

ECG Signal Compression using Compressive Sensing and Wavelet Transform

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Abstract— Compressed Sensing (CS) is a novel approach of reconstructing a sparse signal much below the significant Nyquist rate of sampling. Due to the fact that ECG signals can be well approximated by the few linear combinations of wavelet basis, this work introduces a comparison of the reconstructed 10 ECG signals based on different wavelet families, by evaluating the performance measures as *MSE* (Mean Square Error), *PSNR* (Peak Signal To Noise Ratio), *PRD* (Percentage Root Mean Square Difference) and *CoC* (Correlation Coefficient). Reconstruction of the ECG signal is a linear optimization process which considers the sparsity in the wavelet domain. L1 minimization is used as the recovery algorithm. The reconstruction results are comprehensively analyzed for three compression ratios, i.e. 2:1, 4:1, and 6:1. The results indicate that reverse biorthogonal wavelet family can give better results for all CRs compared to other families.

Index terms: Compressive Sensing; Wavelet transform; Sparsity; Incoherence; L1 minimization

I. INTRODUCTION

Efficient compression of the ECG signal is very important for: 1) storing the large amount of data, particularly the ambulatory ECG data, and 2) transmission over the digital telecommunication network and telephone line [1]. For storing and transmitting of these data, better compression is the obligatory step which can reduce the computational cost.

CS theory [5] is a useful tool for eliminating the inefficiencies caused by traditional signal processing algorithms, because 1) it offers simpler hardware implementation for encoder, as it transforms its computational burden from encoder to decoder, 2) no need to encode the location of the largest coefficients in the wavelet domain, 3) its ability to reconstruct the signal from significantly fewer data samples compared to conventional Nyquist sampling theory. It is a novel technique which

suggests random acquisition of the non adaptive linear projection at lower than the Nyquist rate, which preserves the signal structure. By using an Optimization problem the signal is reconstructed [4].

The ECG signal can be represented in different sparsity levels in different respective wavelet families. Wavelet transform is the best way to represent the signal as it can decompose the signal into number of sub band signals which consists of different spatial resolution, frequency and directional characteristics. CS has been used recently for rapid magnetic resonance imaging [6] and for electroencephalogram signals [7]. In recent work, CS is implemented on three cardiac signals named as ballistocardiogram (BCG), electrocardiogram and Photoplethysmogram (PPG), and reconstructed by TwIST algorithm [8]. Many authors have applied discrete wavelet transform (DWT) in compressed sensing [11]-[14]. But the question is which wavelet is the best one to create sparsity?

In this work, a comparative performance evaluation of on the different wavelet families has been made for CS based compression of the ECG signals. And consequently an attempt is made to select the better wavelet transform for three compression ratios (CRs) i.e. 2:1, 4:1 and 6:1. Performance is analyzed by comparing *MSE*, *PSNR*, *PRD* and *CoC* values. The performance of the signal in terms of quality of reconstruction is evaluated using the MIT-BIH arrhythmia [9]. L1 minimization is used for reconstruction purpose as it is less combinatorial in nature. The organization of the paper is as follows. In the following Section, basics of Compressive Sensing is discussed, in Section III, background of Wavelet transform is discussed, Section IV consists of methodology used in this paper, Section V contains results and in Section VI the paper is concluded.

II. BASICS OF COMPRESSIVE SENSING

Consider a signal $x \in R^N$ and basis of R^N is the matrix $\Psi := [\psi_1, \psi_2, \psi_3, \dots, \psi_N]$. Here, the transform is

$$x = \Psi \theta \quad (1)$$

Where θ signifies transform coefficients of length $N * 1$. If K coefficients of θ are significant than only it is said that x is K sparse.

If we say, $\theta = \theta_k + \theta_e$ where, K significant components are signified by θ_k , and smallest coefficients $N-K$ are denoted by θ_e which are set to zero. We have

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$$x = \Psi \theta = \theta = \Psi \theta_k + \Psi \theta_e = \Psi \theta_k + n_e \quad (2)$$

With appropriate variance each element of n_e can be modeled by a zero mean Gaussian. To measure θ an $M \times N$ matrix Φ is used. The measurement matrix Φ and basis function Ψ should not be correlated with each other in order to reconstruct signal. Φ should satisfy Restricted Isometry Property (RIP) [1]. Iid Gaussian entries are contained in Φ matrix so that Ψ and Φ are incoherent with each other [11]. Computation of the measurements are done by

$$y = \Phi x \quad (3)$$

Where $y \in R^M$ in which $M \ll N$. CS theory says that their exist $c > 1$, an over measuring factor so that to reconstruct x with high probability [1,5] only $M: = c K$ incoherent measurements y are required. Here projections on θ are measured rather than measuring θ directly.

