

Study of Magnetization Evolution by Using Composite Spin-lock Pulses for $T_{1\rho}$ Imaging

Yujia LI, Feng ZHAO, Yi-Xiang WANG, Anil T AHUJA, Jing YUAN

Abstract— B_0 and B_1 field inhomogeneities may generate banding-like artifacts in $T_{1\rho}$ -weighted images and hence result in errors of $T_{1\rho}$ quantification. Several types of composite spin-lock pulses have been proposed to alleviate such artifacts. In this study, magnetization evolution with $T_{1\rho}$ and $T_{2\rho}$ relaxation by using these composite spin-lock pulses are theoretically derived. The effectiveness and limitation of each spin-lock pulse are explicitly illustrated in mathematical forms and phantom $T_{1\rho}$ -weighted images acquired by using each spin-lock pulse are presented. This study also provides a theoretical framework for $T_{1\rho}$ quantification from $T_{1\rho}$ -weighted images even with B_0 and B_1 inhomogeneity artifacts.

I. INTRODUCTION

$T_{1\rho}$ is the time constant of the transverse magnetization decay given the application of a spin-lock pulse, which is aligned with the net magnetization vector. $T_{1\rho}$ relaxation is able to create tissue contrast different from the conventional MRI contrasts based on T_1 and T_2 relaxation. $T_{1\rho}$ relaxation contrast is sensitive to low frequency motional processes, and is potential for many clinical applications [1-6].

Conventionally, a three-pulse cluster including tip-down, spin-lock and tip-up pulse is used to generate $T_{1\rho}$ -weighted images. However, in the presence of inhomogeneous B_0 and B_1 field, the net magnetization may not align with the spin-lock field any longer, and accordingly result in a complicated magnetization evolution and hence banding-like artifact on $T_{1\rho}$ -weighted images. Composite spin-lock pulses have been proposed to reduce the artifacts. In this paper, we present a review of several types of composite spin-lock pulses and the magnetization evolutions with $T_{1\rho}$ and $T_{2\rho}$ relaxation by using each spin-lock pulse are theoretically derived. The relationship between the magnetization and various factors of spin-lock time, spin-lock frequencies, B_0 and B_1 inhomogeneities are illustrated.

II. THEORY

A. Conventional spin-lock pulse: 90_x - TSL_y - 90_x

A conventional spin-lock pulse (Fig. 1) consists of three pulses: an initial hard pulse along x direction to tip-down the net magnetization to the transverse plane, a hard spin-lock pulse with duration of TSL to excite $T_{1\rho}$ relaxation, and a final

hard pulse along $-x$ direction to tip-up the net magnetization from the transverse plane to the longitudinal plane. Principally, the magnetization aligned with the spin-lock field decays with $T_{1\rho}$ and the magnetization perpendicular to the spin-lock field decays with $T_{2\rho}$.

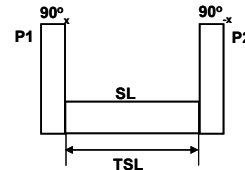


Figure 1. The schematic of conventional spin-lock pulse.

The magnetization evolution during the application of a pulse can be traced by using the Block Equation. For description, an RF pulse is represented in the form of matrix notation $R_\varphi(\Phi)$, where R denotes a rotation matrix, φ is the pulse orientation and Φ is the pulse flip angle. β is the flip angle of tip-down/tip-up pulse and ∂ is the flip angle of each SL segment. Given spin-lock frequency FSL, $\partial = 2\pi \cdot \text{FSL} \cdot \text{TSL}$. E_ρ is a matrix to describe $T_{1\rho}$ and $T_{2\rho}$ relaxation. The basic rotation matrices that rotate spin vectors about the x, y, or z axis by an angle Φ in three dimensions, and E_ρ are expressed as in Equation (1),

$$R_x(\Phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & \sin\Phi \\ 0 & -\sin\Phi & \cos\Phi \end{bmatrix}; R_y(\Phi) = \begin{bmatrix} \cos\Phi & 0 & -\sin\Phi \\ 0 & 1 & 0 \\ \sin\Phi & 0 & \cos\Phi \end{bmatrix};$$

$$R_z(\Phi) = \begin{bmatrix} \cos\Phi & \sin\Phi & 0 \\ -\sin\Phi & \cos\Phi & 0 \\ 0 & 0 & 1 \end{bmatrix}; E_\rho = \begin{bmatrix} e^{-TSL/2T_{2\rho}} & 0 & 0 \\ 0 & e^{-TSL/2T_{2\rho}} & 0 \\ 0 & 0 & e^{-TSL/2T_{1\rho}} \end{bmatrix} \quad (1)$$

