34th Annual International Conference of the IEEE EMBS
San Diego, California USA, 28 August - 1 September, 2012

Realization of Spatial Compliant Virtual Fixture using Eigenscrews

Dongwen Zhang, Lei Wang, Jia Gu, Zhicheng Li, Ken Chen

Abstract—**Virtual fixture is kind of assistance mechanism to limit movement into restricted regions and/or guide movement along desired trajectories in human-machine interactive operation. The structure of geometric and dynamic constraints of reference tasks is analyzed using screw theory. End-effector and the reference frame are elastically coupled by virtual screw springs, which slides along the reference sequences. An allowable motion screw set is constructed, from which the desired spatial compliance and stiffness matrices are synthesized from an allowable motion screw set. The presented virtual fixture are implemented dynamic contour tracking experiment, the effects of control parameters on system performance are also analyzed. The proposed virtual fixtures unites rotation and translation motions, and filter out task-unrelated components from the manual input while augmenting task-related components.**

I. INTRODUCTION

Virtual fixtures (VF) are task-dependent computer-generated mechanisms designed to limit movement into restricted regions and/or influence movement along desired paths [1,2]. VF are commonly used to augment a surgeon's abilities for delicate and difficult surgical tasks. Ming Li proposed constructed a weighted, linearized multiple objective optimization framework to form a virtual fixture library for limiting joint movement[3]. An hierarchical control scheme involves task related information and frees the operator from manipulator details and low-level control of kinematic and motor systems. Device independent bilateral control scheme broadened the application of the scheme to a variety of virtual fixtures, including teleoperative or cooperative control for both admittance and impedance device types[2].

Compliant control facilitates VF by allowing partially constrained end-effectors to comply with the force feedback from general spatial elastic and viscous coupling between the end-effector and the reference configuration. Compliant control is necessary to overcome the uncertainties associated with factors such as registration errors, variations in anatomy, and changes during procedures. Ryo Kikuuwe proposed the concept and control algorithm of new class of virtual fixtures that is based on simulated plasticity [4]. Screw theory has been applied successfully to the analysis of elastically coupled rigid bodies. General spatial stiffness behavior was first analyzed in 1900 when Ball introduced screw theory and used it to describe the motion of a rigid body in a general spatial potential field[8]. The synthesis problem has also been addressed in a number of works[5,6], they examined the synthesis of simple spring systems in depth, and pointed out the structure of spatial stiffness by evaluating the stiffness matrix "primitives" rank-1 matrices that compose a spatial stiffness matrix that refers to screw springs.

In this paper, we analyze the structure of constraints using screw theory, propose a synthesis method for the spatial stiffness and compliance matrices, and define the reference task using a generalized frame curve. A sliding visco-elastic coupled model for the virtual fixtures is also proposed and implemented.

II. COMPLIANCE AND STIFFNESS FOR VIRTUAL FIXTURES

A general manipulation system can be modeled as an elastically suspended rigid body. A wrench applied to the system produces a twist deformation, which is characterized by a 6×6 symmetric positive semidefinite (PSD) compliance matrix. An applied twist deformation causes the elastic suspension to exert a counteracting wrench, which is characterized by a 6×6 symmetric PSD stiffness matrix [5]. A general spring maps a small twist between two elastically coupled bodies into the interactive wrench, which refers to the screw spring [6].

$$\boldsymbol{w}=\left[\mathbf{f}^{T},\boldsymbol{\tau}^{T}\right]^{T},\delta\mathbf{t}=\left[\delta\mathbf{x}^{T},\delta\boldsymbol{\theta}^{T}\right]^{T}$$
$$\boldsymbol{w}=\mathbf{K}\delta\mathbf{t}$$
$$\delta\mathbf{t}=\mathbf{C}\boldsymbol{w} \tag{1}$$

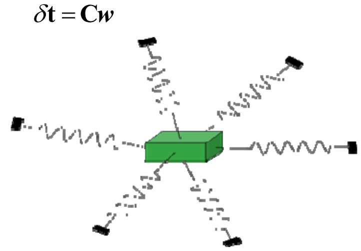

Figure 1. A rigid body elastically suspended by eigenscrew springs.

