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Enhanced SWIFT Acquisition with Chaotic Compressed Sensing by Designing the Measurement Matrix with Hyperbolic-Secant Signals

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Abstract— **Sweep imaging Fourier transform (SWIFT) is an efficient (fast and quiet) specialized magnetic resonance imaging (MRI) method for imaging tissues or organs that give only short-lived signals due to fast spin-spin relaxation rates. Based on the idea of compressed sensing, this paper proposes a novel method for further enhancing SWIFT using chaotic compressed sensing (CCS-SWIFT). With reduced number of measurements, CCS-SWIFT effectively faster than SWIFT. In comparison with a recently proposed chaotic compressed sensing method for standard MRI (CCS-MRI), simulation results showed that CCS-SWIFT outperforms CCS-MRI in terms of the normalized relative error in the image reconstruction and the probability of exact reconstruction.**

I. INTRODUCTION

Fast image acquisition in biomedical imaging is important for reducing scanning time on patients, avoiding physiological effects, overcoming physical constraints of the imaging system, and meeting timing requirements for imaging dynamic structures or processes. With standard magnetic resonance imaging (MRI), it is also challenging to perform imaging of tissues or organs that give only short-lived signals due to fast spin-spin relaxation rates. Another general challenge in biomedical imaging is that an overwhelming amount of acquired data as well as their dimensionality and complexity increase rapidly. These two challenges motivate the development of specialized MRI methods with more efficient acquisition and reconstruction.

With respect to the first challenge, state-of-the-art techniques for fast MRI acquisition follow two approaches: (i) *parallel* imaging and (ii) *frequency-modulated* (FM) excitation. More information on parallel imaging can be consulted in [1]. In this paper, we only consider the latter approach. Data acquired in MRI provide complete Fourier, or k-space, measurements. Image reconstruction can be done in the image domain or in the k-space domain (i.e., Fourier domain).

In the FM excitation approach, with single-coil acquisition, the standard radiofrequency (RF) pulse is pre-modulated by another FM pulse, causing the k-space to spread. The use of linear FM pulses (also called chirp pulses), which have quadratic phase profiles, was proposed in [2]. A recently introduced method called SWIFT (SWeep Imaging with Fourier Transform) exploits a frequency-swept excitation pulse and virtually simultaneous signal acquisition in a time-shared mode [3], able to produce fast and quiet imaging. In SWIFT, the excitation pulses belong to the family of hyperbolic secant pulses, which employ both in the frequency and amplitude modulation, producing uniform and broadband spin excitation. With respect to the second challenge, one way of seeking a solution is to search for the actual amount of information in the data rather than their ambient dimensionality. Fortunately, many of medical images inherently exhibit a *sparse* representation in some transform domains, such as the wavelet domain (a representation of the image using wavelet functions) in which only a small number of wavelet coefficients are significant. Hence, negligible coefficients can be discarded, still the reconstructed image has acceptable quality. Thanks to this sparse structure, *compressed sensing* (CS), proposed by Candes and Donoho (a good tutorial is given in [4]), can provide an efficient way to acquire sparse signals and, thus, helps reduce the dimensionality of the data drastically. Generally, CS senses the signal by random linear projections to produce an observed signal in a special domain (which is often different from the sensing/ambient domain) and the number of samples of the observed signal is far smaller than that obtained as if the signal were sensed in the ambient domain (using, e.g., Nyquist sampling). Exact reconstruction can be achieved by nonlinear sparse approximation algorithms, such as ℓ_1-minimization based algorithms or greed pursuit based algorithms.CS has recently been shown to be successfully applied to standard MRI for fast acquisition by Lustig et al. [5]; the method is called *sparse MRI*. Recently, Puy et al. applied CS to MRI with linear FM excitation [6].

In the present work, inspired by the advantages of SWIFT and CS, we propose to use SWIFT in conjunction with CS in order to tackle both the two abovementioned challenges. Also, we will take a *deterministic* approach in CS, wherein the measurement matrix is deterministically designed. In this case, we follow the deterministic *chaos compressed sensing* (CCS), proposed in [7], which has been applied to standard MRI acquisition (CCS-MRI) in [8].