Aim is to infer θ from y . The major problem is that $y \in R^M$ and $\theta \in R^N$ where $M \ll N$ which creates infinite many solutions due to underdetermined equations.

To eradicate the complexity caused by l_0 , l_1 regularized optimization problem is proposed by Donoho et al. [2].

$$\theta = \arg \min \|\theta\|_1 \quad \text{such that} \quad \Phi \Psi \theta = y \quad (4)$$

If $c \geq O(\log(N/K))$ with the help of l_1 problem θ reconstruction can easily be done.

III. WAVELET TRANSFORM

On 1D finite signals Compressive sensing can be applied. Here, a real valued, finite length, one dimensional, discrete time signal has been considered of ECG. The function x which is being expanded is discrete, in which resulting coefficients are obtained by Discrete Wavelet Transform (DWT) by the use of wavelet Basis function Ψ . Signal is recovered in time domain by taking into consideration the sparsity level for different transforms by minimizing the dot product of estimated signal and wavelet coefficients.

Two different wavelet basis are used by Biorthogonal wavelets $\Psi(x)$ and $\Psi(x)$. One is for analysis (decomposition) and other is for synthesis (reconstruction). That is:

$$\langle \Psi_{j,k}, \Psi_{l,m} \rangle = \delta_{j,l} \delta_{k,m} \quad (5)$$

Then we have

$$c_{j,k} = \langle f(x), \Psi_{j,k}(x) \rangle, d_{j,k} = \langle f(x), \Psi_{j,k}(x) \rangle \quad (6)$$

For the decomposition and

$$f(x) = \sum_{j,k} c_{j,k} \Psi_{j,k}(x) = \sum_{j,k} d_{j,k} \Psi_{j,k}(x) \quad (7)$$

For the reconstruction. In frequency domain the two Scaling functions are given by

$$\Phi(2s) = \prod_{n=0}^{\infty} H_0(s/2^n) \quad \Phi(2s) = \prod_{n=0}^{\infty} H_0(s/2^n) \quad (8)$$

and the wavelets are

$$\Phi(2s) = \prod_{n=0}^{\infty} H_0(s/2^n) \quad \Psi(x) = \sqrt{2} \sum_n h_1(n+1) \phi(2x-n) \quad (9)$$

IV. DESCRIPTION OF THE ALGORITHM

The methodology followed is as shown in Fig. 1 which starts with the analysis of the signal, searching for the operation and finally achieve for the sparse representation. CVX [3] reconstruction tool is used for the reconstruction process. It is done in the signal domain by minimizing the dot product of estimated signal and wavelet coefficients. ECG signals 1-10 represents to records 100, 101, 102, 103, 104, 105, 106, 107, 118, 119 ECG signals taken from MIT-BIH [9] arrhythmia with 1024 point length.

The *MSE* for measuring the performance is stated as:

$$MSE = (\sum (f - fp)^2) / N \quad (10)$$

Where, f is original signal, fp is reconstructed signal, N is total signal length.

The *PSNR* calculated for the comparison of ECG signal is:

$$PSNR = 10 \log_{10}(M^2 / MSE) \quad (11)$$

Where, M is maximum value from the original ECG signal.

The Compression ratio used is:

$$CR = \text{length}(f) / \text{length}(fp) \quad (12)$$

PRD is Percentage root mean square difference which is computed as follows

$$PRD = \sqrt{MSE / (\sum f^2)} \times 100 \quad (13)$$

For highest value of the *PSNR*, the *PRD* value should be minimum. Eventually the CoC value for that ECG signal should tends to 1 more sharply. *CoC* is computed as

$$CoC = \frac{(\sum (f \times fp) \times N) - (\sum f - \sum fp)}{\sqrt{((N \times \sum f^2) - \sum f^2) \times ((N \times \sum fp^2) - \sum fp^2)}} \quad (14)$$

V. RESULTS AND DISCUSSION

Wavelets namely Coiflets, Daubichies, Symlets, Biorthogonal, Reverse biorthogonal etc. have been studied and used to compress the cardiac signals. As shown in Table 1, the most preferable wavelet transforms for efficient reconstruction of the signal based on 10 different ECG signals has been evaluated for three different CRs 2:1, 4:1, 6:1. *MSE*, *PSNR*, *PRD*, CoC value of all signals is computed and by comparing the performance values dominant wavelet basis is decided.

