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Evolutionary Coil Design for Maximally Uniform Magnetic Fields

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Abstract— **Magnetic fields and their applications in medicine and biology are an active area of research in recent years. Some of these applications include chronic wound healing, electrical stimulation of cardiac tissue and medical imaging among others. The effectiveness of these applications strongly relies on the uniformity of the field, a feature closely related to the arrangement of the coils that generates it. In this paper, genetic algorithms are used to create appropriate arrangements of coils in order to generate a field with a wide volume of uniformity. Results show that the proposed methodology is an effective tool for designing coil arrangements capable of generating magnetic field within an arbitrary volume of interest in which field uniformity is higher than in designs using a traditional approach.**

I. Introduction

Magnetic fields have a wide range of applications in medicine and are matter of ongoing research, some examples include nuclear magnetic resonance imaging, magnetic resonance susceptometry, cardiac treatment, cancer treatment, chronic wound healing and electrical stimulation for cardiac tissue [1], in most of medical and biological applications field uniformity is critical for obtaining the desired effect [2].

Obtaining uniform fields requires work of engineering that consists on creating coil arrangements that generate a desired field distribution; some examples contemplate symmetric arrangements by using circular coils [3], square coils [4], near-Helmholtz configurations [5] and designs with multiple coils [6]. The pursue of uniform magnetic fields has driven engineers and scientists to combine simple configurations into complex designs that arise from both, rigorous scientific knowledge and empirical experience. Following this trend, this paper presents a proposal for designing coil arrangements by means of a genetic algorithm that in essence creates random coil arrangements (not necessarily symmetrical) and combines their best attributes in the pursue of a maximally uniform field within a given Volume of Interest (VOI). The VOI may be as large as the required volume to place human body parts for performing In Vivo electromagnetic studies [8] or healing chronic wounds [9].

It is important to notice that the use of genetic algorithms for designing coils has already been studied in the generation of active shielding by designing coils that cancel out an unwanted external field [10].

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II. Methodology

A. Mathematical model

Theoretical computation of the magnetic field due to a finite straight conductor (Fig. 1) has been utilized as a pattern for computing the resulting magnetic field of most of the non-circular symmetric coil arrangemets, common examples include Square Helmholtz coils, Rubens coils, Merritt coils and Alldred and Scollar coils. The key for computing the resulting magnetic field relies on the temporal and spatial summation of transformed versions of the vector field produced by a finite straight conductor (FSC) given by (1), (2) and (3). Such transformations consist of rotating the vector field to the desired tilt and translating it to the desired position; Fig. 2 shows an example for a square coil where the magnetic field due to the segments parallel to the *Y* axis (dashed lines) is obtained by rotating the pattern field $\theta=\pi/2rad$ and displacing it $\Delta_x=\pm l$ units from the origin, whereas the magnetic field due to the segments parallel to the *X* axis (solid line) must only been displaced $\Delta_y=\pm l$ units. These transformations are ruled by the rotation and translation operations given by (4) and (5).

$$|\boldsymbol{B}| = \frac{\mu_0 i}{4\pi d}(\sin\alpha_1 - \sin\alpha_2) \quad (1)$$

$$\boldsymbol{B_x} = \frac{\mu_0 i}{4\pi d}\cdot\frac{z}{d}\cdot\left[\frac{y+a}{\sqrt{d^2+(y+a)}} - \frac{y-a}{\sqrt{d^2+(y-a)}}\right] \quad (2)$$

$$\boldsymbol{B_z} = -\frac{\mu_0 i}{4\pi d}\cdot\frac{x}{d}\cdot\left[\frac{y+a}{\sqrt{d^2+(y+a)}} - \frac{y-a}{\sqrt{d^2+(y-a)}}\right] \quad (3)$$

$$\boldsymbol{B}'(\boldsymbol{x}') = \mathbf{R}^{-1}\boldsymbol{B}(\mathbf{R}\boldsymbol{x}) \quad (4)$$

$$\boldsymbol{B}''(\boldsymbol{x}') = \boldsymbol{B}'(\boldsymbol{x}' + \boldsymbol{x}'') \quad (5)$$