Any rank-m symmetric PSD matrix K can be decomposed into the sum of m rank-1 PSD matrices. The corresponding eigenvalue problem of a compliance matrix is

$$\lambda\mathbf{w}=\mathbf{C}\Lambda\mathbf{w} \tag{2}$$

A wrench applied about an eigenscrew of the compliance matrix yields a twist defor-mation about the same eigenscrew, while a twist deformation about an eigenscrew of the stiffness matrix produces a wrench on the same eigenscrew [7].

Compliant control facilitates virtual fixtures by allowing partially constrained end-effectors to comply with force feedback from the general spatial elastic and viscous coupling between the end-effector and the reference configuration. General virtual fixtures for impedance-type devices generate a haptic force to counteract the unwanted component from

*Resrach supported by ABC Foundation. This paper is supported by the following grants: National Natural Science Foundation of China (61103165), Shenzhen Key Laboratory Project (CXB201005260056A), and Shenzhen Distinguished Young Scholars Fund (JC201005260248A).

Dongwen Zhang is with the Shenzhen Institutes of Advanced Technology, CAS, Shenzhen, 518055, China (e-mail: dw.zhang@siat.ac.cn)

Lei Wang is with the Shenzhen Institutes of Advanced Technology, CAS, Shenzhen, 518055, China (e-mail: wang.lei@siat.ac.cn)

manual input $\boldsymbol{f}_h$ by using the computed elasticity and a viscosity mechanism $\boldsymbol{f}_{VF}$

$$\mathbf{M}(\dot{\mathbf{x}},\mathbf{x})\ddot{\mathbf{x}} + \mathbf{B}(\dot{\mathbf{x}},\mathbf{x})\dot{\mathbf{x}} = \boldsymbol{f}_h + \boldsymbol{f}_{VF} \quad (3)$$

When the slave is admittance-type device, we assume the device is non-backdrivable. The velocity of the end-effector is controlled by a VF controller, which satisfies the quasi-equilibrium condition $\ddot{\mathbf{t}} \approx 0$, such that

$$\dot{\mathbf{x}} = C_{\mathrm{VF}}(\boldsymbol{f}_h, \mathbf{x}, \dot{\mathbf{x}}_r) \quad (4)$$

A. Task Frame Sequence

Surgical reference tasks are defined as a generalized curve $\mathbf{L}_d(\lambda,t)$. The curve is time-varying when considering tissue deformation due to physiological movement. The first-order derivatives for time t and parameter λ exist, so translation and rotation are continuous in the λ-t plane. The generalized frame curve covers a variety of tasks, including forbidden regions, fixed configuration approximations, path following, and active tracking, etc.

The partial derivatives of reference frames and relative displacement of tool frame away the reference construct the allowable motion set, from which the permitted instantaneous motion of the end-effector is defined. The cross-product matrix of the tangent and temporal twists in the reference frame are represented as

$$\begin{aligned} \hat{\boldsymbol{\tau}}_\lambda(\lambda,t) &= \frac{\partial}{\partial\lambda}\mathbf{L}_d(\lambda,t)\mathbf{L}_d(\lambda,t)^T \\ \hat{\mathbf{v}}_t(\lambda,t) &= \frac{\partial}{\partial t}\mathbf{L}_d(\lambda,t)\mathbf{L}_d(\lambda,t)^T \end{aligned} \quad (5)$$

