

# Using Recurrence Network Approach to Quantify Nonlinear Dynamics of Skin Blood Flow in Response to Loading Pressure

Fuyuan Liao and Yih-Kuen Jan

**Abstract**—This paper presents a recurrence network approach to quantify dynamic complexity of skin blood flow oscillations (BFO) in response to loading pressure. This approach consists of three processes, including 1) phase space reconstruction by means of time delay embedding, 2) construction of a recurrence matrix that represents neighboring states in phase space, and 3) consideration of the recurrence matrix as an adjacency matrix representing links in a network and the use of clustering coefficients to characterize phase space properties. By using the Lorenz system and real data, we demonstrate that the global clustering coefficient is robust to the embedding parameters. We applied this approach to study skin BFO at baseline and during loading pressure, a causative factor of skin breakdown. The results showed that global clustering coefficients of BFO significantly decreased in response to loading ( $p < 0.05$ ). Moreover, surrogate tests indicated that such a decrease was associated with a loss of nonlinearity of BFO. Our results suggest that the recurrence network approach can practically quantify the nonlinear dynamics of BFO.

## I. INTRODUCTION

Pressure ulcers are a common but potentially preventable condition in people with physical disabilities [1]. Although the etiology of pressure ulcers has remained unclear, prolonged tissue ischemia has been cited as the most important mechanism [2]. Tissue ischemia occurs when externally applied pressure causes the occlusion of blood vessels, thus reducing blood flow to local tissues [3]. It has been demonstrated that sustained pressure causes not only a decrease in skin blood flow, but also alterations in blood flow oscillations (BFO) [4, 5]. However, the relationship between the alterations in BFO and tissue ischemia is poorly understood.

Skin BFO have been extensively studied by means of laser Doppler flowmetry (LDF) for the assessment of microcirculatory function. Spectral analysis of LDF signals has revealed five characteristic frequencies, including metabolic, neurogenic, myogenic, respiratory, and cardiac frequencies [6]. Usually the power of BFO within each frequency interval is used as a quantitative measure for characterizing the states of the underlying physiological control mechanisms [6, 7]. Based on wavelet analysis, Li et al. [4] showed that pronged compression attenuated the endothelial related metabolic activities. Jan et al. showed that

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blood flow control mechanisms exhibited different responses to alternating pressure and constant pressure [8]. These studies, however, did not provide insights regarding the dynamics of BFO in response to pressure.

The dynamics of BFO can be characterized using nonlinear measures [9, 10]. Most popular nonlinear measures such as Lyapunov exponents, correlation dimension, and entropies have a common feature that they quantify certain invariant properties of phase space. These measures generally require long and stationary data series. Another important method for quantifying phase space properties is the recurrence plots (RPs) [11]. A RP is a graphical representation of a recurrence matrix representing neighboring states in phase space. This method is even useful for the analysis of short and non-stationary data. However, most measures based RPs were found to be sensitive to the embedding parameters [12].

This paper introduces a new conceptual approach, the recurrence network approach, to quantify the nonlinear dynamics of BFO in response to pressure loading. This approach interprets the recurrence matrix of a time series as the adjacency matrix of a network, thus allowing us to study time series using a complex network approach. The new quantitative characteristics related to the dynamic complexity of a time series are not yet provided by the existing nonlinear methods [13].

The remainder of this paper is organized as follows. Section II describes the RP, then the recurrence network approach. We show that the global clustering coefficient is robust to the embedding parameters. Section III presents the results of the approach for skin blood flow data. Finally, section IV gives a brief discussion and our conclusion.

## II. RECURRENCE NETWORK APPROACH

### A. Recurrence Plots (RPs)

Given a time series  $\{x(i), i = 1, \dots, M\}$ , choosing a time delay  $\tau$  and an embedding dimension  $m$ , one constructs a phase space trajectory

$$x_i = (x(i), x(i + \tau), \dots, x(i + (m - 1)\tau)), \\ i = 1, \dots, N = M - (m - 1)\tau. \quad (1)$$