The relaxation during P1 and P2 is usually negligible due to their much shorter pulse duration than the TSL. The magnetization evolution is expressed as in Equation (2),

$$M(t) = R_{-x}(\beta) \cdot R_y(\partial) E_\rho \cdot R_x(\beta) \cdot M(t_0)$$

$$\Rightarrow M_z = M_0 \cdot (\sin^2 \beta \cdot e^{-TSL/T_{1\rho}} + \cos^2 \beta \cos \partial \cdot e^{-TSL/2T_{2\rho}}) \quad (2)$$

Where $M(t_0) = [0 \ 0 \ M_0]^T$. When the tip-down/tip-up pulse has a perfect flip angle of 90° , the resulting longitudinal magnetization is simply related to $T_{1\rho}$ relaxation as the normal mono-exponential relaxation model shown in Equation (3),

$$M_z = M_0 \cdot e^{-TSL/T_{1\rho}} \quad (3)$$

Given imperfect tip-down/tip-up flip angles, a magnetization precession around the spin-lock field occurs (Fig. 2). A flip angle of ∂ is formed as the magnetization precesses from point 1 to point 2 during TSL. As a result, this pulse is B_1 sensitive as ∂ is related to B_1 .

* This work is supported by HK ITF grant ITS/021/10 and RGC grant SEG_CUHK02.

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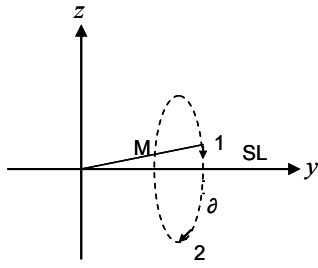


Figure 2. Magnetization precession of conventional spin-lock pulse.

B. Rotary-echo spin-lock pulse: $90_x\text{-TSL}/2_y\text{-TSL}/2_y\text{-}90_x$

This pulse divides a spin-lock pulse into two segments which have opposite phase shifts (Fig. 3) and tends to remove the artifact resulted from the flip angle of SL pulse ∂ [7]. During SL1, the magnetization M precesses from point 1 to point 2 with flip angle of ∂ and then precesses from point 2 to point 3 during SL2 (Fig. 4). Given the same duration of SL1 and SL2, point 3 is identical as point 1. The resulting longitudinal magnetization will be irrelevant with ∂ . The magnetization evolution is expressed as in Equation (4),

$$M(t) = R_{-x}(\beta) \cdot R_{-y}(\partial) E_{\rho} \cdot R_y(\partial) E_{\rho} \cdot R_x(\beta) \cdot M(t_0)$$

$$\Rightarrow M_z = M_0 \cdot (\sin^2 \beta \cdot e^{-TSL/T_{1\rho}} + \cos^2 \beta \cdot e^{-TSL/T_{2\rho}}) \quad (4)$$

When tip-down/tip-up flip angle equals to 90° , the resulting longitudinal magnetization is simplified as Equation (3). Note that image could still be contaminated by $T_{2\rho}$ contrast in the presence of B_1 field inhomogeneity according to Equation (4) when $\beta \neq 90^\circ$.

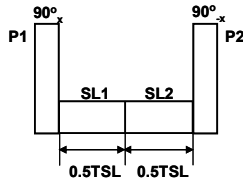


Figure 3. The schematic of rotary-echo spin-lock pulse.

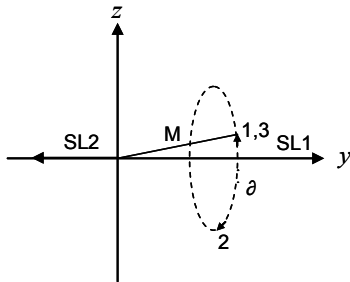


Figure 4. Magnetization precession of rotary-echo spin-lock pulse.

