34th Annual International Conference of the IEEE EMBS
San Diego, California USA, 28 August - 1 September, 2012

A Novel Fast Algorithm for Parallel Excitation Pulse Design in MRI

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Abstract—**Spatially selective excitations with parallel transmitters have been regarded as a key in solving several high field MRI problems such as inhomogeneity correction and reducing specific absorption rate. However, three-dimensional pulse design in general is very time consuming which may prevent it from real-time applications. In this work, we explore the sparsity in the pulse design system equation. The size of system equation is reduced after a sparse transform and therefore design speed can be significantly increased. Computer simulations in several common scenarios show that the proposed design method can achieve up to an order of magnitude speedup than the conventional design methods while maintaining similar excitation accuracy.**

I. INTRODUCTION

Spatially tailored RF (TRF) pulses are widely used in Magnetic Resonance Imaging (MRI) to excite arbitrary shaped spatial patterns. Parallel excitation (pTx) [1-3] with multi-channel transmitters can shorten the RF pulse durations and reduce the specific absorption rate (SAR) [4, 5]. TRF with pTx has been found extremely useful in solving several current challenges in the high field MRI, such as field inhomogeneity and high SAR [6].

One widely used method under the small-tip-angle approximation [7] is the spatial domain method [3]. In the method, a specified target pattern and a k-space trajectory are specified and a set of linear system equations is built. The pulses can be designed by solving the linear system using various numerical methods. One major problem of such pulse design is the high computation cost, especially for multiple-channel three-dimensional (3D) pulse designs. This is due to the large system matrix size, which is proportional to both the size of the target pattern and the total pulse length. Generally, it can take 2~5 minutes [8] to design a pulse, which can prevent parallel excitation technique from being used in real-time applications.

Currently, 3D pulses using spoke trajectories are commonly used to correct the slowly varying B_1 field inhomogeneity in high field MRI while providing slice selection at the same time. In some applications, the pulse shape in the design could be pre-determined (such as Gaussian windowed Sinc function [9, 10]), and only the optimal complex weights for each the spokes are solved. In this case, the design problem can be solved very quickly. However, such methods do not take full advantage of the design freedom and the power of parallel excitation. In addition, this cannot be applied for modified spoke trajectories [11, 12] and other trajectories such as 3D spirals.

In this paper, we propose a novel method to design TRF pulses with pTx based on previous work [13]. By transforming the spatial target pattern into a sparse form, the system equation can be reduced in size to improve the design speed. Computer simulations of the proposed method in several conventional scenarios are performed to validate the proposed method.

II. METHODS

A. RF pulse design using the spatial domain method

In this part, the conventional spatial domain method of RF pulse design for parallel excitation is briefly listed and reviewed.

Under small tip angle assumption [7], the excitation pattern of transverse magnetization and the complex RF pulse are Fourier pairs following the k-space trajectory. Parallel excitation pattern is a linear sum of multi-channel patterns weighted by the coil transmit sensitivity as,

$$M_{xy}(\bar{x}) = i\gamma M_o \sum_l s_l(\bar{x}) \int_0^T B_1(t) e^{-i\cdot\bar{x}\cdot\bar{k}(t,T)} dt \qquad (1)$$

where $M_{xy}(\bar{x})$ is the specified spatial target patter, s_l is the B_1^+ map of the *l*-th channel. After discretizing both the pulse in time and the target pattern in space, the Bloch equation is turned into a linear equation,

$$m = \mathrm{A}b \qquad (2)$$

where *m* is the vector form target pattern, A is the system matrix containing k-space trajectory and coil sensitivities, and *b* is the driving RF waveform vector to be solved as defined in [3].

Then the pulse design problem can be formulated as a minimization problem with a regularization term,

$$\min_b \ \left\|\mathrm{A}b - m\right\|_2 + \lambda \left\|b\right\|_2 \qquad (3)$$

Numerical methods such as conjugate gradient method (CG) can be used to solve the problem. The size of the system matrix A in a typical 3D pulse design with 8-ch and 2msec pulse is on the level of $30k \times 10k$ and takes about 2 min to solve on a i7-core computer.

B. The Novel Fast RF Pulse Design Algorithm

In eq. (2) the resolution of the target pattern can be increased arbitrarily (unlike reconstruction problems) and the

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978-1-4577-1787-1/12/$26.00 ©2012 IEEE

achieved excitation resolution is determined by both the resolution of m and the excitation k-space trajectory. Defining m on a finer grid can improve the design, however this lead to additional cost of design time as the size of A is proportional to the size of m.