B. Search for Reference Configuration

Given a tool frame $\mathbf{L}(t)$, the nearest reference configuration $\mathbf{L}_r(t)$ on a task sequence $\mathbf{L}_d(\lambda,t)$ refers to one with minimum displacement, where

$$\begin{aligned} &\mathbf{L}_r(t) = \underset{\mathbf{L}_d(\lambda,t)}{\operatorname{argmin}}\, \operatorname{error}(\delta\boldsymbol{\tau}) \\ &\mathbf{L}(t)^T\ \mathbf{L}_d(\lambda,t) \Leftrightarrow \delta\boldsymbol{\tau} = \mathbf{s} + \varepsilon\mathbf{r}\times\mathbf{s} + h\mathbf{s} \\ &\operatorname{error}(\delta\boldsymbol{\tau}) = \left\| \mathbf{s}^T, (\mathbf{r}\times\mathbf{s} + h\mathbf{s})^T \right\|_2^1 = \sqrt{\delta\alpha^2(1+h)^2 + d^2} \\ &\delta\alpha = |\mathbf{s}|,\ \ d = |\mathbf{r}\times\mathbf{s}| \end{aligned} \quad (6)$$

In Eq.(14), the error function of a displacement is defined as the 1st norm of the twist, and can be considered as a compound distance consisting of pure rotation, pure translation, and the distance from the origin to the rotation axis. Searching for the reference configuration along the task sequence is a well understood nearest neighbor problem.

C. Elastically Coupled Model

Compliant coupling between the end-effector and the reference configurations enables the end-effector to comply with the force feedback from the virtual fixture based upon general spatial elastic and viscous mechanisms. These mechanisms limit the manipulator's movement into forbidden regions, and/or influence its movement along the desired frame.

For this paper, the end-effector is simplified as a rigid ball independent of manipulator characteristics. The compliant coupling between the end-effector and the reference is shown in Figure 2. In this case, the two frames are coupled by a compliant mechanism consisting of either a stiffness matrix or a compliance matrix.

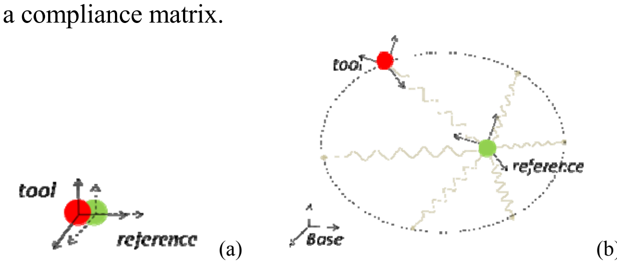

Figure 2. Spatial compliance between the end-effector and the reference frame, for (a) an undeformed configuration; (b) a deformed configuration.

Let 'T' and 'R' index the frames attached to the end-effector and the reference, respectively. Frame 'T' and frame 'R' are assumed to be coincident in the equilibrium state. Let $\mathbf{L}_T(t)$ represent the configuration of the tool, and $\mathbf{L}_R(t)$ represent the configuration of the reference frame. The transformation $\mathbf{L}_T^R(t) = \mathbf{L}_R^{-1}(t)\mathbf{L}_T(t)$ then refers to the relative deformation of the two frames in frame 'R' and the corresponding twist $\mathbf{t}_T^R = \delta\boldsymbol{\theta}_T^R + \varepsilon\delta\mathbf{x}_T^R$ features the screw motion of the relative transformation $\mathbf{L}_T^R(t)$. The elastic wrench $\mathbf{w}_e^T = \mathbf{f}_e^T + \varepsilon\mathbf{m}_e^T$ is exerted on the tool by screw springs in frame 'T' is depicted as

$$\mathbf{w}_e^T = \Gamma_T^R\mathbf{K}\begin{bmatrix}\delta\boldsymbol{\theta}_T^R \\ \delta\mathbf{x}_T^R\end{bmatrix} \quad (7)$$

