

# Optimization of Time-Variant Autoregressive Models for tracking REM - non REM transitions during sleep

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**Abstract**— The aim of this study was the optimization of Time-Variant Autoregressive Models (TVAM) for tracking REM - non REM transitions during sleep, through the analysis of spectral indexes extracted from tachograms. A first improvement of TVAM was achieved by choosing the best typology of forgetting factor in the analysis of a tachogram obtained during a sitting-to-standing test; then, a method for improving robustness of AR recursive identification with respect to outliers was selected by analyzing a tachogram with an ectopic beat. A variable forgetting factor according to the Fortescue method and a specific condition on the prediction error for recursive AR identification gave the best performances. The optimized TVAM was then employed in the analysis of tachograms derived from ECGs recorded during a whole night, through a sensorized T-shirt, from 9 healthy subjects. The spectral indexes (power of tachogram in the LF and HF bands, LF/HF ratio and the absolute value of the spectrum pole in the HF band) were computed from the estimated AR parameters on a beat-to-beat basis. A two groups T-test aimed at comparing values assumed by each spectral index in REM and non-REM sleep epochs was performed. Significant statistical differences (p-value < 0.05) were found in three of the four spectral indexes computed. In conclusion, the combination of the Fortescue variant and of the robustness method based on the prediction error in the TVAM seems to be helpful in the differentiation between REM and non-REM sleep stages.

## I. INTRODUCTION

The evaluation of the quality and quantity of sleep is extremely important, given the role that sleep plays on an individual's everyday life, influencing memorization, learning and concentration processes [1]. The traditional approach to sleep analysis is represented by the polysomnography (PSG). Signals recorded during a whole night, normally including several EEG derivations, ECG, EOG, EMG and respiratory signal, are divided into consecutive 30-s-long epochs classified as Wake, REM or NREM by sleep experts. At the end of this analysis the hypnogram, which represents the sleep macrostructure, is defined [2].

During the last years, the need to simplify the practice of sleep evaluation has led to the development of wearable technologies, to be used in a reliable way also in a domestic environment, and to the search for signals that are easier to

acquire than the EEG, but equally informative about sleep physiology [3]. For this purpose, the ECG has proved to provide valuable information, because of the fluctuations in heart rate produced by the ANS, which is clearly involved in sleep patterns. ECG is then used to derive the tachogram, that represents the series of time intervals between consecutive R peaks (called RR intervals), whose analysis in the frequency domain provides useful parameters for sleep evaluation [4].

Because of the interest in investigating changes on sleep stages, the spectral analysis of the tachogram can be carried out by using a Time-Variant Autoregressive Model (TVAM). Time-variant identification methods, for a given order, update the AR coefficients each time a new sample is available, on the basis of the previous coefficients and of a forgetting factor  $w$ . Different typologies of forgetting factor can be used in order to achieve the best compromise between stability in stationary conditions and speed of adaptation in correspondence of quick variations in the signal dynamics. However, the presence of artifacts in the signal may strongly affect the estimation of the model parameters [5]. For this reason, different methods of robustness for AR recursive identification, based on functions that weigh the prediction error, have been proposed in order to reduce the effect of the outliers [6].

The aim of this study was the optimization of Time-Variant Autoregressive Models (TVAM) for tracking REM - non REM transitions during sleep, through the analysis of spectral indexes extracted from tachograms.

## II. MATERIALS AND METHODS

### A. Clinical protocol

A sitting-to-standing tachogram was obtained on a healthy female volunteer who wore a sensorized T-shirt (SMARTTEX, [7]) during a sitting-to-standing test, while a tracing containing an ectopic beat was obtained as a short segment of the tachogram of a healthy female volunteer who wore the T-shirt during a whole night. The improvement and optimization of TVAM were achieved by using these two signals.

The optimized TVAM was then employed in the analysis of signals recorded from 9 healthy female volunteers, of age between 18 and 45. The recordings were acquired at the FORENAP R&D S.A.S.U. center, in Rouffach (France). From each subject, during a whole night, a continuous recording of ECG, respiratory signal and body accelerations was obtained through the T-shirt. Contemporarily, each subject underwent a standard polysomnographic analysis, with acquisition of EEG, EMG, EOG, ECG, respiratory signal and actigraphy at the wrist and ankle. The scoring of these signals was performed by expert clinicians with

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definition of the hypnogram. The TVAM order was set to 9 for all the signals analyzed in this study.