In this work, we proposed a modified pulse design method with sparse patterns. First, original spatial domain patterns are transformed into a sparse form. Here, we use the Fourier transform as the sparse transform to modify the design equation (1),

$$\int M_{xy}(\vec{x})e^{-i\cdot\vec{x}\cdot\vec{k}}\mathrm{d}\vec{x} = \int\left(i\gamma M_o\sum_l s_l(\vec{x})\int_0^T B_1(t)e^{-i\cdot\vec{x}\cdot\vec{k}(t,T)}dt\right)e^{-i\cdot\vec{x}\cdot\vec{k}}\mathrm{d}\vec{x} \tag{4}$$

where $\vec{k}$ is the spatial frequency defined by the Fourier transform and $\vec{k}(t,T)$ is the specified k-space trajectory. After switching the integral order on the right side, we have

$$\tilde{M}_{xy}(\vec{k}) = i\gamma M_o\int_0^T B_1(t)\sum_l \tilde{S}_l\left(\vec{k}+\vec{k}(t;T)\right)dt \tag{5}$$

where $\tilde{S}_l$ is the spectrum of the B_1^+ map of the l-th channel. After sampling this equation along the time and spatial frequency axis, the modified system equation of pulse design is formulated as,

$$\tilde{M} = \tilde{A}_{full}b \tag{6}$$

Now the spectrum of the original target pattern has concentrated energy at lower frequencies. Then, to speed up the design procedure, a truncation operator is performed on the system equation as,

$$T\tilde{M} = T\tilde{A}_{full}b \tag{7}$$

where T is a truncation matrix that contains only one non-zero element 1 in each row corresponding to the equation kept from (6). This truncation will keep only the equations corresponding to the lower frequency components of $\tilde{M}$ which are dominant coefficients. In practice, the truncation can done very efficiently by equation indexing. The truncation ratio defines the reduction ratio of system size as,

$$R_s = \frac{Num.\ of\ rows\ of\ \tilde{M}}{Num.\ of\ rows\ of\ T\tilde{M}} \tag{8}$$

Then, eq.(7) can be solved using the CG method similar to the one used in conventional method. As the computation cost of solving such a linear system is proportional to the size of system equation, this reduction in system size can directly lead to increased pulse design speed.

To make this method more easily applied to existing designs, a transform of the existing conventional designs can be made. Equation (6) can be taken as multiplying a Fourier transform matrix on both sides of equation (2). The two designs will be completely equivalent before truncation if the this transform matrix is invertible for the chosen spatial frequency k, such as on the Cartesian grid. All the tests of this new pulse design method in this paper will use the Fourier transform on Cartesian grid and the size of system equation in (2) and (6) are kept the same. Thus, this method can be used to increase the design speed of existing pulse designs that use the spatial domain method simply by doing an additional FFT on the original design equation.

C. *Evaluation of the Proposed Pulse Design Algorithm*

To evaluate the performance of this proposed pulse design, several simulations are performed to compare the proposed method with the conventional spatial domain methods. Design time and design accuracy measured by the normalized mean square error (NMSE) of excitation patterns are compared where

$$\mathrm{NMSE} = \frac{\sum_{\vec{r}}\left|I_{\text{Excitated Pattern}}(\vec{r}) - I_{\text{Target Pattern}}(\vec{r})\right|^2}{\sum_{\vec{r}}\left|I_{\text{Target Pattern}}(\vec{r})\right|^2}\times 100\% \tag{9}$$

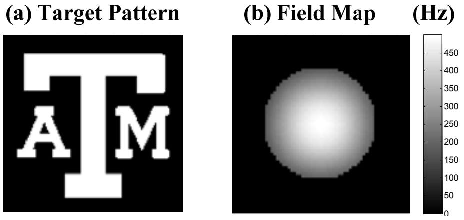

Fig. 1 Target pattern (a) used in the 2D pulse design and the B_0 off-resonance field map (b)

III. EXPERIMENTS

Four experiments are used to evaluate the proposed method. First, 2D complicate target pattern is used to test the design of spatial tailored pulses using both the convention design and the proposed design. Secondly, the stability of the pulse design to inaccurate B_1 map measurement is evaluated and compared between two designs. Then, as a traditional application of spatial tailored RF pulse, the B_0 off-resonance correction via RF pulse design is performed and compared. Finally, tests are made for 3D pulse design.

A. *Simulation I: 2D spatial tailored pulse design*

Spatial tailored pulses can excite pattern of arbitrary spatial shapes. In this simulation, three parallel pulse design experiments will be performed: (1) Conventional pulse design with a high resolution target pattern of size 64x64; (2) Conventional pulse design with a low resolution target pattern of size 32x32 and (3) sparse design with the high resolution target pattern but the system matrix is truncated to the same size as that in (2). All the other factors in three designs are kept the same. A linear transmit array with 8 channels is used for the parallel excitation and the B_1 maps of coils are obtained from simulation following to the Biot-Savart law. These maps are used as ground truth in both pulse design and Bloch simulation. In real implementations, the B1 map should be measured, e.g. using double-angle method. A spiral-in k-space trajectory is used for the pulse design. The low resolution target pattern is directly down-sampled by 2x2 from the high resolution target pattern. Also pulses are designed using the proposed method at different truncation factors R_s. Excitation patterns of the designed pulses are simulated using Bloch simulator on a grid of size 64x64. The high resolution target pattern is used as reference and the excitation errors from these three designs are measured.