Equation (4) is derived with the assumption of homogeneous B_0 . When there is an offset of ΔB_0 , the effective spin-lock field will change to z' and z'' given the application of SL1 and SL2 respectively (Fig. 5), where $\theta = \tan^{-1}(\omega_1/\Delta\omega) = \tan^{-1}(B_1/\Delta B_0)$ is the angle from the effective spin-lock field to z-axis. M precesses initially from point 1 to point 2 during SL1 and then precesses from point 2 to point 3

during SL2. The magnetization evolution becomes much more complicated as shown in Equation (5).

$$M(t) = R_{-x}(\beta) \cdot R_z(\partial) E_{\rho} \cdot R_z(\partial) E_{\rho} \cdot R_x(\beta) \cdot M(t_0)$$

$$= R_{-x}(\beta) \cdot R_x(-\theta) R_z(\partial) E_{\rho} R_x(\theta) \cdot R_x(\theta) R_z(\partial) E_{\rho} R_x(-\theta) \cdot R_x(\beta) \cdot M(t_0)$$

$$\Rightarrow M_z = M_0 \cdot [\cos 2\theta (\cos^2 \beta - \sin^2 \theta) \cdot e^{-TSL/T_{1\rho}} + (\cos^2 \beta - \cos^2 \theta) (\sin^2 \partial - \cos^2 \partial \cos 2\theta) \cdot e^{-TSL/T_{2\rho}} + \cos \partial \sin^2 2\theta \cdot e^{-0.5 \cdot TSL/T_{1\rho}} \cdot e^{-0.5 \cdot TSL/T_{2\rho}}] \quad (5)$$

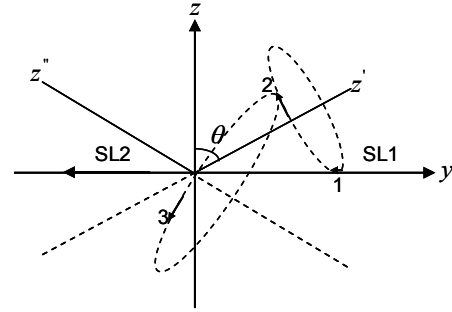


Figure 5. Magnetization precession of rotary-echo spin-lock pulse in inhomogeneous B_0 field.

If $\omega_1 \gg \Delta\omega$, $\theta = 90^\circ$, Equation (5) is identical as Equation (4). If $\omega_1 \ll \Delta\omega$, $\theta = 0^\circ$, Equation (5) is simplified as shown in Equation (6),

$$M_z = M_0 \cdot [\cos^2 \beta \cdot e^{-TSL/T_{1\rho}} + \sin^2 \beta \cos(2\partial) \cdot e^{-TSL/T_{2\rho}}] \quad (6)$$

When $\beta = 90^\circ$, trace of $T_{1\rho}$ relaxation will be lost and only $T_{2\rho}$ relaxation can be traced by Equation (6).

If $\omega_1 \sim \Delta\omega$ and $\beta = 90^\circ$, Equation (5) is simplified as shown in Equation (7) and artifacts will be serious [8],

$$M_z = M_0 \cdot [-\cos 2\theta \sin^2 \theta \cdot e^{-TSL/T_{1\rho}} - \cos^2 \theta (\sin^2 \partial - \cos^2 \partial \cos 2\theta) \cdot e^{-TSL/T_{2\rho}} + \cos \partial \sin^2 2\theta \cdot e^{-0.5 \cdot TSL/T_{1\rho}} \cdot e^{-0.5 \cdot TSL/T_{2\rho}}] \quad (7)$$

C. B_1 and B_0 insensitive composite spin-lock pulse:

$$90_x\text{-TSL}/2_y\text{-}180_y\text{-TSL}/2_y\text{-}90_x$$

This spin-lock pulse inserts a refocusing pulse between rotary-echo spin-lock pulse segments (Fig. 6) so as to cancel the flip angle of spin-lock pulse in imperfect B_0 field [9]. During SL1, M precesses from point 1 to point 2 and then is reversed by the refocusing pulse, precessing 180° around y-axis to point 3. During SL2, M precesses from point 3 to point 4 which is identical as point 1. Hence, the resulting magnetization is irrelevant with ∂ . The magnetization evolution is expressed as in Equation (8),