The paper is organized as follows. Section II describes the principle of two-dimensional (2D) MRI acquisition and the SWIFT method, in view of algebraic formulation. Section III first provides the fundamentals of CS, then describes the proposed method CCS-SWIFT. Section IV shows simulation results to illustrate the effectiveness of CSS-SWIFT, and the superior performance of CCS-SWIFT in terms of probability of exact reconstruction and normalized image reconstruction

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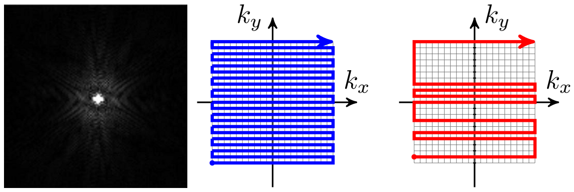

(a) (b) (c)

Fig. 1. k-space of a brain MR image. (a)– analog acquisition, (b)– linear sampling, (c) linear undersampling. In (c), a binary mask (of 128×128 points) is applied to (a), followed by a power decay law along direction k_y.

error, in comparison with CCS-MRI.

II. SWEEP IMAGING WITH FOURIER TRANSFORM

A. Standard 2D-MRI Acquisition

In principle, by exciting the object with a time-varying excitation RF pulse, the resonance information of the nuclei can be picked up by an RF receiving coil. Let us take the simple case of acquisition of a full 2D digital image of an object, e.g., a brain slice, to explain how the image acquisition is done. During a series of RF excitations each of which encodes the 2D location information of a particular point on the brain slice, the receiving coil detects an analog MRI time signal which contains the resonance information at all encoded locations. The encoded locations are represented in a temporary image space, which is called k-space. The changes of locations in the k-space during the acquisition time often form a smooth trajectory (see Fig. 1(a)). Most of the encoded information concentrates around the origin of the k-space, and the density of the k-space approximately follows a power decay law. A digital MRI signal is then obtained by sampling the time (t) and the k-space. Next, the digital MRI image of the brain slice can be reconstructed by applying a reconstruction algorithm on the digital signal. The reconstruction of the image can be done in the image domain or the k-space domain. For example, we apply the 2D Fourier transform on the digital MRI signal from the k-space to the pixel domain.

Consider the imaging of a 2D slice of the object in the 2D plane $\{x, y\}$. Denote $m(x, y)$ the object's image, to be reconstructed. Under discrete formulation, the signal acquired by the receiving coil is given by the following imaging equation:

$$\nu(k_x, k_y) = \sum_{n_x=0}^{N_x-1} \sum_{n_y=0}^{N_y-1} m(n_x, n_y) e^{-j(k_x x + k_y y)}, \quad (1)$$

where k_x and k_y respectively encode the k-space information of locations corresponding to the x and y directions of the image, N_x and N_y respectively are the numbers of pixels along x and y axes of the image. $k = \{k_x, k_y\}$ is said to lie in the k-space. Note that, the discrete representation in (1) corresponds to a linear full-sampling in the k-space along a Cartesian trajectory as shown in Fig. 1(b). In matrix form, (1) is expressed as

$$\boldsymbol{\nu} = \mathbf{F}\mathbf{m}, \quad (2)$$

where $\mathbf{F}$ is the Fourier matrix, $\mathbf{m}$ is the image to be acquired, and $\boldsymbol{\nu}$ is the MR signal obtained in the k-space. Therefore, $\mathbf{m}$ can be reconstructed by applying the inverse Fourier transform on $\boldsymbol{\nu}$. This is why acquisition in MRI is called Fourier imaging.

B. Specialized MRI with SWIFT

The main advantage of SWIFT originates in its nearly simultaneous excitation and acquisition scheme. The scheme employs a sequence of FM RF pulses, each having a duration T_p typically in the millisecond range.

In the present implementation, the RF excitation pulse utilizes both the amplitude modulation, $\omega(t)$, and frequency modulation, $\omega_{\mathrm{RF}}(t)$, expressed by

$$h(t) = \omega(t) \exp\left\{-j \int_0^t (\omega_{\mathrm{RF}}(\tau) - \omega_c)\, d\tau\right\}, \quad (3)$$

where $\omega(t)$ and $\omega_{\mathrm{RF}}(t)$ are designed based on the family of adiabatic hyperbolic secant (HS$_n$) pulses, $f_n(t)$, given by

$$f_n(t) = \operatorname{sech}\left[\beta \left(\frac{2t}{T_p} - 1\right)^n\right], \quad (4)$$

$$\omega(t) = \gamma B_{1\max} f_n(t) \quad (5)$$

$$\omega_{\mathrm{RF}}(t) = \omega_c + 2A \left(\frac{\int_0^t f_n^2(\tau)\, d\tau}{\int_0^{T_p} f_n^2(\tau)\, d\tau} - \frac{1}{2}\right). \quad (6)$$

Above, n is a shape vector (typically, $n \geq 1$), β is a truncation factor (usually, $\beta \approx 5.3$), T_p is the pulse length, γ is the gyromagnetic ratio, $B_{1\max}$ is the maximum amplitude of the RF pulse, ω_c is the center angular frequency, and A represents the bandwidth of the pulse ($-A \leq \omega_{\mathrm{RF}} - \omega_c \leq A$). In SWIFT, during the excitation of the HSn pulse from 0 to T_p seconds, the transmitter is repeatedly turned on and off to enable sampling (acquisition) in short intervals of time, thus the acquisition is virtually performed simultaneously with the excitation. For detailed information and implementation of the SWIFT method and the design of HS$_n$ pulses, the reader is invited to consult references [3].