B. *Simulation II: Stability to inaccurate B_1 measurement*

The stability of the proposed pulse design method is tested in case of inaccurate B_1 sensitivities. Pulses are designed with B_1 sensitivities corrupted by Gaussian noise. Complex Gaussian noise of different amplitude is added to the B_1 sensitivities to mimic the noise corrupted inaccurate measurement. Pulses are designed using the proposed method, the conventional method with the high resolution target pattern and the conventional method with the low resolution target pattern similar to simulation I. Excitation pattern of each designed pulse is simulated using the Bloch simulator on a grid of size 64x64.

C. Simulation III: B_0 Off-resonance correction

One common application of spatial tailored pulse is to correct the off-resonance effect caused by the main field inhomogeneity. An off-resonance field map is incorporated in the pulse design as in the conventional method to achieve the target excitation pattern. Also an additional phase map is modulated to the target pattern to compensate the dephasing caused by the off-resonance with a TE of 5 msec during data acquisition using gradient echo type of off-resonance sensitive sequences. Pulses are designed using the proposed method and the conventional method with the same size of system matrix. Amplitude and phase map of excitation pattern are compared. The Bloch simulator used in this test incorporates the specified off-resonance map by using the total effective field of $B_{eff}(r) = \left(\Delta f^2(r)/\gamma^2 + B_1^2(r)\right)^{1/2}$ instead of B_1 alone, where Δf is the off-resonance frequency.

D. Simulation IV: 3D spatial tailored pulse design for B_1 inhomogeneity correction

The proposed method is also evaluated for 3D pulse design. A real valued volume of size 20x20x30 cm^3 is used as the target pattern with resolution 32x32x32. A slice of 7cm thickness is set in the target volume and the in-slice amplitude is smoothly varying to compensate the mimic high-field inhomogeneity. Simulated coil B_1 map is generated using the model of an 16-element array of planar pair coils [14] wrapped around a cylinder surface. A five-spoke trajectory is used for the pulse design. The proposed design is implemented at different truncation ratios. Excitation patterns are obtained using the Bloch simulator and numerical excitation errors are evaluated and compared referring to the original target pattern.

IV. RESULTS

A. Simulation I:

The excitation patterns of pulses designed in the aforementioned three ways are shown in the first row in Fig.2 along with the corresponding error maps in the second row. The NMSE and the design time for each design are labeled. As expected, the design time using the conventional method with low resolution target pattern and the proposed method are about 4 times faster than using the conventional method with high resolution target pattern due to the one quarter reduced size of the system matrix. However, the excitation error of the proposed design is only slightly higher than the error of the conventional design with high resolution target pattern and only half of the error in (b).

B. Simulation II:

The Gaussian noise are added to the correct B_1 sensitivity with different amplitude. Excitation errors of pulses design with these inaccurate sensitivities are obtained and the excitation errors are evaluated. The design error curve is shown in Fig. 3. For all three designs, the excitation error goes down with the increasing SNR (accuracy) of the sensitivity used in pulse design. And the conventional design with high resolution target pattern shows lowest excitation error when the sensitivity is accurate. However, the trend of error curve shows that the proposed method is more stable to Gaussian noise type of inaccuracy in B_1 measurement than the conventional design.

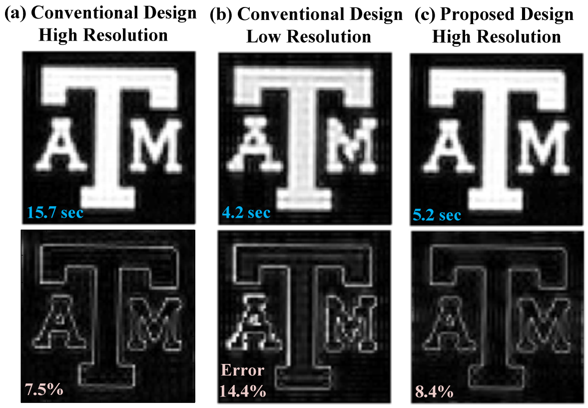
(a) Conventional Design High Resolution **(b) Conventional Design Low Resolution** **(c) Proposed Design High Resolution**

Fig. 2 Excitation patterns and the corresponding error maps of pulses using three different designs

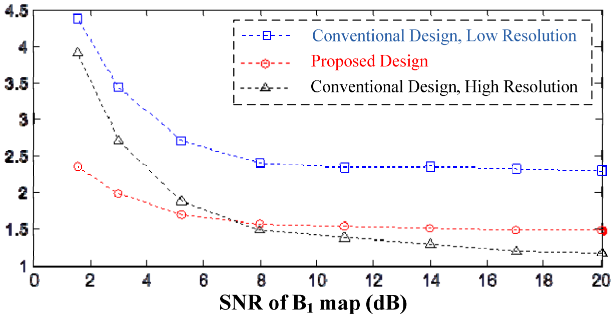

Fig. 3 Error curve of different design versus SNR of Gaussian noise corrupted B_1 map.