$$\begin{aligned}
M(t) &= R_x(\beta) \cdot R_z(\partial) E_\rho \cdot R_y(2\beta) \cdot R_z(\partial) E_\rho \cdot R_x(\beta) \cdot M(t_0) \\
&= R_x(\beta) \cdot R_x(-\theta) R_z(\partial) E_\rho R_x(\theta) \cdot R_y(2\beta) \cdot R_x(\theta) R_z(\partial) E_\rho R_x(-\theta) \cdot R_x(\beta) \cdot M(t_0) \\
\Rightarrow M_z &= M_0 \cdot [e^{-TSL/T_{1\rho}} \cdot (\cos^2 \beta - \sin^2 \theta) (\cos^2 \partial \cos 2\beta - \sin^2 \theta) \\
&\quad + e^{-TSL/T_{2\rho}} \cdot (\cos^2 \beta - \cos^2 \theta) \\
&\quad \cdot (-\cos^2 \partial \cos^2 \theta + \cos(2\beta) \sin^2 \partial + \cos(2\beta) \cos^2 \partial \sin^2 \theta) \\
&\quad + \frac{1}{2} e^{-0.5TSL/T_{1\rho}} e^{-0.5TSL/T_{2\rho}} \cdot (1 + \cos(2\beta)) \\
&\quad \cdot (\cos \partial \sin^2(2\theta) + 2 \sin \partial \cos \theta - 2 \cos(2\beta) \sin \partial \cos \theta)]
\end{aligned} \tag{8}$$

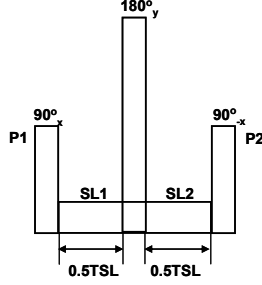


Figure 6. The schematic of B_1 and B_0 insensitive composite spin-lock pulse.

If $\beta=90^\circ$, Equation (8) is simplified as Equation (9),

$$M_z = M_0 (\sin^2 \theta \cdot e^{-TSL/T_{1\rho}} + \cos^2 \theta \cdot e^{-TSL/T_{2\rho}}) \tag{9}$$

The resulting longitudinal magnetization will be irrelevant with ∂ . If $\omega_1 \gg \Delta\omega$, $\theta=90^\circ$, Equation (9) is simplified as Equation (3). If $\omega_1 \ll \Delta\omega$, $\theta=0^\circ$, Equation (9) is simplified as Equation (10) which is a mono-exponential $T_{2\rho}$ relaxation model,

$$M_z = M_0 \cdot e^{-TSL/T_{2\rho}} \tag{10}$$

If $\beta \neq 90^\circ$, provided $\omega_1 \gg \Delta\omega$, $\theta=90^\circ$ and $\omega_1 \ll \Delta\omega$, $\theta=0^\circ$, Equation (10) is simplified as Equation (11) and (12) respectively and artifacts will occur.

$$M_z = M_0 \cdot [e^{-TSL/T_{1\rho}} \cdot \sin^2 \beta + e^{-TSL/T_{2\rho}} \cdot \cos^2 \beta \cos(2\beta)] \tag{11}$$

$$\begin{aligned}
M_z &= M_0 \cdot [e^{-TSL/T_{1\rho}} \cdot \cos^2 \beta \cos(2\beta) \\
&\quad + e^{-TSL/T_{2\rho}} \cdot \sin^2 \beta (\cos^2 \partial - \sin^2 \partial \cos(2\beta)) \\
&\quad + e^{-0.5TSL/T_{1\rho}} e^{-0.5TSL/T_{2\rho}} \cdot \sin^2(2\beta) \sin \partial]
\end{aligned} \tag{12}$$

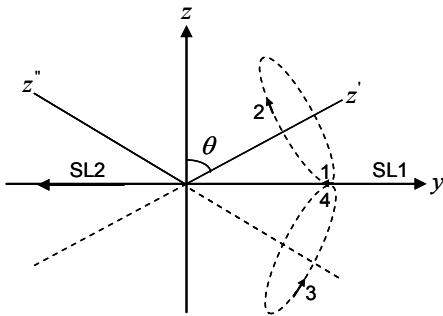


Figure 7. Magnetization precession of B_1 and B_0 insensitive composite spin-lock pulse.