A RP is a representation of a recurrence matrix

$$R_{ij} = \Theta(\varepsilon - d(x_i - x_j)), \quad (2)$$

where  $\Theta(\cdot)$  is the Heaviside function,  $d$  is the distance between  $x_i$  and  $x_j$ , and  $\varepsilon$  is a predefined threshold. Using a spatial distance as the recurrence criterion, the matrix  $R$  is symmetric with  $R_{ij}=1$  if the state  $x_j$  is a neighbor of  $x_i$  in

phase space, and  $R_{ij}=0$  otherwise. Figure 1 shows the RPs of a sine wave,  $x = \sin(0.1\pi t)$  (a), the  $x$ -coordinate of Lorenz system

$$\begin{aligned} dx/dt &= \sigma(y - x), \\ dy/dt &= x(r - y), \\ dz/dt &= xy - \beta, \end{aligned} \quad (3)$$

with the parameters  $r=28$ ,  $\sigma=10$ ,  $\beta=8/3$ , and sampling time  $\Delta t=0.03$  (b), uniformly distributed white noise (c), and a blood flow signal (d). The embedding parameters were separately selected for each time series. Time delay  $\tau$  was selected as the first minimum of the auto mutual information function, and the optimal value of  $m$  was determined using the Cao's method [14]. The recurrence threshold  $\varepsilon$  was selected such that the global recurrence rate

$$RR = \frac{1}{N^2} \sum_{i,j=1}^N R_{i,j} \quad (4)$$

is fixed at the same value [15].

The structural features of a RP can be quantified in terms of diagonal and vertical lines. The lengths of diagonal lines are related to the predictability of the data series, and the presence of vertical lines indicates slowly changing states. Recurrence quantification measures based on the distribution of lengths of diagonal and vertical lines have been used to characterize different aspects of dynamic complexity of the time series. However, these measures were found to be sensitive to time delay  $\tau$  and embedding dimension  $m$  [12].

### B. Recurrence Network Approach

A new conceptual approach for analyzing structural features of time series is to consider the phase space vectors as nodes of an undirected and unweighted network and quantify the topological features of the network [13]. For this purpose, we define an adjacency matrix

$$A_{ij} = R_{ij} - \delta_{ij}, \quad (5)$$

where  $\delta_{ij}$  is the Kronecker delta for excluding the line of identity from the RP. To illustrate the potential of the

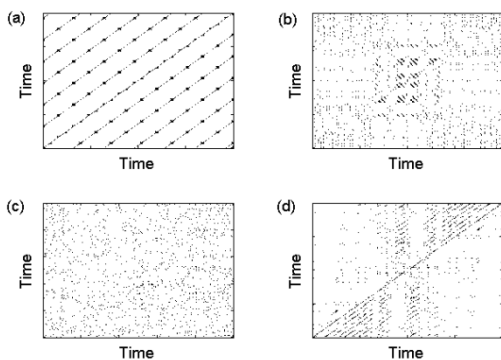


Figure 1. Recurrence plots of (a) a sine wave,  $x = \sin(0.1\pi t)$ , (b) the  $x$ -coordinate of the Lorenz system (3), (c) uniformly distributed white noise, and (d) a skin blood flow signal. For all the series, the threshold  $\varepsilon$  was selected preserving  $RR=0.03$ .

recurrence network approach for quantifying dynamic properties of BFO, we consider only the clustering coefficient of vertex  $x_i$  [16]

$$C_i = \sum_{k>j} A_{ij} A_{ik} A_{jk} / \sum_{k>j} A_{ij} A_{ik} \quad (6)$$

Note that if  $x_i$  has no or only one neighbor, i.e.  $\sum_{k>j} A_{ij} A_{ik} = 0$ ,  $C_i$  is defined as  $C_i=0$  [13].  $C_i$  gives the probability that two neighbors (recurrences) of the state  $x_i$  are also neighbors. Donner et al. [13] suggested that  $C_i$  characterizes higher order properties related to the heterogeneity of the phase space density in the vicinity of the vertex  $x_i$ . It was found that high values of  $C_i$  often coincide with dynamically invariant objects such as invariant manifolds [15]. The global clustering coefficient is defined as