C. Simulation III:

The off-resonance field map with a maximum of 500Hz is shown in Fig.1 (b) and is used in the pulse design. The dephasing phase map caused by this off-resonance during the acquisition with 5 msec TE is shown in Fig. 4 (a) and is modulated to a real valued face pattern as the final complex valued target pattern. The pulse design with this setup is able to achieve excitation pattern with amplitude equal to the real valued pattern with additional phase that will counter the off-resonance dephasing in acquisition as well. The amplitude excitation patterns of the conventional design (b) and the proposed design (c) with the same system size are provided along with error maps in Fig. 4. The phase excitation patterns using two designs are shown in (d) and (e). The time cost for two designs are similar while the proposed method provided lower excitation error.

D. *Simulation IV:*

The excitation error curve (a) and design time curve (b) of the proposed sparse design versus R_s are shown in Fig.5. By comparing the results of $R_s = 0.1$ and $R_s = 1$, it can be found that the design time is reduced by 10 fold with a sacrifice of increased excitation error around 1%.

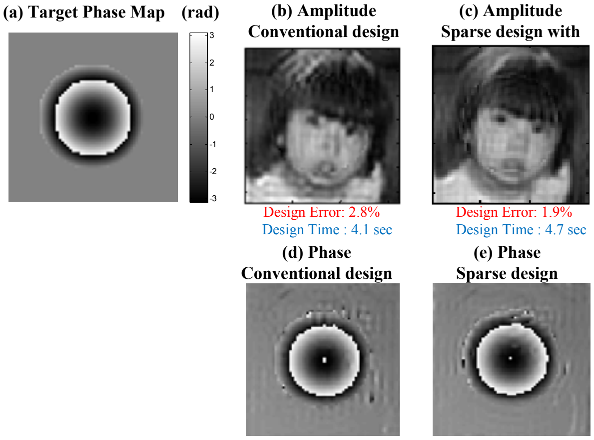

Fig. 4 The phase map of the target pattern is given in (a). The excitation patterns using the conventional design (b) and the sparse design (c) along with the corresponding excitation phase maps (d-e).

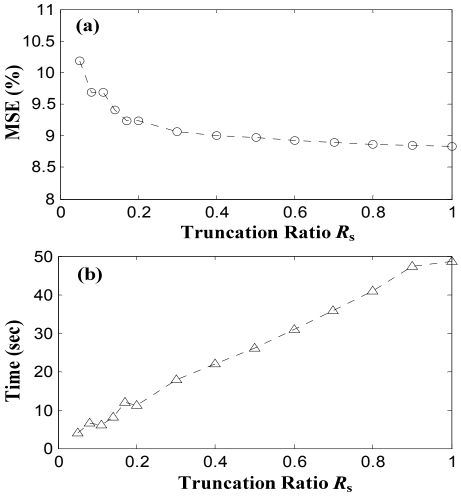

Fig.5 Excitation error curve (a) and design time curve (b) of the proposed design method at different truncation ratios.

V. CONCLUSIONS AND DISCUSSION

In this work, a novel fast algorithm was proposed for designing parallel excitation pulses in MRI. The new method explores the sparsity of the excitation pattern in the pulse design system equations where the system size can be reduced after a sparse transform and truncation. Simulation results have shown that the proposed design method can achieve up to an order of magnitude faster design speed than the conventional design methods while maintaining similar excitation accuracy in 3D applications.

This fast design method can be easily incorporated into existing pulse designs that use the spatial domain method. A fast Fourier transform (FFT) is used to quickly transform the equations. The transform on the right side of the equations can be pre-calculated as long as the k-space trajectory is given. Other sparse transforms such as those based on singular value decomposition (SVD) can also be used. However, SDV must be adaptive to target patterns and it is not as efficient as FFT. This pulse method is suitable for tip angle up to 90 degrees, similar to that of the conventional method. For real large tip angle pulse designs, our method can be incorporated in exiting methods that employ the similar system equations. Our future work includes using parallel computation hardware such as using graphic processing unit (GPU) [15] to further accelerate this pulse design algorithm.

ACKNOWLEDGMENT

This work was supported in part by the National Science Foundation under award number 0748180. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect those of the National Science Foundation.

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