D. Revised B_1 and B_0 insensitive composite spin-lock pulse: 90_x - $TSL/2_y$ - 180_y - $TSL/2_y$ - 90_x

As shown in Section II.C, B_1 and B_0 insensitive composite spin-lock pulse loses its effectiveness when β is not 90° . The revised composite spin-lock pulse is robust for imperfect tip-down/tip-up flip angle [10]. The last tip-up pulse is modified to be aligned with x-axis (Fig. 8) and tips the magnetization to $-z$ -axis. Given imperfect tip-down/tip-up flip angle, M precesses around z' from point 1 to point 2 during SL1 and then precesses from point 2 to point 3 with the function of refocusing pulse. During SL2, M precesses from point 3 to point 4 around z'' . Point 4 and point 1 should be symmetric about y-axis. Lastly, with the tip-up pulse, M will perfectly return to $-z$ -axis. The process can remove the contaminations from imperfect tip-down/tip-up flip angle, while 180° flip angle of refocusing pulse is still necessary. Supposing δ is the flip angle of refocusing pulse, the magnetization evolution is expressed as in Equation (13),

$$\begin{aligned}
M(t) &= R_x(\beta) \cdot R_z(\partial) E_\rho \cdot R_y(\delta) \cdot R_z(\partial) E_\rho \cdot R_x(\beta) \cdot M(t_0) \\
&= R_x(\beta) \cdot R_x(-\theta) R_z(\partial) E_\rho R_x(\theta) \cdot R_y(\delta) \cdot R_x(\theta) R_z(\partial) E_\rho R_x(-\theta) \cdot R_x(\beta) \cdot M(t_0) \\
\Rightarrow M_z &= M_0 \cdot [e^{-TSL/T_{1\rho}} \cdot \cos^2(\beta - \theta) (\cos \delta \cos^2 \theta - \sin^2 \theta) \\
&\quad + e^{-TSL/T_{2\rho}} \cdot \sin^2(\beta - \theta) (-\cos^2 \partial \cos^2 \theta + \cos \delta \sin^2 \partial + \cos \delta \cos^2 \partial \sin^2 \theta) \\
&\quad + \frac{1}{2} e^{-0.5TSL/T_{1\rho}} e^{-0.5TSL/T_{2\rho}} \cdot (1 + \cos \delta) \cos \delta \cos \partial \sin(2\theta) (\sin(2\theta) - \cos(2\theta))]
\end{aligned} \tag{13}$$

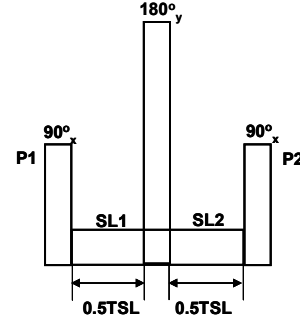


Figure 8. The schematic of revised B_1 and B_0 insensitive composite spin-lock pulse.

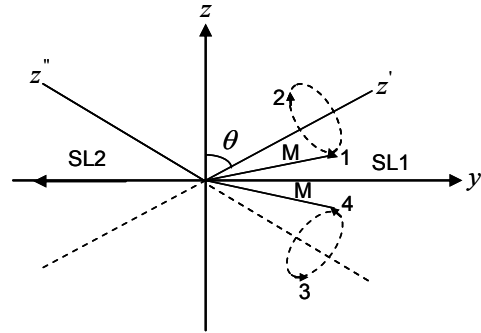


Figure 9. Magnetization precession of revised B_1 and B_0 insensitive composite spin-lock pulse.

If $\delta=180^\circ$, Equation (13) is simplified as Equation (14),

$$M_z = -M_0 \cdot [e^{-TSL/T_{1\rho}} \cdot \cos^2(\beta - \theta) + e^{-TSL/T_{2\rho}} \cdot \sin^2(\beta - \theta)] \tag{14}$$

M returns to $-z$ -axis, with the contamination from $T_{2\rho}$ relaxation.

