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Dual Energy Pulses for Electrical Impedance Spectroscopy with the Stochastic Gabor Function.*

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Abstract— **This paper introduces the stochastic Gabor function (SGF), an excitation waveform that can be used to design optimal excitation pulses for Electrical Impedance Spectroscopy (EIS) of the brain. The SGF is a Gaussian function modulated by uniformly distributed noise; it has wide frequency spectrum representation regardless of the stimuli pulse length. The SGF was studied in the time-frequency domain. As shown by frequency concentration measurements, the SGF is least compact in the sample frequency phase plane. Numerical results obtained by using a realistic human head model indicate that the SGF may allow for both shallow and deeper tissue penetration than is currently obtainable with conventional stimulus paradigms, potentially facilitating tissue subtraction assessment of parenchymal dielectric changes in frequency. This could be of value in advancing EIS of stroke and hemorrhage.**

I. Introduction

Electrical impedance spectroscopy (EIS) is being studied as a diagnostic tool for the evaluation and characterization of ischemic and hemorrhagic tissues [1, 2]. EIS estimates the macroscopic dielectric constants from surface voltage measurements between electrode pairs positioned on the surface of an object in response to the applied probe current, typically using the four-terminal system [3]. Unfortunately EIS is maximally sensitive to skin/bone and is not very sensitive to brain parenchymal changes due to the limited penetration of the probe current. We propose an ideal probe current design based on the concept of dual energy. In Computed Tomography (CT) dual energy is a relatively new imaging technique that uses two different x-ray tubes in a single CT unit. Bone can be identified through the use of dual energy CT based on its spectral properties and can be removed from an angiogram [4]. This paper illustrates, both in theory and with numerical examples, the design of a dual energy pulse for EIS based on the stochastic Gabor function (SGF) [5]. The SGF is a Gaussian function which has been modulated by uniformly distributed noise. The SGF reaps the benefits of a very wide frequency bandwidth while retaining a non-narrow pulsed envelope in time. The behavior and propagation have been studied with Finite Differences Time Domain (FDTD) [5] and we show examples of pulse penetration using the SGF in a realistic human head model.

This paper is organized as follows. Section II defines the stochastic Gabor function. In section III, statistical parameters of the Gaussian, Gabor and stochastic Gabor function stimuli are delineated. Finally, Section IV shows the dual energy example stimuli using a realistic head model.

II. The Stochastic Gabor Function

The stochastic Gabor function (**Fig. 1**) is defined as [5]:

$$\lambda_n = \xi_n g_n^\sigma \tag{1}$$

where $n \in [1:N]$, ξ_n is a random Gaussian white noise process uniformly distributed in [-1; 1], and $g_n^\sigma = \frac{\exp[-\frac{n^2}{2\sigma^2}]}{\sigma\sqrt{2\pi}}$ is the Gaussian function. The ξ_n set is valid only if the resulting stochastic Gabor function is zero-mean (i.e., $\langle \lambda_n \rangle = 0$). The power spectral density [6] of

$$\wp_k = S_k^{\xi\xi} * \left|G_k^{1/\sigma}\right|^2 \cong c \tag{2}$$

where $k \in [1:N]$ is the frequency variable and $S_k^{\xi\xi}$ is the discrete Fourier transform, or FFT, of the autocorrelation function of the white noise process ξ_n; $G_k^{1/\sigma}$ is the FFT of g_n^σ. The whitening of the Gaussian in eq. (1) flattens the frequency response. The short-time Fourier Transform is used to determine the sinusoidal frequency and phase content of a signal inside a time window, following the spectral changes of the signal over time. The short-time Fourier Transform of the stochastic Gabor function is:

$$\Gamma_{k,m} = \sum_{n=1}^{N} \lambda_n w_{n-m} \exp[-j2\pi nk] \tag{3}$$

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where $m \in [1:N]$ specifies the position of the time window and w_n is the time window function such that $\sum_{n=1}^{N} |w_n|^2 = 1$. By selecting a Gaussian, $w_n = g_n^{\hat{\sigma}}$, as the window function,

$$\Gamma_{k,m} = \sum_{n=1}^{N} \xi_n g_n^{\sigma} g_{n-m}^{\hat{\sigma}} \exp[-j2\pi nk] \tag{4}$$

and the short-time power spectral density becomes:

$$\wp_{k,m} = S_k^{\xi\xi} * \left| G_k^{1/\sigma} * G_k^{1/\hat{\sigma}} \exp[j2\pi mk] \right|^2 \cong c g_m^{2(\sigma+\hat{\sigma})} \tag{5}$$

The stochastic Gabor function has a Gaussian envelope in the time domain; its frequency representation (**Fig. 2,** bottom) is very uniform. A Gaussian (**Fig. 2**, top) has a Gauss function representation in both time and frequency domains. The Gabor function (**Fig. 2,** middle), a harmonic function with frequency ω_0 multiplied by a Gaussian, has the same time-frequency distribution as the Gaussian but shifts in frequency by ω_0.

