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Smoothed Random-Like Trajectory for Compressed Sensing MRI

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Abstract— **In this paper, we explores a rapid imaging method based on a proposed random-like trajectory for compressed sensing (CS) which requires the sampling trajectory should satisfy the Restricted Isometry Property (RIP) condition. In the existing CS literature, the attentions are on randomly sampling points on the conventional trajectories. However, the proposed trajectory is a random-like trajectory generated based on the High Order Chirp (HOC) sequences, which use the Traveling Salesman Problem (TSP) solver to choose a "short" trajectory and design a time optimal gradient waveforms to satisfy the gradient amplitude and slew rate limitation. The MR physical feasibility of the proposed method is verified by the Bloch simulation, and the simulations show that the proposed method can reduce artifacts than conventional Spiral trajectory under the CS framework.**

I. INTRODUCTION

Recently, compressed sensing (CS) [1,2] has been studied as an alternative sampling theory in many clinic applications and successfully applied to accelerate conventional magnetic resonance imaging (MRI) [3,4]. This theory allows sparse or compressible signals to be sampled at a rate that is close to their intrinsic information rate and well below their Nyquist rate, and still allows the signal to be recovered exactly from randomly under-sampled frequency measurements by a non-linear procedure. The sampling trajectories, such as, Radial and Spiral, can directly be reconstructed by the CS framework [5,6]. Some researches [5-8] found that it can improve reconstructions to add random perturbed factors into the conventional sampling trajectories. Their results show that using randomly perturbed *k*-space trajectories enables more sparsely sampled image reconstruction with higher quality and fewer artifacts compared to using non-randomly sampled trajectories in CS MRI.

Theoretically, the CS framework requires more randomly sampling in frequency domain to satisfy the RIP condition [1]. However, random sampling in frequency domain is impractical for MRI hardware. Therefore, random sampling trajectories are studied to reconstruct better using the CS framework. A. Curtis et al. [9] have researched random volumetric MRI trajectories for CG-SENSE reconstruction, but he mentioned that it could be recovered by CS. Their simulations show the random volumetric trajectories can get good detail and acceptable noise for large-volume imaging with 32 coils. M. Seeger et al. [10] tried to optimize MRI trajectories by Bayesian experimental design. Their results show that it can improve the recon results through optimizing the existing trajectories. R. Willett et. al. [11] made the proposed trajectories to satisfy the RIP condition [1]. Their proposed trajectories approximate to the space-filling Hilbert curve, but they possibly have very long path-lengths.

To directly use the CS framework to recover, we propose a rapid imaging scheme which can generate a random-like trajectory that obeys the RIP condition [1]. Firstly, the random-like trajectory is created based on the High Order Chirp (HOC) sequences [12]. Secondly, the Traveling Salesman Problem (TSP) solver is used to choose a "short" trajectory. Here, we use the Simulated Annealing (SA) algorithm [13, 14] as the TSP solver. Thirdly, we use a fast algorithm for designing a time optimal gradient waveforms to mainly satisfy the gradient amplitude and slew rate limitation [15]. Fourthly, Non-uniform Fast Fourier Transformation (NUFFT) [16] and Nonlinear Conjugate Gradient (NCG) are used to reconstruct under the CS framework. The MR physical feasibility of the proposed random-like trajectory is verified by the Bloch simulation, and the simulations show the proposed method can reduce artifacts than conventional Spiral under the CS framework.

II. BACKGROUND

A. Compressed Sensing (CS)

In conventional MRI, the CS application is made possible by the facts [3] that (1) most MR images are compressible by certain transforms and (2) the desired image is Fourier encoded in the measurement (so called k-space) which allows incoherent sampling. If image f is given by [3], the image is recovered by solving a constrained convex minimization problem,

$$\arg\min_{\mathbf{f}}\{\|\mathbf{b}-\mathbf{F}_u\mathbf{f}\|_2^2+\lambda_1\|\mathbf{W}\mathbf{f}\|_1+\lambda_2\,\mathrm{TV}(\mathbf{f})\} \qquad (1)$$

where b is the measured *k*-space data; $\mathbf{F}_u$ is the random subset of the rows of the Fourier encoding matrix; W is the sparsifying transform matrix, such as wavelet, DCT, etc.; $\mathrm{TV}(\cdot)$ expresses total variation; λ_1 and λ_2 are constant regularization parameters.