In the present work, the HS$_n$ pulse can be viewed as such it is used to excite the FID (free-induction decay) system in MRI, instead of being excited by the impulse function as in standard MRI acquisition. In other words, the image is premodulated by the hyperbolic secant pulse. Therefore, the resulting imaging equation becomes

$$\nu(k_x, k_y) = \sum_{n_x=0}^{N_x-1} \sum_{n_y=0}^{N_y-1} m(n_x, n_y)\, h(n_x, n_y)\, e^{-j(k_x x + k_y y)}, \quad (7)$$

which can be expressed in matrix form as

$$\boldsymbol{\nu} = \mathbf{F}\mathbf{H}\mathbf{m}. \quad (8)$$

where $\mathbf{H}$ is a diagonal matrix whose diagonal elements are obtained from the hyperbolic secant pulse $h(t)$. Note that, matrix representation of SWIFT has been shown in [9].

III. CHAOTIC COMPRESSED SENSING FOR SWIFT

A. Compressed Sensing Fundamentals

To simply illustrate the concept of CS, we describe the discrete-to-discrete formulation of CS as follows. Let $\mathbf{x} \in \mathbb{R}^N$ be the signal of interest and suppose that $\mathbf{x}$ admits a sparse representation by a known transform in a proper basis $\mathbf{\Psi} = [\psi_1, \ldots, \psi_N]$ as given by $\mathbf{x} = \mathbf{\Psi}\boldsymbol{\alpha}$, where $\boldsymbol{\alpha} \in \mathbb{R}^N$ is a K-sparse vector (i.e., containing exactly K nonzero values) and the transform matrix $\mathbf{\Psi}$, used to represent $\mathbf{x}$ in the sparsity basis, is called the sparsifying matrix or representation basis. In CS, $\mathbf{x}$ is acquired by:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} = \mathbf{\Phi}\mathbf{\Psi}\boldsymbol{\alpha} = \mathbf{\Theta}\boldsymbol{\alpha} \tag{9}$$

where $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$ is the linear sensing matrix, and $\mathbf{y} \in R^M$ is the vector contain the measurements, $M < N$. We wish to have M as small as possible and the reconstruction algorithm as efficient as possible. When $M \ll N$, to reconstruct $\mathbf{x}$ from $\mathbf{y}$ we need to solve an under-determined linear system of equations. In this case, CS theory interestingly states that signals that admit a sparse representation in a known basis can be uniquely reconstructed from M measurements in an incoherent domain. Exact reconstruction is feasible based on two principles: sparsity and incoherence. Sparsity is related to the signal of interest while incoherence is related to the sensing modality. They admit a duality relationship in CS such that signals having a sparse signal representation must have a very dense acquisition representation. Thanks to the sparse structure of $\boldsymbol{\alpha}$, the recovery of $\boldsymbol{\alpha}$ is made possible if $\mathbf{\Theta}$ is constructed as an almost orthonormal system when restricted to sparse linear combinations. Specifically $\mathbf{\Theta}$ must satisfy the so-called Restricted Isometry Property (RIP) of order K. In other words, $\mathbf{\Theta}$ approximately preserves the length of K-sparse signals; all subsets of K columns of $\mathbf{\Theta}$ are near orthogonal. One way to satisfy RIP is incoherence, denoted as $\mu(\mathbf{\Phi}, \mathbf{\Psi})$, measuring the largest correlation between any two columns of $\mathbf{\Phi}$ and $\mathbf{\Psi}$. The coherence is constrained by $1 \le \mu \le \sqrt{N}$ and compressed sensing takes place when μ is small. When entries of $\mathbf{\Phi}$ are independently and identically Gaussian distributed with zero mean and variance of $1/M$ (in turns, μ is small), if $M \ge CK\log(N/K)$ for some positive constant C, then exact reconstruction of $\boldsymbol{\alpha}$ (or, essentially, $\mathbf{x}$ since $\mathbf{\Psi}$ is known) is achieved with overwhelming probability using the following $\ell_1$minimization problem:

$$\boldsymbol{\alpha} = \arg\min_{\boldsymbol{\alpha}'} \|\boldsymbol{\alpha}'\|_1 \qquad \text{subject to} \quad \mathbf{\Theta}\boldsymbol{\alpha}' = \mathbf{y}. \tag{10}$$