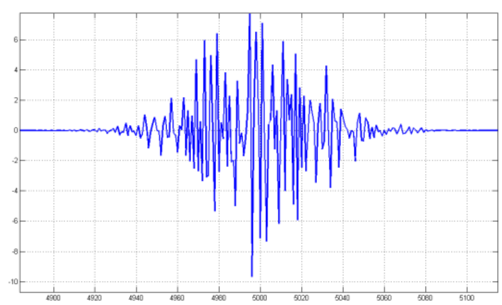

Fig. 1: *The stochastic Gabor function* [5] .

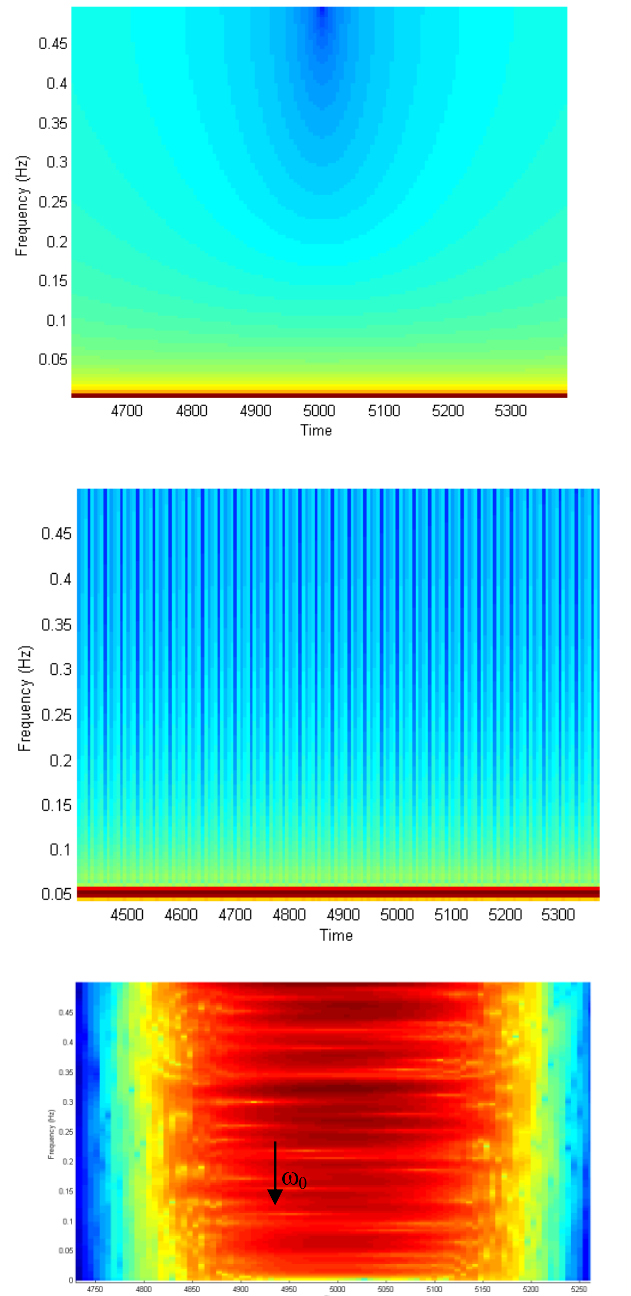

Fig. 2: *Spectrograms of: (top) Gaussian (σ=12.8, N=64,000), (middle) Gabor (ω_0=0.1) and (bottom) stochastic Gabor function [5] .*