### B. Data pre-processing

In the present study, among all the recorded signals, only the ECG was employed. The hypnogram was used as reference for the sleep stages. An algorithm for the detection of the QRS complex was applied to each ECG signal, in order to obtain the tachogram. The identification and removal of outliers in each tachogram were performed by applying the method described by Kemper et al. [8].

### C. Time-variant identification methods with different typologies of forgetting factor

The comparison between time-variant identification methods employing different typologies of forgetting factor was performed by analyzing the sitting-to-standing tachogram. The first method used a constant forgetting factor  $w$  (classical Recursive Least Squares form, RLS), whose value was set to 0.98 in accordance with preceding examinations [9]. The equations used in the recursive identification of AR models are listed below:

$$\begin{cases} \mathbf{a}(t) = \mathbf{a}(t-1) + \mathbf{K}(t)e(t) \\ \mathbf{K}(t) = \frac{\mathbf{P}(t-1)\varphi(t)}{w + \varphi(t)^T \mathbf{P}(t-1)\varphi(t)} \\ e(t) = y(t) - \varphi(t)^T \mathbf{a}(t-1) \\ \mathbf{P}(t) = \frac{1}{w} \left[ \mathbf{P}(t-1) - \frac{\mathbf{P}(t-1)\varphi(t)\varphi(t)^T \mathbf{P}(t-1)}{w + \varphi(t)^T \mathbf{P}(t-1)\varphi(t)} \right] \end{cases} \quad (1)$$

Here,  $\mathbf{a}(t)$  is the vector of model coefficients and  $\varphi(t)$  represents the observation vector of the signal  $y$ .  $\mathbf{K}(t)$  and  $\mathbf{P}(t)$  are the time-variant gain and the covariance matrix of data respectively, while  $e(t)$  represents the prediction error of the model and  $w$  is the constant forgetting factor.

In the second method a variable forgetting factor, according to the Fortescue variant [10], was used. The forgetting factor  $w(t)$  is updated, each time a new sample is available, by using the following expression:

$$w(t) = \frac{1 - \varphi(t)^T \mathbf{K}(t) e(t)^2}{N \sigma_0} \quad (2)$$

The parameters  $\varphi(t)$ ,  $\mathbf{K}(t)$  and  $e(t)$  have already been defined. The term  $N$ , which represents the length of the running window of data, was set to 50, corresponding to a mean forgetting factor of 0.98. The parameter  $\sigma_0$  was set to the mean value of the variance of the prediction error obtained with the method implemented in the classical RLS form. The variable forgetting factor  $w(t)$  was confined to the interval 0.97 - 0.99 because of the signal dynamics, that does not require the adoption of values smaller than 0.97. AR coefficients, computed on a beat-to-beat basis, were then used to obtain an automatic spectral decomposition of the signal, based on a residual integration algorithm.

Fig. 1 shows the tachogram and the spectral indexes obtained with time-variant identification methods employing both the constant and the variable forgetting factor. The power in the LF band (in the range 0.04 - 0.15 Hz) and in the HF band (in the range 0.15 - 0.4 Hz) and the LF/HF ratio are represented in this figure.