When there is noise in the measurements, the optimization problem is reformulated as follows:

$$\boldsymbol{\alpha} = \arg\min_{\boldsymbol{\alpha}'} \|\boldsymbol{\alpha}'\|_1 \quad \text{subject to} \quad \|\mathbf{\Theta}\boldsymbol{\alpha}' - \mathbf{y}\|_2 < \epsilon, \tag{11}$$

where ϵ is a constant related to the variance of the noise.

B. Proposed Chaotic Compressed Sensing for SWIFT

The above works using CS for MRI use random sensing. However, random sensing has some drawbacks in comparison to deterministic sensing: less efficient recovery time, no explicit constructions, larger storage, looser recovery bounds [10]. As we have mentioned in the introduction, chaotic design for the measurement matrix in CS has been proposed in [7]. In this method, a sampled logistic sequence is generated by a deterministic chaotic system called Logistic Map. Then, the measurement matrix is created column by column with this sequence. One of the advantages of using chaotic sequences instead of random ones is due to its simpler hardware implementation.

Chaotic design for the measurement matrix has been reformulated to apply CS in standard MRI by under-sampling the k-space in a chaotic manner [8]. Mathematically put, the imaging equation with incomplete measurements in the k-space becomes

$$\boldsymbol{\nu} = \mathbf{PFm}, \tag{12}$$

where $\mathbf{P} \in \mathbb{R}^{M\times N}$ is a rectangular binary matrix containing only one non-zero value on each row, representing the action of selecting only M rows out of $\mathbf{F}$ where the indices of these rows are obtained chaotically. By corresponding the CS model in (9) and the imaging equation in standard MRI in (12), one can see that the CS incomplete measurements $\mathbf{y} \equiv \boldsymbol{\nu}$, the measurement matrix $\mathbf{\Phi} \equiv \mathbf{PF}$ and the underlying signal to be reconstructed $\mathbf{x} \equiv \mathbf{m}$. To obtain $\mathbf{P}$, we generate the values of k_x by the Logistic Map:

$$s(n+1) = as(n)(1 - s(n)), \tag{13}$$

where parameter a controls the chaotic behavior of $s(n)$. The resulting values are used to determine which values of k_y are to be obtained from sampling in the k-space. In other words, we have set up a binary mask $\mathbf{P}$ for chaotically under-sampling the k-space (see Fig. 1(c)); note that linear sampling is assumed along k_x.

Now, by incorporating CS in the specialized MRI designed by SWIFT, the equivalent matrix representation of the imaging equation in SWIFT becomes

$$\boldsymbol{\nu} = \mathbf{PFHm}, \tag{14}$$

where $\mathbf{H}$ is the hypersecant matrix. The image is reconstructed by solving the following optimization using the non-linear conjugate gradient (NCG) algorithm:

$$\hat{\mathbf{m}} = \arg\min_{\mathbf{m}} \left\{ \|\mathbf{PFHm} - \boldsymbol{\nu}\|_2^2 + \lambda \|\mathbf{\Psi}\mathbf{Hm}\|_1 \right\} \tag{15}$$

where λ is a tuning constant for the trade-off between fidelity term and the sparsity, ϵ controls the fidelity term, and $\mathbf{\Psi}$ represents the sparsifying matrix in the wavelet domain.

IV. SIMULATION

In the simulation, the data source in use is a brain slice of 128×128 pixels, as shown in Fig. 2(a). Define a compression radio $r = M/N$. The logistic map was simulated with $a = 4$ and the initial condition of $s(0) = 3$. For the HS_n pulse, we set $n = 1$. Fig. 2(b) shows the reconstructed image when the k-space was under-sampled linearly at the ratio of $r = 0.3$. The ringing in this image reflects the aliasing effect due to under-sampling. Figs. 2(c) and 2(d), respectively, show the reconstructed images when CCS is applied for