$$C = \sum_i C_i / N. \quad (7)$$

To investigate the influences of time delay  $\tau$  and embedding dimension  $m$  on global clustering coefficient  $C$ , we calculated  $C$  for the  $x$ -coordinate of the Lorenz system (3) and the blood flow signal (same as that used in Fig. 1d) for wide ranges of  $\tau$  and  $m$  around their optimal values. The threshold  $\varepsilon$  was selected such that  $RR=0.03$ . For the Lorenz system (3), the optimal values of  $\tau$  and  $m$  are 5 and 3, respectively; for the blood flow signal, the optimal values of  $\tau$  and  $m$  are 30 and 7, respectively. Thus, for the Lorenz system (3) we calculated  $C$  for  $m=1\sim 10$  while  $\tau=5$  and  $\tau=1\sim 10$  while  $m=3$ ; for the blood flow signal, we calculated  $C$  for  $m=1\sim 15$  while  $\tau=7$  and  $\tau=10\sim 50$  while  $m=3$ . The results showed that global clustering coefficient  $C$  is robust to the embedding parameters (Fig. 2).

## III. APPLICATION TO SKIN BLOOD FLOW DATA

### A. Experimental Protocols

We recruited 10 healthy, young subjects (4 male and 6 female) for this study. The demographic data were: age  $23.7\pm 2.4$  (mean  $\pm$  standard deviation) years, height  $1.69\pm 0.09$  m, and weight  $59.8\pm 6.3$  kg. The exclusion

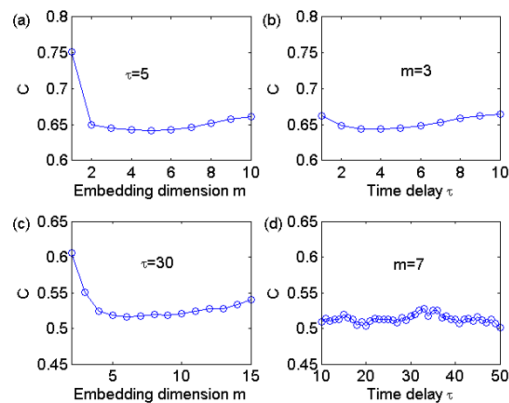


Figure 2. Influence of time delay  $\tau$  and embedding dimension  $m$  on the global clustering coefficient  $C$  for (a)(b) the Lorenz system (3) and (c)(d) the blood flow signal (same as that used in Fig. 1d). For the Lorenz system, the optimal values of  $\tau$  and  $m$  are 5 and 3, respectively; for the blood flow signal, the optimal values of  $\tau$  and  $m$  are 30 and 7, respectively. The threshold  $\varepsilon$  was selected such that  $RR=0.03$ .

criteria included the presence of pressure ulcers on the sacrum, diabetes, vascular diseases, hypertension, or use of vasoactive medicines. This study was approved by the University Institutional Review Board. Informed consent was obtained from each subject before any testing. After at least a 30 min quiet rest period to become acclimated to the room temperature ( $24\pm 2^\circ\text{C}$ ), the subject was positioned in a prone posture. Sacral skin blood flow was recorded using a laser Doppler flowmetry (PF 5001, Perimed AB, Sweden) at a sampling frequency of 32 Hz. An indenter was used to apply 60 mmHg loading to the sacral skin through the probe. The protocol included a 10 min baseline, a 20 min loading and a 10 min recovery period. Figure 3 shows skin blood flow response to applied pressure.

### B. Data Analysis

We calculated global clustering coefficient  $C$  for the blood flow data at baseline and during loading. First, the embedding parameters, time delay  $\tau$  and embedding dimension  $m$ , were separately selected for each data set. Then, to avoid the possible influence of non-stationary, the data series was subdivided into non-overlapping epochs consisting of  $4000+(m-1)\tau$  samples, i.e. 4000 phase space vectors. Given the sampling rate of 32 Hz and assuming that the lower frequency bound of the characteristic frequency associated with endothelial-related metabolic activity is 0.0095 Hz [6], such an epoch contains at least one cycle of the metabolic frequency. This means that such an epoch preserves all power of the five characteristic components embedded in skin BFO. Next, for each epoch, the parameter  $\varepsilon$  was selected such that the global recurrence rate  $RR=0.03$ . The final result was the averaged global clustering coefficient over all the epochs. W

To investigate whether pressure loading causes a loss of nonlinearity of BFO, we performed the following tests using surrogate time series. For each original data series, 30 surrogate series were generated using the iterative amplitude adjusted Fourier transform approach [17]. This approach eliminates nonlinearity, preserving the linear features of the original time series, e.g. power spectrum and signal distribution [17]. Then we calculated global clustering coefficient  $C$  for the surrogate series for a range of  $RR=0.01\sim 0.08$ .