In which the matrix $\boldsymbol{\Gamma}_T^R$ transforms the infinitesimal change of twist of the tool in frame 'T' to reference frame 'R' [9]. Let the twist rate be $\dot{\mathbf{t}}_T^R = \dot{\boldsymbol{\theta}}_T^R + \varepsilon\dot{\mathbf{x}}_T^R$, $\boldsymbol{\omega}_T$ and $\mathbf{v}_T$ be the angular and linear velocity of the end-effector in frame 'T', and $\boldsymbol{\omega}_R$ and $\mathbf{v}_R$ be the angular and linear velocity of the task in frame 'R'. The twist rate can be written as

$$\begin{bmatrix}\dot{\boldsymbol{\theta}}_T^R \\ \dot{\mathbf{x}}_T^R\end{bmatrix} = \left(\boldsymbol{\Gamma}_T^R\right)^T\begin{bmatrix}\boldsymbol{\omega}_T \\ \mathbf{v}_T\end{bmatrix} - \left(\boldsymbol{\Gamma}_R^T\right)^T\begin{bmatrix}\boldsymbol{\omega}_R \\ \mathbf{v}_R\end{bmatrix} \quad (8)$$

If $\mathbf{w}_v^T = \mathbf{f}_v^T + \varepsilon\mathbf{m}_v^T$ is the viscous wrench in frame 'T' exerted by springs on the tool,

$$\begin{bmatrix}\mathbf{m}_v^T \\ \mathbf{f}_v^T\end{bmatrix} \approx \Gamma_T^R\mathbf{B}\begin{bmatrix}\dot{\boldsymbol{\theta}}_T^R \\ \dot{\mathbf{x}}_T^R\end{bmatrix} \quad (9)$$

For simplicity, we construct the damping matrix using the Rayleigh damping model, in which the damping matrix is linear to the stiffness matrix

$$\mathbf{B} = \alpha\mathbf{K} \quad (10)$$

The derivation of compliant behavior for the compliance coupled model is similar to the elastic wrench mentioned above. A manual input wrench $\mathbf{w}_h^R$ applied to the tool yields a deformation twist For simplicity, we construct the damping matrix using the Rayleigh damping model, in which the damping matrix is linear to the stiffness matrix

$$\begin{bmatrix}\delta\boldsymbol{\theta}_T \\ \delta\mathbf{x}_T\end{bmatrix} = \left(\boldsymbol{\Gamma}_T^R\right)^{-T}\mathbf{C}\begin{bmatrix}\mathbf{m}_h^R \\ \mathbf{f}_h^R\end{bmatrix} \quad (11)$$

The wrench $\mathbf{w}^R$ in frame 'R' can be transformed to $\mathbf{w}_R^B$ in base frame 'B' using the adjoint matrix $Ad_{\Sigma_R^B}$

$$\mathbf{w}_R^B = Ad_{\Sigma_R^B}\mathbf{w}_R \tag{12}$$

The resultant force generated at the end-effector to counteract the manual input is actuated by separated motors, which are assigned from the feedback force $\mathbf{w}^B$ by a force Jacobian matrix.

D. A Sliding Elastically Coupling Virtual Fixture

Given a reference task, the spatial elastically coupling guides the manipulator from start to end, while varies the stiffness according to the reference configuration and operation error. The spatial elastically coupling is realized using virtual screw springs between the reference and a virtual rigid shell, on which the end-effector is mounted. These screw springs stop the end-effector deviating from the reference and push the end-effector back to an equilibrium state, while still allow tangential movement along the reference. When the tool moves, the elastically coupling smoothly slides along the reference as well, helps the operator intuitively to complete their task.

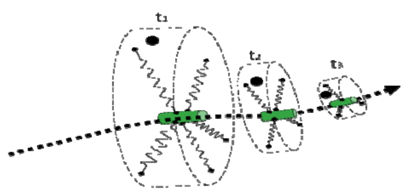

Figure 3. A sliding elastically coupled model.