B. High Order Chirp (HOC) Sequence

The High Order Chirp (HOC) sequences are mentioned by V. Saligrama in Ref. [12], who proposes a family of

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discrete sequences have the special "random-like" uniformly decaying auto-correlation properties. His results show a type of High-Order-Chirps (HOC) sequences satisfied RIP property under some conditions, and he proves the spatial deterministic signals can be recovered by the CS theory. For example, one of HOC sequences can mathematically be expressed as [12],

$$u(p) = e^{i \cdot 2\alpha\pi \cdot p^3}. \tag{2}$$

Here, p are the sampling points; α is the irrational coefficient, such as, golden ratio, π , e , etc..

C. Traveling Salesman Problem (TSP)

Traveling Salesman Problem (TSP) is the problem of a salesperson trying to visit *N* cities with as little time on the road as possible. As a touchstone for many general heuristics, the TSP is concerned with find the "shortest" path connecting N points, so this is known to be an NP-Hard problem, the exact solutions require $O(N!)$ computation time [11]. But, the approximate solutions can be computed much more rapidly by some algorithms , such as, genetic algorithms (GA) [17], simulated annealing (SA) [13,14], Tabu search [18], ant colony optimization (ACO) [19], etc., who are theoretically a little longer than the shortest possible length with high probability. Here, we suppose the sampling points on the proposed trajectory are the cities of the TSP, and use the SA algorithm [13, 14] as the TSP solver, whose complexity computation is well known as $O(N^2)$. These approximate solutions would certainly satisfy the RIP condition and ensure the success of sparse recovery algorithms [11].

III. PROPOSED METHOD

The proposed method has total four steps: firstly, the random-like trajectory is generated; secondly, an approximation "short" trajectory is solved; thirdly, the time optimal gradient waveforms are designed; fourthly, final image results are recovered under the CS framework.

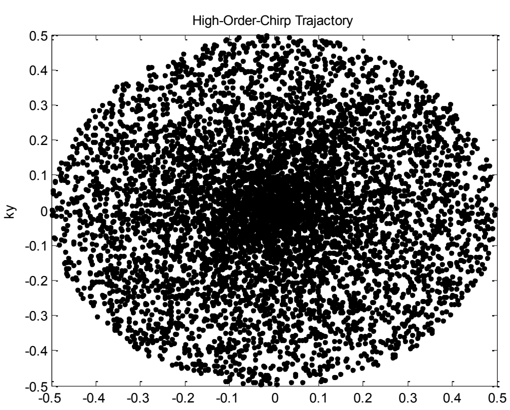

Fig.1 the proposed random-like trajectory smoothed from HOC sequences.

A. Random-Like Trajectory

The conventional Spiral trajectory that is expressed as,

$$u(p) = \beta \cdot s \cdot e^{i \cdot 2\alpha\pi \cdot p}. \tag{3}$$

Based on Eq. (2) and (3), we explore a type of random-like trajectory is,

$$u(p) = \beta \cdot s \cdot e^{i \cdot 2\alpha\pi \cdot p^m}. \tag{4}$$

where, β is the decay coefficient; m is larger than 2. For example，if $\alpha = \left(1+\sqrt{5}\right)/2$, $\beta = 1/1000$, $m = 3$ and p increases from 0 to 1000, the proposed random-like trajectory is illustrated as seen as the Figure 1. But, this trajectory cannot satisfy the maximum gradient and slew rate limitations, to be implemented by the pulse sequences on MRI scanners.