Where:

$d = \sqrt{x^2+z^2}$

x,y,z = spatial coordinates o which the field is computed
a = conductor length
$|\boldsymbol{B}|$ = magnitude of the magnetic field vector
$\boldsymbol{B}_x$ = field component over the *X* axis
$\boldsymbol{B}_z$ = field component over the *Z* axis
$\boldsymbol{B}'$ = rotated magnetic field
$\boldsymbol{B}''$ = translated magnetic field
$\boldsymbol{x}'$ = rotated domain
$\boldsymbol{x}''$ = translated domain
$\mathbf{R} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ 2-D rotation matrix
θ = rotation angle

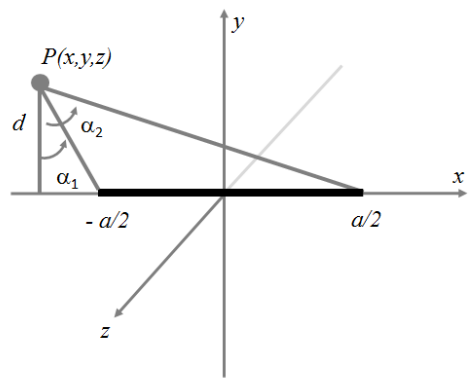

Fig. 1. Magnetic field due to a finite straight conductor at point *P(x,y,z)*, the conductor lies over the *X* axis extending from $x = -a$ to $x = +a$.

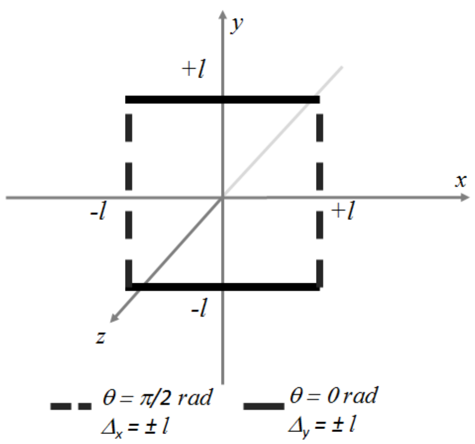

Fig. 2. Transformations to the magnetic field due to a finite straight conductor to obtain the resulting field from a squared coil.

The idea of creating random coil arrangements and combining their best attributes in the pursue of a maximally uniform field is depicted in Fig. 3, where two coils were created by connecting randomly scattered points in two parallel planes of the $\boldsymbol{R}^3$ Cartesian coordinate system; for each coil the resulting magnetic field is given by the vector summation of the individual contributions of each straight conductor and for the coil arrangement the resulting magnetic field is obtained by the vector summation of the field due to each coil. By creating and evaluating a number of random arrangements it is possible to identify their best characteristics and iteratively combine them until a quality criterion is met.

Random coils are designed obeying the pattern described in Fig. 3 where axis are normalized to the interval *[-1, 1]*, coils are spaced by $d=0.5$ distance units, magnetic permeability is normalized to $\mu=1$, each coil carries a current $i_x=1$A and coils are built from $N=1$ turns. Since under ideal conditions these quantities contribute only on scaling the magnitude of the field, they are not rigorously considered for evaluating uniformity, nonetheless they may be considered for optimizing manufacturability issues on a physical prototype. Notice that Fig. 3 highlights distances h_1 and h_2 that have been defined as the longest distance between edges of each coil, so the mean distance $h = (h_1+h_2)/2$ can be used for comparing the size of the resulting arrangement against the size of an standard configuration such as Square Helmholtz coils. On the other hand the VOI has been defined as a circular cylinder whose center is located at the origin of the *XY* plane and its length is extended along the *Z* axis. It is worth to mention that the VOI could be defined as any other arbitrary volume but it is kept as a straight circular cylinder in order to preserve simplicity for the presented implementation. Finally, since a high number of edges could create polygons that could be well approached using fewer points, coils have been limited to have up to $N_e=10$ edges.