E. Synthesis of a Spatial Compliance Matrix

Given an allowable motion set, the desired spatial compliance matrix and stiffness matrix can be synthesized, partially preserving the clarity and the characteristics of the eigenscrews. For an arbitrary twist set $\boldsymbol{V}$, the synthesized matrix $\boldsymbol{K}$ is symmetric and semidefinite. Components in $\boldsymbol{V}$ do not necessarily need to be reciprocal; independent restriction is sufficiently satisfying.

$$\begin{aligned}&\mathbf{V} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots \mathbf{v}_r\}, r \le 6\\ &\mathbf{K} = span(\mathbf{V}) = \sum k_i \mathbf{v}_i \mathbf{v}_i^T,\ \ k_i > 0\end{aligned} \tag{13}$$

Proof:

$\forall\ \mathbf{x} \in \mathbf{R}^{6\times1}$

$$\mathbf{xKx^T} = \sum k_i \mathbf{x}\mathbf{v_i}\mathbf{v_i}^T\mathbf{x}^T = \sum k_i(\mathbf{x}\mathbf{v_i})(\mathbf{x}\mathbf{v_i})^T = \sum k_i\left|\mathbf{x}\mathbf{v_i}\right|^2 \ge \mathbf{0}$$

When using a virtual fixture for guidance, motion along the preferred direction occurs with high compliance, while low compliance occurs along non-preferred directions. The partial derivatives $(\boldsymbol{\tau}_t, \boldsymbol{\tau}_n)$ and displacement $\boldsymbol{\tau}_d$ of the tool in frame 'R' construct the allowable motion set $\boldsymbol{V}$, from which the instantaneous compliance matrix is synthesized. A complete constraint corresponds to a full-rank compliance matrix; a deficient- rank matrix indicates an incomplete constraint.

III. Numerical Experiments

In this section, the presented virtual fixture were implemented for dynamic contour tracking (DCT) experiment, the effects of control parameters on system performance were also analyzed. In this simulation, we assume that the impedance device runs quasi-statically, where a constant force results in a constant velocity [10]. The ratio M is a diagonal matrix, independent of coordinate selection.

$$\begin{bmatrix}\boldsymbol{\omega}_T\\ \mathbf{v}_T\end{bmatrix} = \mathbf{M}\begin{bmatrix}\mathbf{m}_h^T\\ \mathbf{f}_h^T\end{bmatrix} \tag{14}$$

The compound acceleration of rotation and translation are represented in frame 'T' , and external applied wrench is linearly related by the ratio matrix M.

Numerous surgical tasks require a surgeon to follow a predetermined path on a deformable tissue surface while still maintaining other constraints. The task can be simplified if we consider the task as moving a tool from north to south on a deformable ball while the radius r of the ball varies periodically, such that

$$r = r_0 + \delta r \sin \omega t$$

The reference sequence is generated by moving the tool along a deformable semicircle with an angle from 0 to π, while keeping the y-axis of frame 'T' coincident with the x-axis of the reference frame. The allowable motion set $\boldsymbol{V}$ consists of a unit twist of relative displacement, a tangential twist, and the deformed velocity of the reference, from which the compliance and stiffness matrices are constructed.The manual input is linearly represented by the allowable motion basis.

$$\begin{aligned}&\mathbf{L}_r(t) = \arg\min_{\mathbf{L}_d(\alpha,t)} \text{error}(\delta\mathbf{t})\\ &\mathbf{V}_R = \left\{\delta\mathbf{t}_T^R, \boldsymbol{\tau}^R, \mathbf{v}^R\right\}\\ &\boldsymbol{\tau}^R(\alpha,t) = \begin{bmatrix}0 & 0 & 1 & 0 & r & 0\end{bmatrix}^T\\ &\mathbf{v}^R(\alpha,t) = \begin{bmatrix}0 & 0 & 0 & \delta r\omega\cos\omega t & 0 & 0\end{bmatrix}^T\\ &\begin{bmatrix}\mathbf{m}_m^T\\ \mathbf{f}_m^T\end{bmatrix} = -\lambda\delta\mathbf{t}_T^R + \gamma\hat{\boldsymbol{\tau}}^R + \xi\mathbf{v}^R + noise\end{aligned}$$

