with nonoverlapping CIs and p Value = 0 for any R .

The last segmentation considered is presented in Fig.4. This segmentation is between side brain EEG signals, namely, EEG coming from the hippocampal epileptogenic zone and EEG coming from the hippocampal region on the opposite side of the brain. On a first sight, both measures can enable segmentation but when proceeding with the Student T-Test statistical analysis, it can be seen that p -values for SampEn are higher than 0.05 for any R , stating no segmentation is possible. For DFA, the p -value yields lower results than 0.022 for any R , presenting only overlapping CI for $R=60\%$ ($CI = [0.529, 0.559]$ for epileptogenic zone and $[0.555, 0.576]$ for the opposite side) but allowing segmentation.

Finally, Fig.5 shows the variability of both entropy measures, SampEn (Sp) and DFA (α), along with the samples loss ratio (R). DFA always exhibits a lower variability range than SampEn, for any type of signal considered.

IV. DISCUSSION

Both entropy measures enable clinical validity, since for any sample loss ratio, R considered, the CC is higher than 0.8 for any type of EEG in this study.

In Fig.1, it can be observed that SampEn has a more stable behavior, as it is an increasing function of R , which can be modeled by a low order polynomial function, whereas DFA has an unpredictable behavior, since for some R control signals, it yields lower values than pathologic signals and

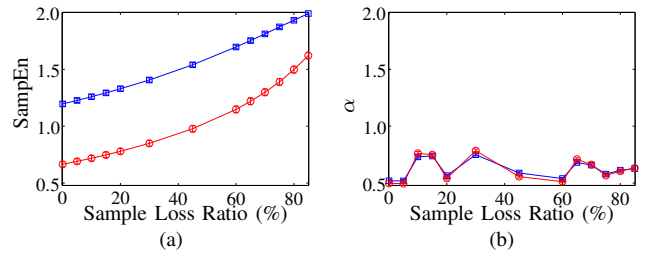


Fig. 1. Segmentation between control (CTRL, blue squares) vs. epileptic (PAT, red circles) subjects in terms of the sample loss ratio. (a) SampEn, (b) DFA scaling exponent (α).

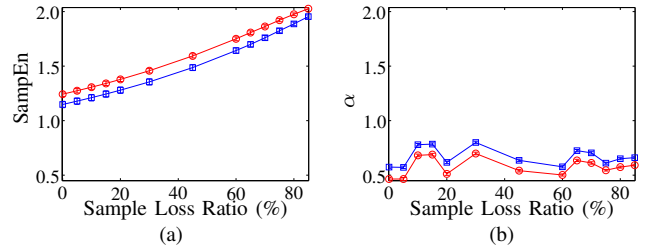


Fig. 2. Segmentation between control subjects with open eyes (blue squares) vs. control subjects with closed eyes (red circles), in terms of the sample loss ratio. (a) SampEn, (b) DFA scaling exponent (α).

vice-versa. This makes SampEn a more robust measure when comparing control vs. pathologic signals. With an adaptive threshold algorithm, defined in terms of the signal and R , identification and classification of epileptic patients could be performed, showing how the regularity of the signal is increased for epileptic patients, conclusions that can no be derived by just observing α .

The same reasoning can be done when comparing control EEG recorded with open or closed eyes (Fig.2). Both measures enable segmentation with nonoverlapping CIs and low Student T-test for any R , but SampEn enables an easier segmentation than DFA.

When analyzing epileptic patients (Fig.3 and Fig.4) DFA performance is higher. SampEn only enables segmentation between seizure free and seizure intervals for $R \leq 60\%$, while DFA enables it not only for them, but also between areas of the brain. What is even more interesting is that DFA behaviour is almost uniform, just showing an increment in α for seizure free intervals or the opposite hippocampal area. The similar behaviour in the scaling coefficient α in both

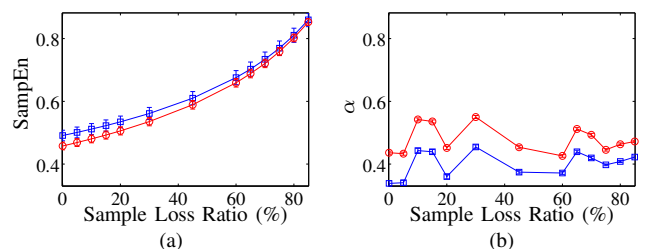


Fig. 3. Segmentation between seizure free intervals (red circle) and during seizure intervals (blue square) in terms of the sample loss ratio. (a) SampEn, (b) DFA scaling exponent (α).