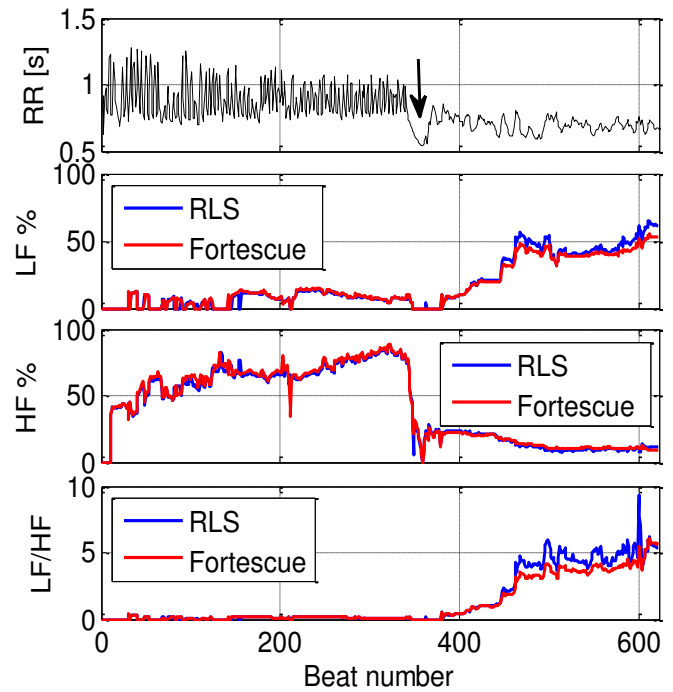


Figure 1: sitting-to-standing tachogram and spectral indexes. Power in the LF and HF bands (expressed in percentage units with respect to the total power) and the LF/HF ratio are obtained with time-variant identification methods using constant (RLS) and variable (Fortescue) forgetting factor. An arrow on the tachogram marks the transition between sitting and standing position.

As expected, the transition to the standing position is characterized by an increase in the LF component and in the LF/HF ratio. On the other hand, the HF component shows a decrease in correspondence of the transition. Even if in the Fortescue variant the power in the LF band seems to be underestimated during the standing position, this method is more stable in stationary conditions and reacts faster to abrupt changes in the signal dynamics, allowing for a better tracking of the signal dynamics.

### D. Methods of robustness for AR recursive identification

The comparison between different methods of robustness for AR recursive identification, aimed at reducing the effect of outliers, was performed by using the tachogram segment containing an ectopic beat. In the first method a specific condition on the prediction error was applied. The model parameters were updated only if the absolute value of the current prediction error  $e(t)$  was lower than a term including the absolute value of the previous error  $e(t-1)$  and its standard deviation  $\sigma_e(t-1)$  according to the following expression:

$$|e(t)| < F \sigma_e(t-1) + |e(t-1)| \quad (3)$$

This condition acts more or less restrictively on the basis of the value assigned to the parameter  $F$ . If a very small value is used, the robustness to outliers increases, but the method results too conservative. In this study, the best compromise was reached by setting  $F$  to the value 4. In the second method, among different weighting functions proposed in literature, the Hard Rejection function  $\rho(e)$  was adopted because it was considered more suitable to the analysis of the tachogram [6].

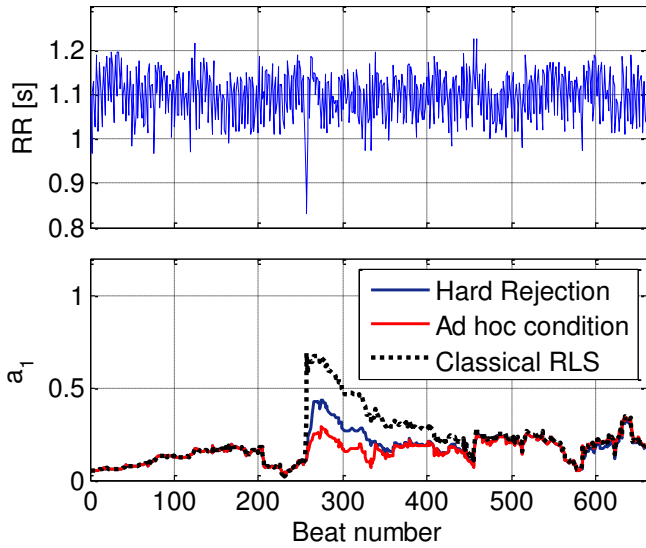


Figure 2: tachogram segment containing the ectopic beat and the model coefficient  $a_1$  computed by using the specific condition on the prediction error (red line), the Hard Rejection function (blue line) and the classical RLS algorithm (black dotted line).