The tracking error oscillates at the same frequency as the reference, with the peaks appearing at values of $n\pi$ for n = 0, 1, 2, …, and corresponding to the maximum deformed velocity. Figure 4 shows the results of the simulation using an admittance type device with compliance values of (70, 1, 0) mm/N and angle velocity π rad/s. Figure 4(b) shows the trajectory of point (1,1,1) on the tool in base frame 'B' and, apart from the initial error, the tool closely follows the reference path through the entire task. Note that an increase in compliance cannot eliminate the dynamic error, only reduce it to as small a value as possible.

Figure 5 shows the results of a dynamic contour tracking experiment (DCT) using an impedance type device with a stiffness of 70 N/mm and manual input ratios of (-0.2,0.5,0). Figure 5(b) shows the trajectory of point (1,1,1) on the tool in base frame 'B'. The same situation holds as with the admittance type device, where increasing stiffness can only reduce the dynamic error, not eliminate it.

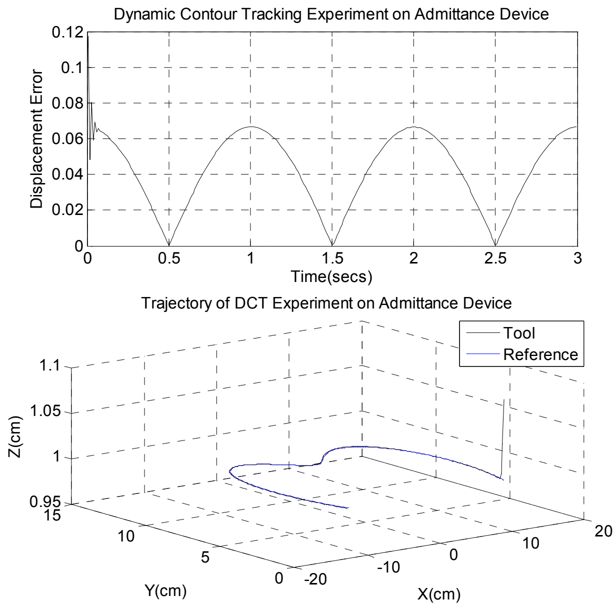

Figure 4. DCT experiment using an admittance type device: (a) the displacement error; and (b) the trajectory of point (1,1,1) on the tool.

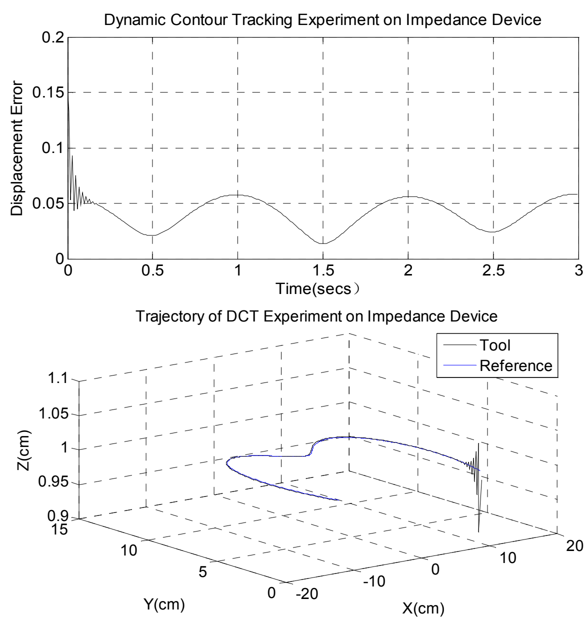

Figure 5. DCT experiment using an admittance type device: (a) the displacement error; and (b) the trajectory of point (1,1,1) on the tool.