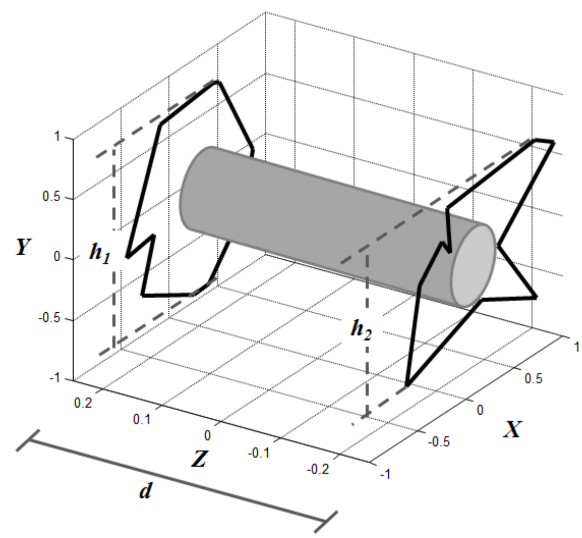

Fig. 3. Coil design pattern.

B. Genetic Algorithm

In order to iteratively combine arrangements with good attributes, an implementation of the simple genetic algorithm [11] was coded considering the following steps:

1. Create a random population of arrangements (individuals), including the elitist elements from the previous iteration, for the very first iteration use random individuals.
2. Perform random crossover and mutation with high and low probabilities of occurrence respectively.
3. Compute the resulting magnetic field for each arrangement by combining the individual contribution of straight wire segments and then by combining the resulting field from each coil.
4. Evaluate the uniformity of the magnetic field within the VOI by means of a suitable quality function.
5. Identify the fittest arrangements (elitist elements) and preserve them for the next iteration
6. If the stop criterion has not been reached yet, go to step 1, else stop the algorithm and present the fittest individual.

For implementing the algorithm individuals were defined as a set of ten random vectors $\{\boldsymbol{v_1}, .., \boldsymbol{v_{10}}\}, \boldsymbol{v_i} \in \boldsymbol{R^2}$ with uniform probability distribution, these vectors describe the edges of the coil so conductor segments are built by connecting them to create a simple irregular polygon shape. Considering this, chromosomes were defined as each pair of (x_i, y_i) coordinates of said vectors and crossover was implemented by randomly interchanging vectors within individuals, whereas mutation was implemented by adding white noise ($\sigma = 0.01$) to the whole individual.

Field quality Q was measured as the inverse of the standard deviation of the magnitude of the vectors contained inside the VOI (6), by doing so it is expected to obtain an uniform field whose magnitude could be controlled by increasing the number of turns of each coil and by increasing the current carried by them. Finally, stop criterion was defined as reaching 100 iterations.

$$Q = \frac{1}{std(\mathbf{B})}; \ \mathbf{B} \in \mathbf{VOI} \quad (6)$$

C. Comparison against common configurations

In order to assess the value of the proposed methodology, homogeneity of the resulting field was compared against the field produced by Square Helmholtz coils. Comparison is based on estimating the uniform volume obtained for each coil at different uniformity thresholds of 2%, 5% and 10%. Volumes were computed by segmenting uniform areas of each slice at each threshold and then by rendering the volume using the open source, free available Visualization Toolkit (VTK) [7]. In order to make a fair comparison, Square Helmholtz coils were scaled to the same dimensions of the genetically designed arrangement, this means that spacing between coils, number of turns, magnetic permeability and average longest distance between edges h were preserved.

As a complement to the comparison based on the resulting uniform field, the total perimeter (*TP*) defined as the sum of the perimeters of each coil was also considered, this characteristic is of high relevance since it is closely related to the amount of conductor and hence the overall cost of manufacturing the coils.