III. The Time-Frequency Resolution

One of the main advantages of the Gaussian and Gabor functions is their time-frequency localization. In this section, the stochastic Gabor function is studied in terms of localization in the time domain, which can be measured by estimating the time-frequency resolution to select the value for σ, or pulse width of the stochastic Gabor function. A more uniform sampling in frequency corresponds to a source excitation with lower concentration in the sample frequency phase plane [7]:

$$H(\Gamma_n) = \sum_{n=1}^{N} \|\Gamma_n\|^2 \log\left(\|\Gamma_n\|^2 + \varepsilon\right) \tag{7}$$

where ε is an arbitrarily small constant introduced for regularization. Eq. (7) has a form similar to the entropy function, $E(p_i) = -p_i \log(p_i)$; however, the resulting quantity is an estimate of frequency concentration when the hermitian vector Γ_n is transformed into a real vector using the square norm. When all frequency values of Γ_n are constant, $H(\Gamma_n) = 0$. Conversely, $H(\Gamma_n)$ reaches maximal value when the function Γ_n is concentrated at a single frequency point. For instance, zero frequency concentration occurs when $\lambda_n = [1 \quad 0 \quad 0 \quad 0]$, where $\Gamma_n = [1 \quad 1 \quad 1 \quad 1]$ results in $H(\Gamma_n) = 0$. High frequency concentration occurs when $\lambda_n = [1 \quad 1 \quad 1 \quad 1]$, in which case $\Gamma_n = [4 \quad 0 \quad 0 \quad 0]$ and results in $H(\Gamma_n) \cong 44.4$ (with $\varepsilon = 10^{-10}$).

Fig. 3 shows the difference in frequency concentration as a function of the pulse width σ. In the case of larger σ values, the stochastic Gabor function --virtually flat in frequency --exhibits much lower concentration in the sample frequency phase plane than either the Gaussian or

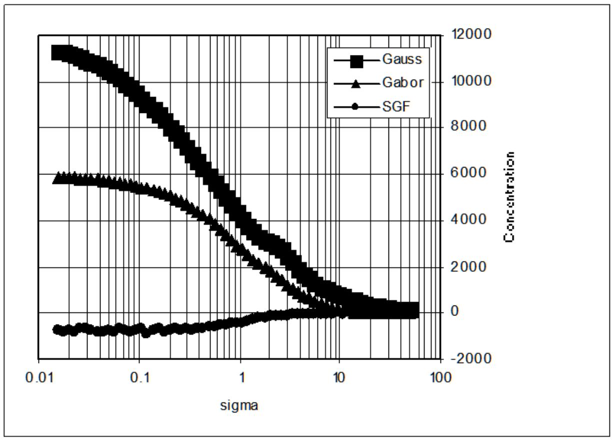

Fig. 3: *Frequency concentration with N=512 and ε = 10^{-10} of the three functions for different values of σ: the Gaussian (◆, on top), Gabor with ω=6π (■, in the middle), and stochastic Gabor (▲, on the bottom) functions [5] .*

Gabor functions. The Gabor is less concentrated in frequency than the Gaussian, as it is composed of two Gaussians centered at ω_0 and $-\omega_0$. As the pulse width approaches zero, all functions approach $\lambda_n = \delta_n$ such that the frequency concentration also approaches zero. The concentration property allow for shorter exciting pulses have the advantage of reducing the number of time steps needed for FDTD convergence [5].

IV. NUMERICAL RESULTS

Fig. 4 illustrates how the stochastic Gabor function can modulate the penetration depth in a realistic head model [8] when used as a probe current pulse in EIS. The results are shown in terms of the Electric Field in and around the head using two different values of the σ for the SGFs (top and middle), which resulted in a different current density penetration profile between the two SGFs (bottom). The low energy SGF was defined with σ = 128 and the high energy with σ = 12.8 both with $N_s = 10^5$.

The computation times for both SGF stimuli were 5 minutes for Ns = 105, respectively, using an eight cores Dell Precision T7500 desktop computer with 48 gigabyte of RAM.

V. CONCLUSION

A new excitation pulse, the stochastic Gabor function, has been introduced. It has a marked cylindrical shape in the time-frequency domain, produces steady values in frequency, and has a Gaussian shape in time. The stochastic Gabor function can be used to design pulses for electrical impedance spectroscopy with greater penetrating depth than is currently standard. Examples of SGF with different energies exhibit different penetration in the head and thus may be used to estimate more focused parenchymal tissue impedances. This has the potential to help make portable, noninvasive detection and monitoring of stroke and intracranial hemorrhage a clinically useful tool in the ambulance, battlefield, or intensive care unit settings.