B. Simulated Annealing Algorithm

In the SA method [13, 14], each points of the search space is analogous to a temperature state of a thermo-dynamics system, and the function to be minimized is analogous to the internal energy of the thermodynamics system in that temperature state. To incorporate temperature parameters into the minimization procedure, explore the parameter space at the high temperature and restrict exploration at lower temperatures. The goal is to bring the thermodynamics system, from an arbitrary initial temperature state, to a temperature state with the minimum possible energy. Although, the SA algorithm cannot get the optimal solution for the TSP, but it can fast compute an approximation solution. To short the path length of the proposed trajectory, the SA algorithm is applied to minimize the total arc length between the sampling points. Actually, the SA algorithm rearranges the order of the sampling points on the random-like trajectory.

C. Time Optimal Gradient Waveforms

An algorithm based on optimal control theory [15] is applied to compute the time-optimal gradient waveform of the paths optimized by the SA algorithm. Here, we design a gradient waveform as a function of time equivalent to a time parameterization function of time $p = s(t)$ in the arc-length parameterization, such that,

$$s(0) = 0, s(T) = L$$

where L is the length of the path; T is the traversal time. The time trajectory in k-space is given by the composite function $u(p) = u(s(t))$. The time optimized problem can be formulated in the arc-length parameterization as,

$$\underset{s(t)}{\arg\min} \quad T \tag{5}$$

$$s.t. \quad s(0) = 0, \dot{s}(0) = 0, s(t) = L$$

$$\left|\ddot{s}(t)\right| \le \left[\gamma^2 S_{\max}^2 - \kappa^2\left(s(t)\right) \cdot \dot{s}(t)^4\right]^{1/2} \quad t \in [0, T]$$

$$s(t) \le \min\left\{\gamma G_{\max} \quad \sqrt{\frac{\gamma S_{\max}}{\kappa\left(s(t)\right)}}\right\} \quad t \in [0, T]$$

where γ is the gyro-magnetic ratio; $s(t)$ is the variable, which is a time function; $\dot{s}(t)$ is the 1-order time derivative of $s(t)$; $\ddot{s}(t)$ is the 2-order time derivative of $s(t)$; $G_{\max}$ is the maximum gradient of the system ; $S_{\max}$ is the maximum slew rate of the system; $\kappa(s(t))$ is the magnitude of the acceleration is the curvature of the curve $u(s(t))$. If we know the time optimal solution $s^*(t)$ of problem (5), we can find the time gradient waveform solution is,

$$g^*(t) = \gamma^{-1} \cdot \frac{du(s^*(t))}{dt}. \tag{6}$$

To solve the optimal problem (5), a 4-order Runge-Kutte method [20] and the cubic-spline interpolation method for interpolating the curve are applied when needed [15]. Figure 2 illustrates the smoothed proposed random-like trajectory, whose parameters are as same as Fig. 1.

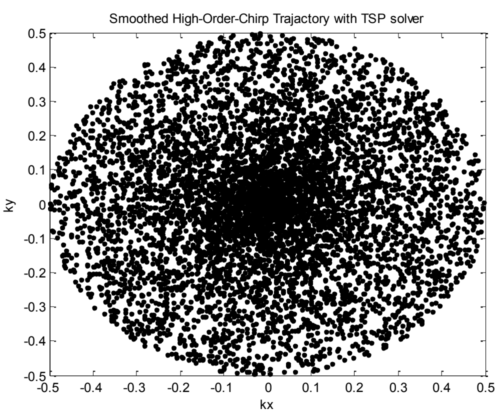

Fig.2 the proposed random-like trajectory smoothed by time optimization

D. Image Reconstruction

The smoothed random-like sampling points are one type of non-Cartesian sampling, which can be recovered by NUFFT and NCG under the CS framework.