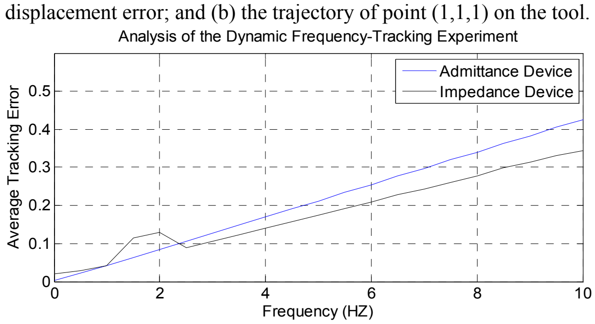

Figure 6. An analysis of the dynamic frequency-tracking characteristics of spatial virtual fixtures.

To discuss the dynamic properties of the proposed virtual fixtures, the end-effector is made to follow the contour of a sinusoidally deforming circle as the frequency varies from 0.5 to 10 HZ, with the control parameters remaining unchanged. As shown in Figure 6, the average tracking error linearly increases as the deforming frequency grows. Thus, fixed or self-adaptive compliance/stiffness settings do not effectively eliminate the drift error over frequency.

IV. Discussion and Conclusion

We have analyzed the structure of the necessary geometrical and dynamic constraints of medical robots using screw theory, which unites rotational and translational motions into one set. Given a reference task, the desired virtual fixture can be constructed using spatial compliance and stiffness matrices from the allowable set of screw motions. The end-effector and the reference frame are elastically coupled by virtual screw springs, and the end-effector slides along the reference sequences. The results show that the static properties of the system remain stable for a wide range of noise disturbances and stable dynamic characteristics at low frequencies. A remaining question concerns conducting a thorough stability analysis of these algorithms. Under a quasi-static assumption, a first order dynamic model was employed for these experiments, which directly led to the observed poor dynamic tracking performance at high frequency.

References

[1] Panadda Marayong,etc. Spatial Motion Constraints: Theory and Demonstrations for Robot Guidance Using Virtual Fixture. IEEE International Conference on Robotics an Automation. vol.2,1954 – 1959(2003)

[2] Kapoor, A., Ming Li, Taylor, R.H.,Constrained control for surgical assistant robots, Proceedings 2006 IEEE International Conference on Robotics and Automation(ICRA 2006), 231 – 236(2006)

[3] Ming Li,etc. Spatial Motion Constraints Using Virtual Fixtures Generated by Anatomy, IEEE Trans. on Robotics, 23 (1), 4-19(2007)

[4] Ryo Kikuuwe,etc. A control Framework to Generate Nonenergy-Storing Virtual Fixture: Use of Simulated Plasticity. IEEE Trans. on Robotics, 24(4), 781-793(2008)

[5] Patterson, T. and H. Lipkin, A classiffcation of robot compliance. ASME J. of Mechanical Design 115, 581-584(1993)

[6] Shuguang Huang, Joseph M. Schimmels,The Bounds and Realization of Spatial Stiffnesses Achieved with Simple Spring Connected in Parallel, IEEE Transactions on Robotics and Automation， 14(3), 466-475(1998)

[7] Shuguang Huang, Joseph M. Schimmels, The Eigenscrew Decomposition of Spatial Stiffness Matrices, IEEE Transactions on Robotics and Automation， 6(2),146-156(2000)

[8] Ball, R., A Treatise on the Theory of Screws. Cambridge University Press. 1998 paperback edition(1900).

[9] Ernest D. Fasse, Some Applications of Screw Theory to Lumped-Parameter Modeling of Viscous-Elastically Coupled Rigid Bodies. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science , 216(1), 105-121(2002)

[10] Jake J. Abbot,G D Hager, A M Okamura, Steady-hand teleoperation with virtual fixtures, The 12th IEEE International Workshop on Robot and Human Interactive Communication 2003 Proceedings, 145-151(2003)