III. Results

Table I shows the *(x,y)* coordinates for three coil arrangements designed by the genetic algorithm as well as coordinates for the comparison Square Helmholtz coils. For each arrangement, *Coil 1* and *Coil 2* coordinates lie in the planes $z = \pm 0.25$. On the other hand Fig. 4 shows field slices for the first arrangement emphasizing uniform regions for the different thresholds of 2%, 5% and 10% depicted as black, dark gray and light gray areas respectively, finally Fig. 5 and Fig. 6 show the corresponding rendered volume at each threshold for the first coil arrangement created by the genetic algorithm and for the Square Helmholtz coil respectively.

Square Helmholtz coils for comparison have been designed considering $h = 2.22$ that corresponds to the mean longest distance of coils given by Arrangement 2 in table I, this yields to a square coil whose sides have a length of $a = 0.78$.

Table II show the comparison between the uniform volume obtained by each arrangement casted by the genetic algorithm and the uniform volume obtained by the Helmholtz coil, volumes are normalized so 1 corresponds to the volume obtained by the Square coil.

Results show that arrangements designed by the genetic algorithm consistently offer higher uniform volumes at every threshold, improvements rise from 14% at the most restrictive threshold of 2% for the first arrangement up to 50 and 60% for the least restrictive thresholds of 5 and 10%. On the other hand, the comparison of the total perimeter and the distance h of the genetically designed arrangements against the Square Helmholtz configuration show that the proposed methodology casts designs that may require lager amounts of conductor for manufacturing the coils, however depending on the specific design criteria for each specific application this drawback may be worth in order to achieve uniform magnetic fields.

It is worth to mention that volumes shown in Fig. 5 are not perfectly symmetric along the Z axis as those shown in Fig. 6, depending on the application this situation may or not represent a concern, however it also worth to mention that increasing the number of iterations, enriching the quality function or modifying the volume of interest can be used for manipulating the morphology of the resulting field.

TABLE I. RESULTING ARRANGEMENT CONFIGURATIONS

Arrangement 1			
Coil 1		*Coil 2*	
0.74	0.28	0.64	0.31
0.85	0.96	0.57	0.44
-0.86	0.57	-0.28	0.91
-0.78	0.41	-0.54	0.61
-0.78	-0.27	-0.99	-0.8
-0.95	-0.45	-0.42	-0.4
-0.42	-0.62	-0.43	-0.92
-0.33	-0.83	-0.24	-0.89
0.24	-0.71	0.65	-0.64
0.75	-0.95	0.63	-0.35
h = 2.13 TP =12.33			

Arrangement 2			
Coil 1		*Coil 2*	
0.74	0.28	0.64	0.31
0.93	1.05	0.57	0.44
-0.86	0.57	-0.28	0.91
-0.86	0.45	-0.54	0.61
-1	-0.37	-0.99	-0.8
-0.95	-0.45	-0.42	-0.4
-0.09	-0.94	-0.43	-0.92
0.58	-0.73	-0.43	-0.92
0.53	-0.61	-0.24	-0.89
0.97	-0.67	0.87	-0.61
h = 2.22 TP = 13.43			

Arrangement 3			
Coil 1		*Coil 2*	
0.93	1	0.97	0.44
0.69	0.94	0.93	0.53
-1	0.66	0.93	0.89
-0.86	0.45	-0.78	0.62
-1	-0.38	-0.99	-0.8
-1	-0.59	-0.42	-0.4
-0.09	-0.94	-0.43	-0.92
0.63	-0.79	-0.43	-0.92
0.54	-0.62	-0.24	-0.89
0.97	-0.67	0.52	-0.89
h = 2.64 TP = 14.47			

Square Helmholtz	
Coils	
-0.78	-0.78
-0.78	0.78
0.78	-0.78
0.78	0.78
h = 2.22	
a = 0.78	
TP = 12.48	
Coils located at z = ±0.25	

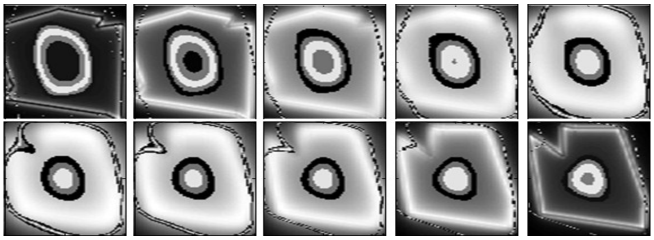

Fig. 4. Field slices across the *Z* axis. Light gray, gray and black areas on the middle of each slice highlight regions of 2, 5 and 10% of uniformity respectively.