The image is recovered by solving a minimization problem as similar as the problem of the Eq. (1),

$$\arg\min_{\mathbf{f}} \{\|\mathbf{b} - \mathbf{F}_{NUFFT}\mathbf{f}\|_2^2 + \lambda_1 \|\mathbf{Wf}\|_1 + \lambda_2 \,\mathrm{TV}(\mathbf{f})\} \tag{7}$$

where f is the desired image; b is the measured non-Cartesian *k*-space data; $\mathbf{F}_{NUFFT}$ is the NUFFT encoding matrix; W is the wavelet transform matrix; $\mathrm{TV}(\cdot)$ is total variation; λ_1 and λ_2 are constant regularization parameters.

IV. RESULTS

We conducted three simulations to demonstrate the performance of the proposed random-like trajectory method. The results were compared with those of the conventional Spiral Trajectories in the context of CS.

A. Bloch Simulation

In the Bloch simulation [21], Shepp-Logan phantom was used to generate the simulated data. Generally, we assumed gradients capable of 40 mT/m, slew-rate of 200 mT/m/ms, and a sampling rate of 4 µs. Figure 3 compares the images recovered by gridding and CS algorithms between Spiral and proposed trajectory. The size of the simulated image is 256×256. The total 1024 points are separately sampled by the Spiral and proposed trajectories (m=3), as seen as Fig. 3, 4 and 5. The proposed method for Shepp-Logan phantom is seen to as similar as Spiral under the Bloch simulation.

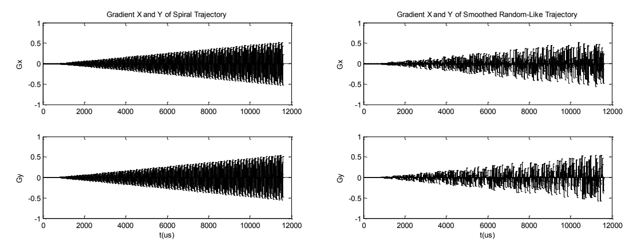

Fig. 3 comparison of gradient x and y between Spiral (left) and proposed trajectory (right)

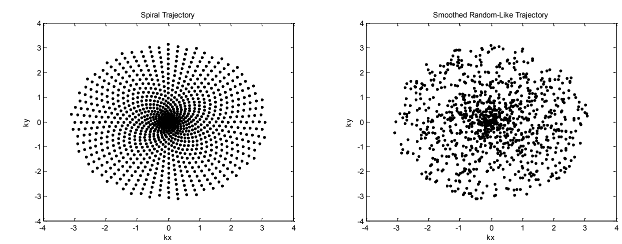

Fig. 4 comparison of sampling points in k-space between Spiral (left) and proposed trajectory (right)

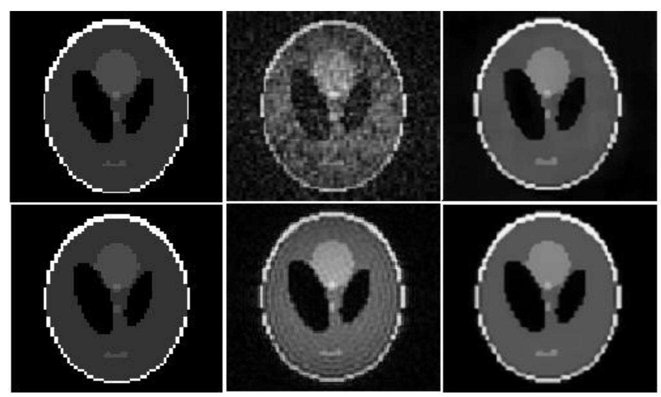

Fig. 5 comparison of the reference (left), zf-w/dc (middle) and CS recon (right) between Spiral (1024 points, second row) and the proposed trajectory (1024 points, first row)

B. Phantom

The phantom dataset was sampled by the spin echo sequence (TE/TR: 40/1000ms; RBW: 8.4 kHz; Flip angle: FOV: 220mm^2). The data size is 256×256. The 7010 points are sampled on the Spiral and proposed trajectory (m=2), as seen as Fig.6. Figure 7 compares the images reconstructed by reference (left), zf-w/dc (middle) and CS recon (right) images between Spiral (1st row) and proposed trajectory (2nd row). The proposed method is seen to reduce the latticed artifacts, comparing the CS recon of Spiral trajectory.