If $\delta \neq 180^\circ$, provided $\omega_1 \gg \Delta\omega$, $\theta = 90^\circ$ and $\omega_1 \ll \Delta\omega$, $\theta = 0^\circ$, Equation (13) is simplified as Equation (15) and Equation (16) respectively and artifacts will occur.

$$M_z = M_0(-e^{-TSL/T_{1\rho}} \cdot \sin^2 \beta + e^{-TSL/T_{2\rho}} \cdot \cos^2 \beta \cos \delta) \quad (15)$$

$$M_z = M_0[e^{-TSL/T_{1\rho}} \cdot \cos^2 \beta \cos \delta - e^{-TSL/T_{2\rho}} \cdot \sin^2 \beta (\cos^2 \delta - \cos \delta \sin^2 \delta)] \quad (16)$$

III. IMAGING EXPERIMENT

The rotary-echo spin-lock pulse (SL pulse B), B_1 and B_0 insensitive composite spin-lock pulse (SL pulse C) and revised B_1 and B_0 insensitive composite spin-lock pulse (SL pulse D) were experimentally tested. Each spin-lock pulse was implemented with Turbo spin-echo (TSE) sequence on a 3T MRI scanner (Philips Medical Systems, Best, The Netherlands). A homogeneous agar phantom was imaged. Body coil was used for excitation so homogeneous B_1 field was assumed. A birdcage head coil was used as a receiver. The parameters were set as following: FSL = 50, 250 and 500Hz; TSL = 20ms; TR/TE = 4000/17ms; image matrix size = 144×144 ; FOV = 20cm^2 ; slice thickness = 8mm; echo train length = 6; BW = 217Hz/pixel. An interval of 5000ms was inserted after each shot to allow fully recovery of longitudinal magnetization. Fig. 10 presents the B_0 map acquired by the normal dual-TE method ($\Delta\text{TE} = 1\text{ms}$) imaging results. SL pulse C and D are obviously less sensitive to B_0 inhomogeneity.

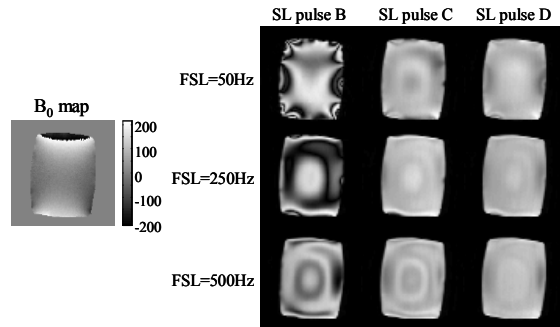


Figure 10. $T_{1\rho}$ weighted images obtained by using SL pulse B, C, D respectively.

IV. DISCUSSION

As identified from the magnetization evolution and imaging results of the investigated spin-lock pulses, imperfect flip angle of tip-down/tip-up/refocus pulse due to inhomogeneous B_1 field will complicate the orientation of the net magnetization. The deviation of the effective spin-lock field from the nominal spin-lock field due to inhomogeneous B_0 further complicates the magnetization evolution and leads to signal null (banding artifact) and/or signal contamination from $T_{2\rho}$ relaxation.

Although composite spin-lock pulses can be used to reduce the spin-lock artifacts associated with B_0 and B_1 field imperfections to different extents, complete elimination of such artifacts is still technically challenging in practice.

Improvement of $T_{1\rho}$ quantification can be realized either by reducing artifacts from $T_{1\rho}$ -weighted images and then following the simple mono-exponential relaxation model for fitting, or by fitting the signal intensity of $T_{1\rho}$ -weighted images even with artifacts to a more complicated magnetization model which is able to quantify $T_{1\rho}$ and $T_{2\rho}$ relaxation even in the presence of field imperfections, as demonstrated in the literature [8]. The former one works at an image acquisition stage; the latter one works at an image post-processing stage. Our study provides a theoretical framework beneficial for both.

This study has some limitations. Phase cycling technique [11] was not analyzed in this study. The theoretical derivation does not account for the transient effect during the spin-lock time. The transient effect may partially explain the incomplete banding compensation by composite spin-lock pulses, in particular for tissues with short T_1 relaxation time. Besides, the phantom imaging demonstration in this study is rather qualitative without taking B_1 inhomogeneities into account. Quantitative evaluation and *in vivo* validation should be further performed in the future studies.

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