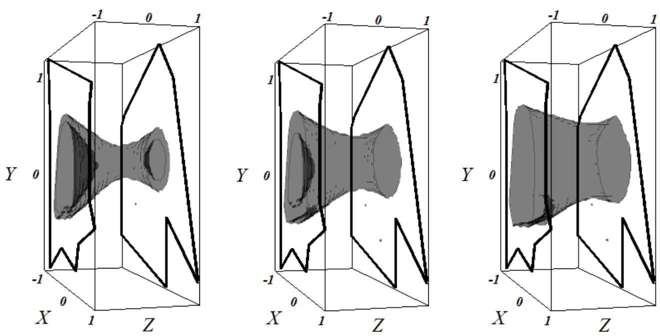

Fig. 5. Rendered uniform volumes for the first genetically designed arrangement (from left to right 2%, 5% and 10%).

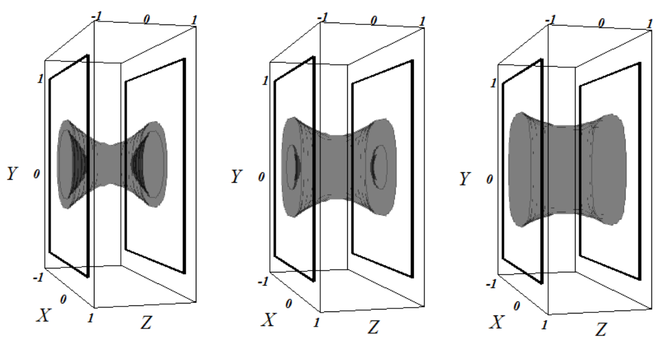

Fig. 6. Rendered uniform volumes for the Square Helmholtz arrangement (from left to right 2%, 5% and 10%).

TABLE II. UNIFORMITY COMPARISON

Threshold	Arrangement 1	Arrangement 2	Arrangement 3	Square Helmholtz
2%	1.14	1.25	1.26	1
5%	1.22	1.60	1.48	1
10%	1.30	1.49	1.44	1

Considering applications like In Vivo electromagnetic studies [8] or chronic wound treatment [9], where a large volume of the body must be exposed to the magnetic field, the goal VOI can be established as a volume large enough to cover the human trunk. A reasonable design criterion is to establish the spacing between coils as $d = 66\ cm$ that corresponds to the 95th percentile of the forearm – forearm breath [12]. Transforming the normalized arbitrary units to centimeters, the h distance for each of the genetically designed arrangements correspond to 281, 293 and 348 cm respectively whereas the h distance for the square Helmholtz coil corresponds to 293 cm. On the other hand the equivalent total perimeter for each arrangement is of 16.27, 17.73 and 19.10 m respectively, whereas the Square configuration requires 16.47 m.

IV. Conclusion

Results show that genetic algorithms cast promissory results on designing coils for generating uniform magnetic fields. Genetically designed coil arrangements utilizing the proposed quality function consistently exhibit higher field uniformity with the tradeoff of needing higher amounts of conductor and generating non – perfectly symmetrical regions of uniformity, this behavior may not represent a major drawback for applications in which field uniformity has the highest priority.

It is important to emphasize that the presented approach considers simplified conditions where no other properties except field uniformity were included, the quality function Q can be enriched to incorporate physical and manufacturability attributes such as inductance, frequency response, resistance, weight and length that can lessen undesired attributes on the design. At the same time, the proposed methodology could also be applied on determining design attributes for known multiple coil configurations such as Merritt coils, Rubens coils and Alldred and Scollar coils, quality function could consider properties such as number of coils, coil length, coil spacing and Ampere – Turn ratios.

V. References

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