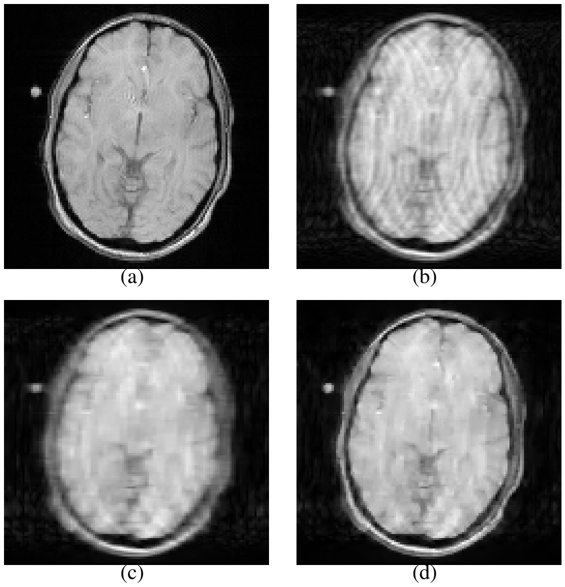

Fig. 2. Original brain slice image (a), and its reconstructed images (b) without using CS, (c) with CCS-MRI and (d) with CCS-SWIFT, at $r = 0.3$.

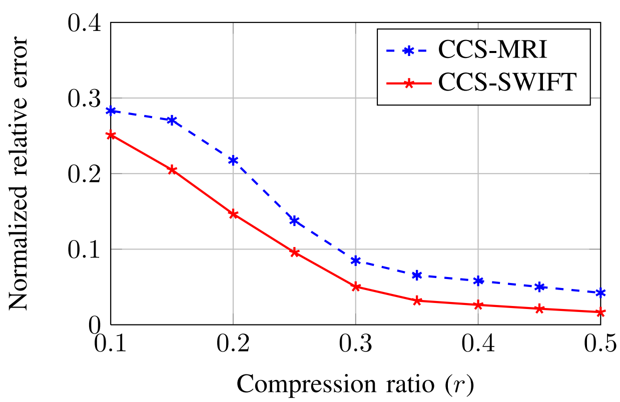

Fig. 3. Normalized image relative error performance.

standard MRI and for specialized MRI using SWIFT, both with the same ratio $r = 0.3$. They indicate that CCS-SWIFT offers a higher quality of reconstruction than CCS-MRI. This better performance is because the broadband HS_n pulse has spread the k-space. In other words, it reduces the mutual coherence $\mu(\mathbf{\Phi}, \mathbf{\Psi})$. Performance comparison between CCS-MRI and CCS-SWIFT is also shown in Fig. 3 with a series of compression ratios from 0.1 to 0.5. We use the normalized image relative error metric for reconstruction performance

$$e = \frac{1}{N_x \times N_y} \frac{\sum_{i=1}^{N_x} \sum_{j=1}^{N_y} |\mathbf{m}_{ij} - \hat{\mathbf{m}}_{ij}|}{\sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \mathbf{m}_{ij}} \tag{16}$$

As can be seen in Fig. 3, it is obvious that CCS-SWIFT outperforms CCS-MRI. Another performance study based on the probability of exact reconstruction is also in Fig. 4. This reconfirms the superior performance of reconstruction of CCS-SWIFT.

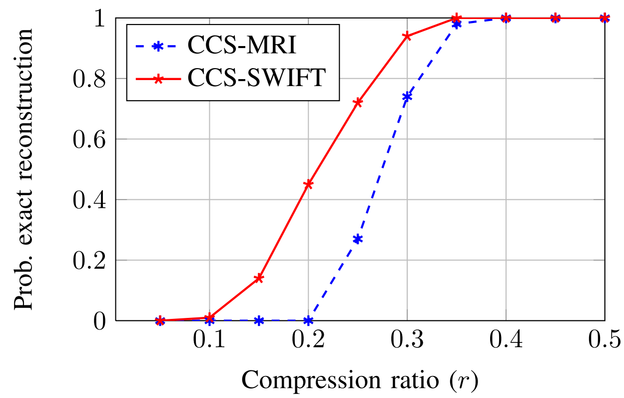

Fig. 4. Probability of exact reconstruction performance.

V. Conclusions

By combining CS in SWIFT, this paper presents a novel method to enhance the speed of acquisition in SWIFT for MRI applications. For the sake of simplicity in presenting the idea of CS, this paper only considered 2D SWIFT (using horizontal trajectories in the k-space) rather than 3D SWIFT (using radial trajectories) as implemented in [3].With this presentation, instead of fully sampling the k-space, we only selected M out of N horizontal trajectories in the k-space and the selection of these trajectories was done using the values generated from the chaotic sequence (Logistic map). The good performance of CSS-SWIFT at the ratio of $r = M/N = 0.5$ implies that the speed of acquisition in SWIFT can be enhanced by a factor of 2 when combined with CS. Also, CSS-SWIFT performed better than CSS-MRI because the use of the hyperbolic secant pulses effectively reduces the coherence between the measurement matrix and the sparsifying matrix in CS.

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