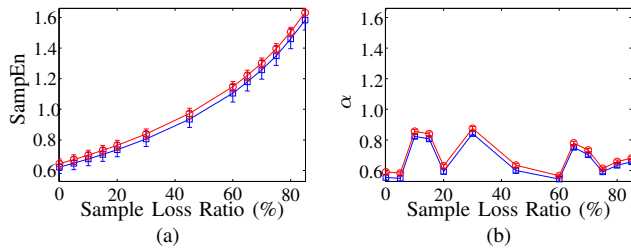


Fig. 4. Segmentation between recording areas, epileptogenic formation hippocampal area (blue squares) and the opposite hippocampal area (red circles) in epileptic patients. (a) SampEn, (b) DFA scaling exponent (α).

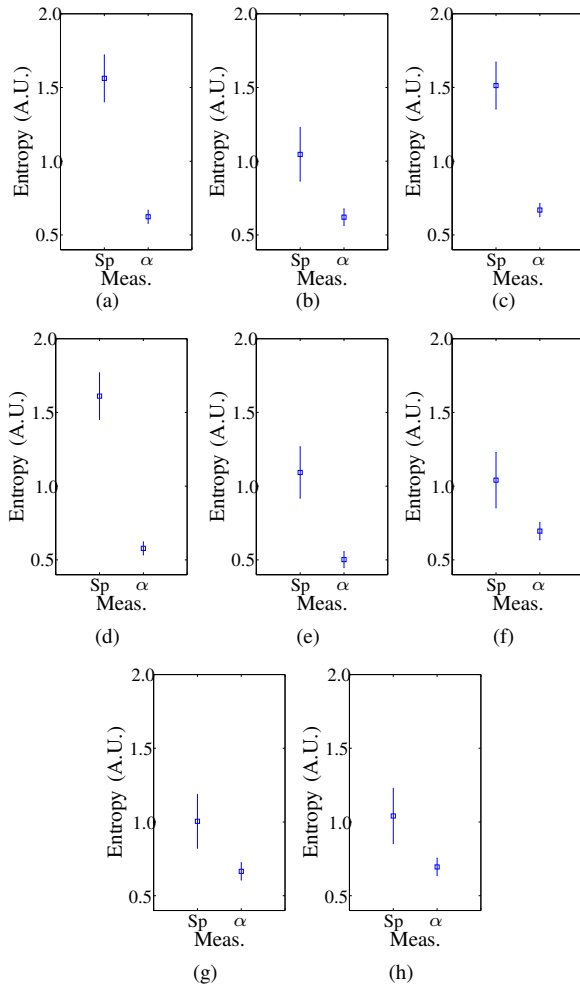


Fig. 5. Variability of the entropy measures (Meas.) considered SampEn (Sp) and DFA (α) for (a) Control EEG, (b) epileptic EEG, (c) control opened eyes EEG, (d) control closed eyes EEG, (e) during epileptic seizures, (f) seizure free EEG on epileptic patients, (g) epileptogenic zone of the hippocampal formation, seizure free EEG and (h) opposite to the hippocampal formation, seizure free EEG.

experiments, could enable the prediction of a seizure taking place.

Finally, for any of the signal groups considered, the variability of the entropy measure in terms of R is lower in DFA, which makes it a more robust measure in general. On the contrary, DFA does not allow segmentation for any case of the study.

V. CONCLUSION

This work presents a characterization and comparison study between SampEn and DFA in terms of signal type segmentation capabilities.

SampEn has demonstrated to perform better on discerning control vs. pathologic (epileptic) EEG records and when analyzing different types of control EEG signals. DFA performs better when analyzing seizure free and seizure signal segments, both coming from epileptic patients, or even between the recording areas.

We can conclude that even though DFA shows a lower variability scaling factor for any R considered, SampEn performs better and provides a better segmentation with a clearer boundary when comparing control vs. epileptic signals and when studying control EEG. For the study of pathologic signals, we encourage the use of DFA, as it allows better discrimination and enables the possibility of temporally anticipating when a seizure might occur (due to a decrease in α).

As current and future work, we are now characterizing additional entropy estimation measures by using the same database and comparing the results with previous ones. A seizure prediction algorithm is being studied by using DFA in a pseudo-real time implementation. Additionally, a classification algorithm based on SampEn is being assessed, for identification of epileptic subjects in an automatic way before seizures occur.

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