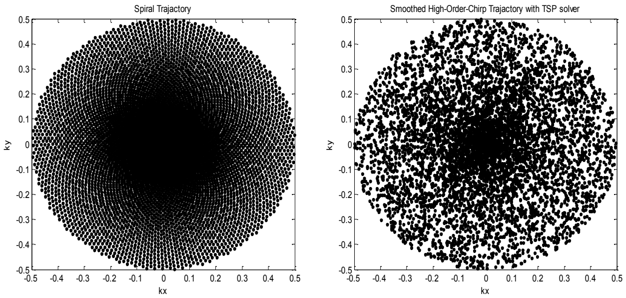

Fig. 6 comparison the trajectories (7010 points) between Spiral (left) and proposed method (right)

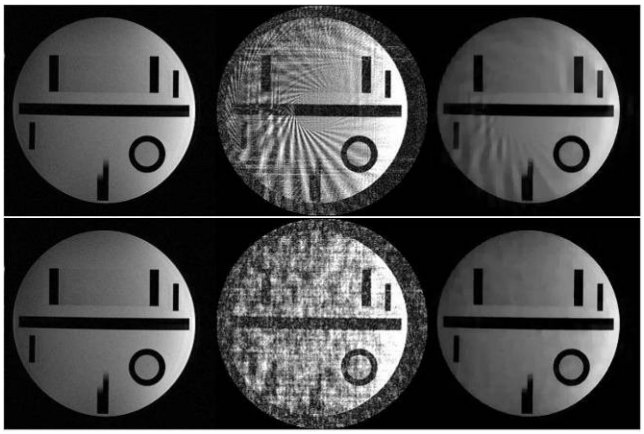

Fig. 7 comparison the reference (left), zf-w/dc (middle) and CS recon (right) images between Spiral (1st row) and proposed trajectory (2nd row)

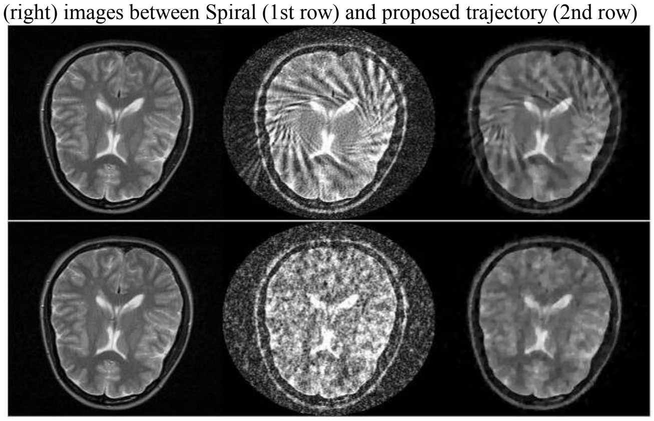

Fig. 8 comparison the reference (left), zf-w/dc (middle) and CS recon (right) images between Spiral (1st row) and proposed trajectory (2nd row)

C. Human Brain

An axial human brain data was sampled in the Ref. [3]. The size is 256×256. The 7010 points are sampled as same as Fig.6. Figure 8 compares the reference (left), zf-w/dc (middle) and CS recon (right) between Spiral (1st row) and proposed trajectory (2nd row). The proposed method can reduce more aliasing artifacts than Spiral under the CS framework.

V. Conclusion

In this paper, we propose a novel smoothed random-like trajectory scheme recovered under the CS framework. The simulations show the proposed trajectory outperforms the conventional Spiral trajectory in reducing aliasing artifacts.