### C. Statistical analysis

We compared  $C$  between baseline and during loading using the Wilcoxon signed-rank tests. One sample t-tests were used to compare  $C$  of an original data series and those of surrogate series. The significance level was set at  $p<0.05$ .

### D. Results

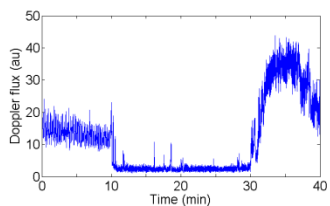


Figure 3. Skin blood flow response to pressure loading.

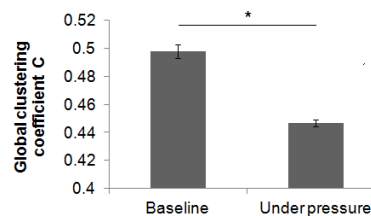


Figure 4. Comparison of global clustering coefficient  $C$  of BFO between baseline and loading period. Values are means  $\pm$  standard errors.  $*p<0.05$ .

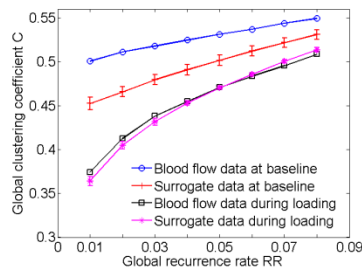


Figure 5. Results of surrogate tests in a typical subject. For each original data set, 30 surrogate series were generated. The centers of the error bars represent the mean values of  $C$  for 30 surrogate series, and the lengths of the error bars represent the standard deviations.

The results showed that global clustering coefficient  $C$  of BFO significantly decreased in response to loading ( $p<0.05$ ) (Fig. 4). Surrogate tests indicated that  $C$  of BFO at baseline was significantly larger than that of surrogate data ( $p<10^{-4}$ ), whereas  $C$  of BFO during loading was almost equal to that of surrogate data ( $p>0.05$ ).

## IV. DISCUSSIONS AND CONCLUSIONS

We have demonstrated that skin BFO exhibited altered structural properties when tissues were subjected to external loading and that the alterations were associated with a loss of nonlinearity of BFO. To our knowledge, this is the first study investigating the effect of external loading on nonlinear properties of skin BFO.

Skin blood flow is controlled by a number of physiological mechanisms, e.g., metabolic, neurogenic, and myogenic mechanisms [18]. Each mechanism manifests as an almost periodic process, which corresponds to a specific frequency range [6]. Thus, spectral analysis of skin BFO has been widely used to assess the mechanisms' activities [6]. In the literature, much attention has been paid to post-occlusive hyperemia and thermal stress-induced responses. Only a few studies focused on skin BFO during external loading. The major findings of these studies were: (i) in healthy young adults, incremental pressure resulted in an increase in power of the myogenic component and a decrease in power of the metabolic component [19]; (ii) in anesthetized rats, prolonged pressure resulted in a decrease in the contribution of the metabolic activity to skin BFO [4]. Our results confirmed that surface pressure causes changes in BFO but in terms of nonlinear properties.

In fact, spectral analysis provides information only on magnitude properties of a signal but not on structural

properties. The later could be studied using nonlinear methods [10]. In this study, a measure derived from the theory of complex networks, clustering coefficient, was used to capture the structural features of BFO. This measure characterizes higher-order properties related to the heterogeneity of phase space density [13]. However, recurrence network approach does not take the temporal ordering of data points into account, thus leading to a loss of temporal information of real-world data [15]. Additionally, the relationship between this measure and other nonlinear characteristics such as Lyapunov exponent remains unknown, which may be a topic for future work.

In summary, recurrence network approach provides new characteristics such as clustering coefficient that are not yet provided by the existing methods for time series analysis. The simulation results showed that the global clustering coefficient is robust to the embedding parameters. Using this measure, we found that skin BFO showed an altered dynamic property in response to pressure loading, which was associated with a loss of nonlinearity of BFO. Our results suggested that recurrence network approach is a promising tool for quantifying the nonlinear dynamics of BFO.

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