VI. REFERENCES

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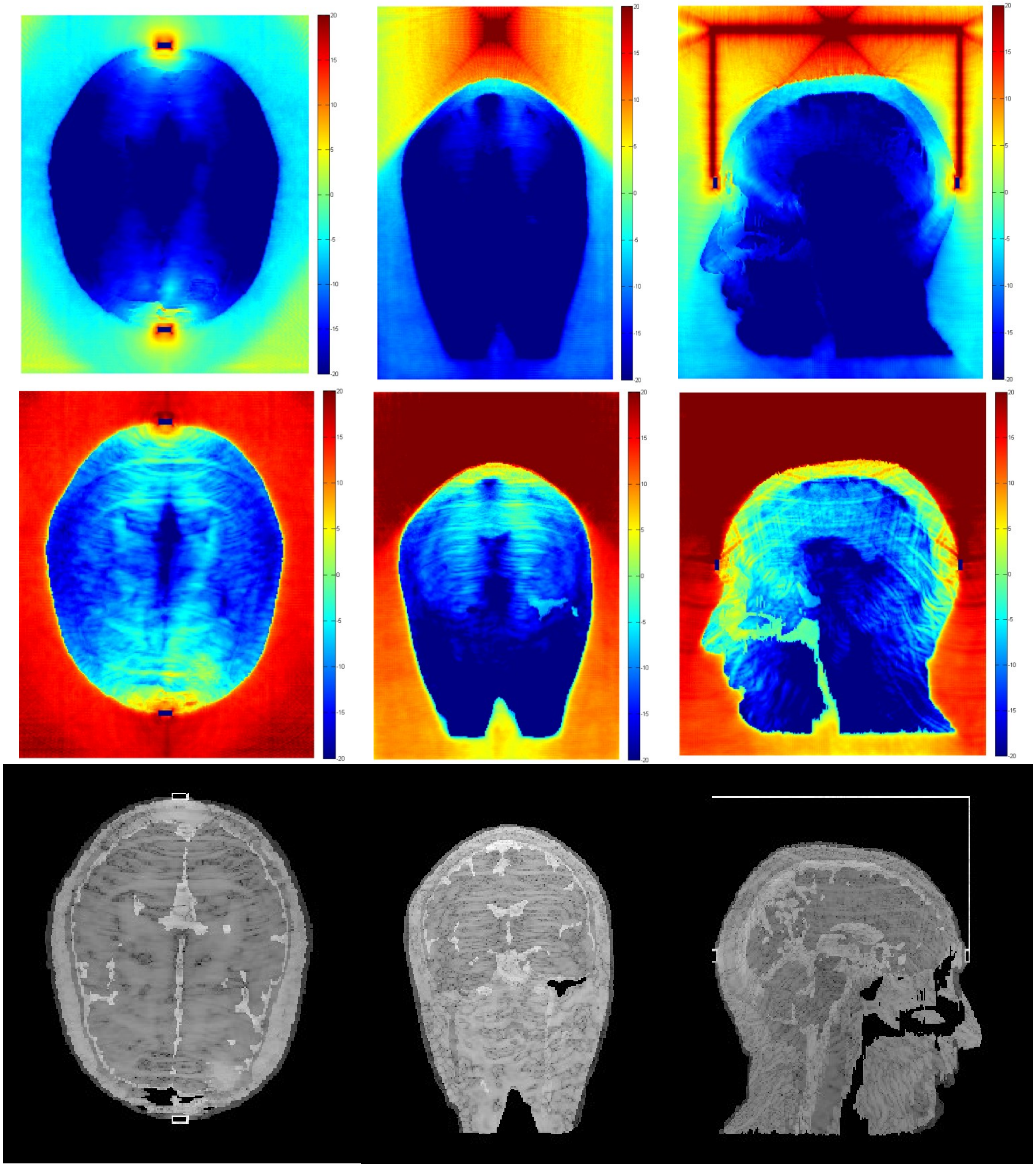

Fig. 4: *Example of a low energy SGF (top) and a high energy SGF (middle) with a marked increase of tissue penetration at high energy compared to the lower energy pulse for the magnitude of the electric field integrated over time in the same logarithmic scale. (Bottom) Normalized difference map for the current densities Jr integrated over time between the low energy and high energy SGFs.*