For compression ratio 2:1 for most of the ECG signals rbio 3.9 is dominant. It creates the highest sparsity which is being utilized at the time of reconstruction for finding the sparsest signal from infinite many solutions of original signal. By comparing the performance parameters for 10 ECG signals we observed *PSNR* values 58.88db, 56.51db, 58.89db, 57.52db for ECG6, ECG7, ECG8 and ECG10 signals, respectively at reverse biorthogonal basis (rbio3.9), is

maximum compared to other basis. Mean square error for four signals out of 10 signals is minimum for rbio3.9 wavelet family. *MSE* values for these signals are 1.43, 2.12, 1.58 and 1.65 respectively. Noise is very less in the reconstructed signal. Similarly, as shown in Table 2, there correlation coefficient (CoC) values are 0.9998, 0.9996, 1 and 0.9997. As most of the values are close to 1 it gives an idea of strong correlation between original and reconstructed signal. *PRD* values for these signals are .01% which shows the lowest rate distortion among others. Next, preferable basis can be rbio3.7 as for ECG1 and ECG5 mean square error is minimum.

For compression ratio 4:1, *PSNR* values 44.24db, 45.00db, 43.60db and 45.29db of ECG1, ECG2, ECG3 and ECG5 signals respectively are maximum for rbio3.7 which proves this wavelet suitable for 4:1 CR. During L1 minimization lowest error is indicated by rbio3.7 basis which creates more number of zeros at the reconstruction step. Consequently, *MSE* values for these signals are 7.26, 6.88, 7.84, and 6.49. Similarly, *CoC* values as shown in Table 3, for these signals are 0.9875, 0.9911, 0.9934, and 0.9946. These values close to 1 proves this wavelet as the most favorable basis. Second favorable one for 4:1 CR is rbio3.9 as out of 10 ECG signals three ECG signals are sparse under this wavelet family. *PRD* values are .03% for ECG1, ECG2 and ECG5, whereas it is .04% for ECG5 signal.

For 6:1 CR, again rbio3.9 is the most occurring wavelet family which gives the highest sparsest representation of the signal and reconstruct with much less error among others. It leads the role against its counterparts. Mean square errors (*MSE*), for ECG1, ECG5 and ECG8 signals are 18.74, 21.85, and 43.29 respectively. For, three signals the reconstruction error is the lowest for rbio3.9. *PSNR* values for these signals are 36.00db, 34.74db and 30.14db respectively. *CoC* values as shown in Table 4, for these signals are 0.91018, 0.936612, and 0.969895. *PRD* values for these signals are 0.09%, 0.10% and 0.19% respectively. The two wavelets which can be considered as the second most dominant one are the rbio3.7 and rbio3.3 as there reconstruction error is lower after rbio3.9 basis. Original and reconstructed signals for 2:1, 4:1 and 6:1 are shown in Fig 2.

VI. CONCLUSION

In this paper, we test the quality of reconstructed ECG signals by using 29 wavelets for three different compression ratios. With the help of performance measures such as *MSE*, *PSNR*, *PRD* and *CoC* we found that for 2:1 compression ratio rbio3.9 is the best basis as it creates more sparsity for most of the ECG signals. As we increase the CR to 4:1 we observed again rbio3.7 is the more efficient one as its *PSNR* values are the highest among its counterparts. For 6:1 CR rbio3.9 is the again best choice and consequently rbio3.7 and rbio3.3 is the second better choice. For all the three CR's bior3.1 is the worst among different wavelet families. In future, we intend to explore the use of other transforms like Curvelet and Ridgelet transform in order to generate sparsity in ECG signal.

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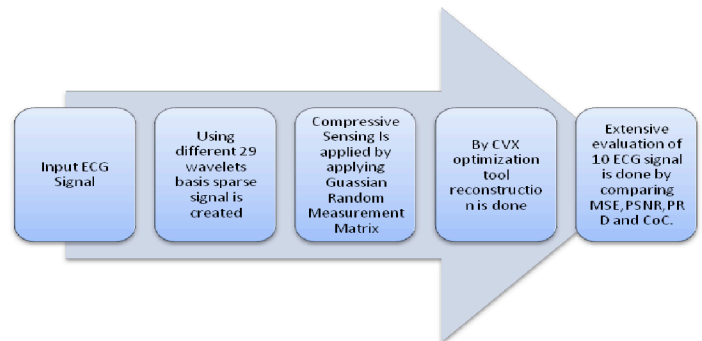
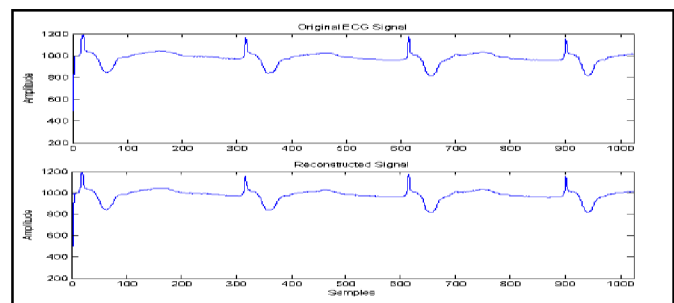


Fig. 1: Methodology for evaluating ECG signal compression by CS and WT.



(a)