According to the following expression, limited weights are assigned to large prediction errors:

$$\rho(e) = \begin{cases} \frac{1}{2} e^2, & |e| \leq \Delta\sigma \\ \frac{1}{2} (\Delta\sigma)^2, & |e| > \Delta\sigma \end{cases} \quad (4)$$

$\Delta$  represents a user-chosen positive constant for balancing the efficiency and robustness of the algorithm and it is usually set to 3, while  $e$  is the prediction error of the time-variant model. The parameter  $\sigma$  is the standard deviation of the prediction error and is strongly affected by outliers, if computed with the classical recursive expression. To face this problem,  $\sigma$  can be estimated by using a median filter whose order is chosen on the basis of the length  $N$  of the running window of data ( $N = 50$  in this study). The Hard Rejection Function  $\rho(e)$ , as well as its first and second derivatives, are then used in the expressions for the computation of AR coefficients as described by [11].

Fig. 2 shows the tachogram segment containing the ectopic beat and the trend of the model coefficient  $a_1$  computed by using the two different methods of robustness, in comparison with values obtained with the classical RLS algorithm. Both methods of robustness for AR recursive identification result less sensitive to the presence of artifacts than the classical RLS algorithm. In addition, the specific condition on the prediction error allows to achieve a better reduction of the effect of the outliers.

### III. RESULTS

According to the previous section, the optimization of TVAM was achieved by the employment of a variable forgetting factor (Fortescue method) and through the adoption of a specific condition on the prediction error as a method of robustness for AR recursive identification. The former allows a better tracking of the signal dynamics, while the latter allows the achievement of a better reduction of the effect of the outliers.

The optimized TVAM, whose order was set to 9, was then employed in the analysis of tachograms derived from ECGs recorded during a whole night through the sensorized T-shirt from 9 healthy subjects. The spectral indexes were computed from the estimated AR parameters, on a beat-to-beat basis, by using a residual integration algorithm.

Fig. 3 shows a tachogram segment, relative to 1 subject among the 9 considered in this study, corresponding to a transition between REM and NREM sleep stages. Power in the LF and HF bands (expressed in percentage units with respect to the total power), LF/HF ratio and the absolute value of the spectrum pole in HF band [4] are also represented in the same figure. The power in the HF band shows a clear decrease in correspondence of the transition between NREM and REM episodes, while the LF component does not show a clear variation in the values assumed in the two sleep stages. The LF/HF ratio, which combines powers in both the relevant frequency ranges, remarks a clear difference between REM and NREM phases. In addition, the absolute value of the spectrum pole in the HF band characterizes the transition between NREM and REM stages very clearly.

A statistical analysis aimed at corroborating the conclusions drawn from the qualitative observation of Fig. 3 was performed. The mean value of each spectral index on consecutive 30-s-long windows was computed in order to have the same time scale of the hypnogram.

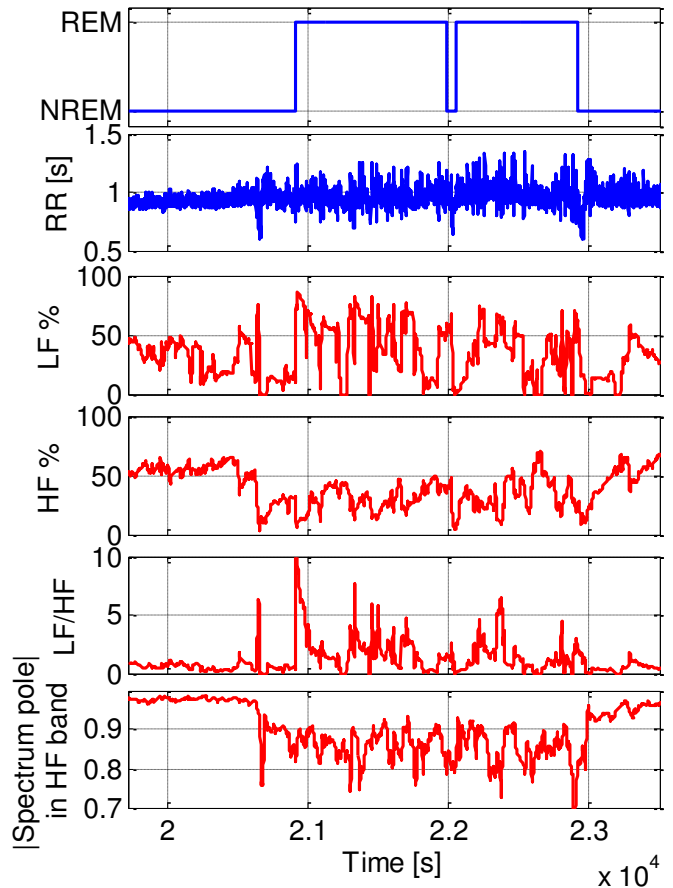


Figure 3: hypnogram and tachogram segments selected during a transition between NREM and REM sleep stages, power in the LF band expressed in percentage units, power in the HF band expressed in percentage units, LF/HF ratio and the absolute value of the spectrum pole in the HF band.