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References

[1] E. J. Candès, J. Romberg and T. Tao, “Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information,” *IEEE Trans. on Information Theory*, vol. 52, pp. 489 – 509, Apr. 2006.

[2] D. Donoho, “Compressed sensing,” *IEEE Trans. on Information Theory*, vol. 52, pp. 1289 - 1306, Apr. 2006.

[3] M. Lustig, D. Donoho, and J.M. Pauly, “Sparse MRI: The application of compressed sensing for rapid MR imaging,” *Magn. Reson. Med.*, vol. 58, pp. 1182-1195, Dec. 2007.

[4] M. Lustig, D. Donoho and J. Pauly, "Rapid MR imaging with compressed sensing and randomly undersampled 3DFT trajectories," Proc. of the ISMRM, Seattle, WA, USA, May 2006.

[5] M. Lustig, J. Lee, D. Donoho and J. Pauly, "Faster imaging with randomly perturbed undersampled spirals and L1 reconstruction," Proc. of the ISMRM, Miami, FL USA, May 2005.

[6] T. Chang, L. He, T. Fang, “MR image reconstruction from sparse radial samples using Bregman iteration,” Proc. of the ISMRM, Seattle, WA, USA, May 2006.

[7] A. Bilgin, T. Trouard, A Gmitro, M Altbach, “Randomly perturbed radial trajectories for compressed sensing MRI,” Proc. of the ISMRM, Seattle, Toronto, Canada, May 2008.

[8] S. El-Metwally, Z. Abd-Elmoniem, M. Youssef, M. Kadah, " Rapid cardiac MRI using random Radial trajectories,” Proc. of the NRSC, Tanta, Egypt, Mar. 2008

[9] A. Curtis and C. Anand, “Random volumetric MRI trajectories via genetic algorithms,” *Int. J. of Biomed. Imaging*, Vol. 2008(2008), 297089.

[10] M. Seeger, H. Nickisch, R. Pohmann, B. Scholkopf, “Optimization of *k*-space trajectories for compressed sensing by Bayesian experimental design,” *Magn. Reson. Med.*, vol. 63(1), pp. 116-126, Jan. 2010.

[11] R. Willett, “Sampling trajectories for sparse image recovery,” Proc. of the ISBI, Chicago, IL, USA, Jun. 2011.

[12] V. Saligrama, “Deterministic designs with deterministic guarantees: Toeplitz compressed sensing matrices, sequence design and system,” http://arxiv.org/abs/0806.4958v2.pdf

[13] S. Kirkpatrick, D. C. Gelatt, P. M. Vecchi, "Optimization by Simulated Annealing," *Science* 220 (4598): 671–680, 1983.

[14] V. Černý, "Thermodynamical approach to the traveling salesman problem: An efficient simulation algorithm," *Journal of Optimization Theory and Applications*，45: 41–51, 1985.

[15] M. Lustig, S-J Kim, J. Pauly, “A fast method for designing time-optimal gradient waveforms for arbitary k-space trajectories,” *IEEE Trans. Med. Image*, vol. 27(6), pp. 866-873, Jun. 2008.

[16] J. Fessler, “On NUFFT-basesd gridding for non-Cartesian MRI,” *J. Magn. Reson.*, 188:191-195, 2007.

[17] A. Fraser, “Simulation of genetic systems by automatic digital computers. I. Introduction". *Aust. J. Biol. Sci.* 10: 484–491, 1957.

[18] F. Glover, "Tabu Search - Part 1". *ORSA Journal on Computing* 1(2): 190-206, 1989.

[19] M. Dorigo, “*Optimization, Learning and Natural Algorithms*,” PhD thesis, Politecnico di Milano, Italy, 1992.

[20] W Boyce, R. DiPrima, “*Elementary Differential Equations and Boundary Value Problems*,” 6th ed. New York: Wiley, 1997.

[21] Z. P. Liang, P. C. Lauterbur, “Principles of magnetic resonance imaging, A signal processing perspective,” IEEE Press, New York, 2000.