TABLE I:  
MEAN AND STANDARD DEVIATION OF SPECTRAL INDEXES  
COMPUTED DURING REM AND NREM SLEEP.  
\*REPRESENTS SIGNIFICANT DIFFERENCES (p-value < 0.05)

	NREM ( $\mu \pm \sigma$ )	REM ( $\mu \pm \sigma$ )
LF %	31.2 ± 9.9	32.5 ± 12.1
HF %	37.9 ± 12.4*	27.1 ± 9.5*
LF/HF	1.2 ± 1.1*	1.7 ± 1.1*
Spectrum pole  in HF band	0.91 ± 0.04*	0.84 ± 0.03*

Whole night data coming from 9 healthy subjects were analyzed. A total of 5763 and 1581 epochs of NREM and REM respectively were included in the analysis. A two groups T-test aimed at evaluating significant statistical differences (p-value < 0.05) between REM and NREM sleep stages in each spectral index was performed. The significant statistical differences, as reported in Table I, were found in HF%, LF/HF and in the absolute value of the spectrum pole in the HF band. These results confirm the conclusions drawn by the qualitative observation of Fig.3.

#### IV. DISCUSSION AND CONCLUSION

In this study, a TVAM was optimized in order to achieve a better discrimination between REM and NREM sleep stages, through the analysis of spectral indexes extracted from tachograms obtained on 9 healthy subjects during a whole night. In previous works the update of the TVAM coefficients was performed by employing the classical RLS algorithm with a constant forgetting factor  $w$  [4, 12]. However, a constant value assigned to this parameter may not always be suitable to the analysis of a signal characterized by transient phenomena. In fact, lower values of  $w$  should be used to follow the fastest dynamics of the signal (e.g. a transition between two different sleep stages), while higher values may seem to be more adequate to the analysis of stationary portions of the signal (e.g. NREM sleep stages). For these reasons, in this study, a first improvement of the TVAM was achieved by choosing the Fortescue method because a variable forgetting factor seemed to be more appropriate to follow the signal dynamics and so to track REM - NREM transitions during sleep. A second improvement of the TVAM was obtained by employing a method of robustness for AR recursive identification. In this study, a better reduction of the effect of the outliers was achieved by using the specific condition on the prediction error. In this way, as shown in Fig. 2, the computation of the model coefficients, influencing the spectral parameters estimate, can be carried out in a more reliable way.

According to a qualitative observation of Fig. 3, NREM stages are characterized by a prevalent parasympathetic activity with a more regular respiratory frequency with respect to REM sleep. In fact, power in the HF band assumes, in this phase, higher values than in REM episodes. In the same way, the absolute value of the spectrum pole in the HF band is closer to the unitary circle by demonstrating a more regular breathing than in REM sleep. On the contrary, during REM sleep the sympathetic activation reaches values similar to wake and the respiratory activity becomes irregular. In fact, in REM episodes the LF/HF ratio shows an increase with respect to NREM sleep. Lower values assumed

by the absolute value of the spectrum pole in the HF band and its oscillatory trend demonstrate a more irregular respiratory frequency than in NREM sleep stages. As reported in Table I, the spectral indexes HF%, LF/HF ratio and absolute value of the spectrum pole in the HF band are useful features that can be used in the discrimination between REM and NREM sleep stages.

In conclusion, the analysis of whole night tachograms by means of the optimized TVAM allows to obtain spectral indexes whose values differ significantly during REM and NREM sleep stages, as reported in Table I. So the employment of a variable forgetting factor (Fortescue method) and the adoption of the specific condition on the prediction error allow for a real optimization of the TVAM in tracking REM - NREM transitions during sleep.

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