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MEDICAL IMAGE DENOISING USING LOW PASS FILTERING IN SPARSE DOMAIN

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Abstract— **In this work, we introduce a new approach for medical image denoising. An innovative method is proposed to extend the concept of low-pass filtering to the sparse representation framework. A weight matrix is applied to the definition of the sparse coding optimization problem intended to reduce coefficients corresponding to atoms with higher frequency contents, which dominantly represent the image noise. In parallel, a new overcomplete Discrete Cosine Transform (DCT) dictionary is constructed to include both frequency and phase information, aiming to remove blocking artifacts without considering patch-overlap. The proposed denoising approach was applied on low-dose Computed Tomography (CT) phantoms. The resultant observations demonstrate qualitative and quantitative improvements, in terms of peak signal to noise ratio (PSNR), in comparison to some previous approaches.**

I. INTRODUCTION

In the literature, many denoising methods in spatial and various transform domains such as frequency, wavelets, curvelets and contourlets have been proposed [1]. In recent years compressed sensing theory [2] has proven to have great potential in signal and image processing. Noise reduction is one application of sparse representation and has shown some promising results [3,4,5].

The key idea behind sparse coding and representations is that signals can be efficiently represented as a linear combination of a set of prototype elementary vectors $\{d_i\} \in \mathbb{R}^m$. These basic vectors are called atoms and form a dictionary $D \in \mathbb{R}^{m \times n}$, where n represents the number of atoms in the dictionary. A given signal $Y = \{y_1, ..., y_k\} \in \mathbb{R}^{m \times k}$ can be characterized by a linear combination of a few atoms in the dictionary. For a signal Y and a given dictionary D it is desired to find a sparse matrix α containing the coefficients for the linear combination such that $Y \approx D\alpha$. Determining the sparsest representation of a signal is an NP hard combinational problem [6]. Thus the problem of finding the sparsest solutions among infinitely many solutions can be formulated as an optimization problem satisfying,

$$\min_{\alpha} \|\alpha\|_0 \ \textit{subject to} \ \|Y - D\alpha\| < \varepsilon \tag{1}$$

Where $\|.\|_0$ is the l_0 norm that is equivalent to the number of non-zero entries of a vector. There exist many methods in the literature for finding the solution to (1). The main goal in denoising using sparse representation is to efficiently separate noisy signals into signal and noise components, while the image is not distorted (such as blurring effect caused by using low-pass filtering). This advantage stimulates the motivation to use the sparse representation, instead of using conventional low-pass filtering methods. The sparse model of a noisy image is,

$$Y = D\alpha + \varepsilon_n \tag{2}$$

where the D, α, Y and ε_n correspond to dictionary, sparse vector, image patch, and the image noise, respectively. It is assumed that $\tilde{Y} = D\alpha$ approximately extracts the original from the noisy image.

To the best of our knowledge, the following work are the only ones investigating the application of the sparse representation to the noise reduction in medical images, specifically CT. The work presented in [4] applies the K-SVD denoising algorithm to reduce the noise of CT images. The work presented by Rubinstein *et. al.* [5] proposes a double sparsity dictionary learning method and investigates its application to reduce the CT image noise. A fixed dictionary, DCT to be more specific, is used to sparsely define dictionary atoms during the modified K-SVD learning process based on a set of training patches. In other words, a learn-based dictionary, is created to provide a better sparse representation for a specific sort of images, is it sparsely represented based on a fixed dictionary. Although, the dual-sparsity method uses high quality images to derive the learnt dictionary and is supposed to be not representative for image noise, it does not consider structural discrimination between noise and signal. Moreover, these methods are time consuming and computationally complex, which makes them not suitable for some clinical purposes.

In this paper, a new scheme to apply the sparse coding for the image noise reduction is introduced. The proposed approach is inspired from the concept of low-pass filtering and is based on a modification to the general formula of the sparse coding problem, aiming to reduce non-zero coefficients representing the image noise. The modified sparse coding is accompanied with a new overcomplete DCT dictionary which maintains the frequency domain characteristic of the image while interestingly eliminates the blocking artifact problem of using the conventional DCT dictionary. This is the key of our approach to apply the low-pass filtering in the sparsely represented domain.

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II. METHODOLOGY

A. New DCT Dictionary Construction

Currently the most commonly used dictionaries for sparsely representing images, due to their viable and fast implementation, are the Discrete Cosine Transform (DCT) and Discrete Wavelet Transform (DWT) dictionaries. Although these transforms are effective tools for signal and image processing, they have two main shortcomings: lack of phase information and shift sensitivity. To overcome these disadvantages, we introduce a dictionary (transform) that incorporates phase information with the frequency characteristics. For a two dimensional space, the proposed dictionary can be written along the rows and columns as:

$$\phi_{m_1,\varphi_1,m_2,\varphi_2}(n_1,n_2)=\cos\left(\frac{\pi n_1 m_1}{N_1}+\varphi_1\right)\cdot\cos\left(\frac{\pi n_2 m_2}{N_2}+\varphi_2\right)$$
$$\varphi_{1,2}\in\left\{0,\frac{\pi}{2},\pi,\frac{3\pi}{2}\right\} \tag{3}$$

and,

$$d_j=\left[\phi_j(1,1),...,\phi_j(1,N_2),\phi_j(2,1),...,\phi_j(2,N_2),....,\phi_j(N_1,1),...,\phi_j(N_1,N_2)\right] \tag{4}$$

N_1 and N_2 represent the number of samples taken in row-wise and column-wise directions for each patch. n_1 and n_2 each increment from zero to N_1-1 and N_2-1, respectively within each patch. and take care of patch count in either direction.

Unlike other popular dictionaries that take overlapping patches from the image to overcome the mentioned weaknesses, our new proposed DCT dictionary has two outstanding advantages: 1. The need for overlapped data selection is eliminated without sacrificing the reconstruction quality. 2. The number of patches chosen in the image are decreased, hence the computation cost and complexity is significantly reduced. For $N_1=N_2=8$, the conventional DCT versus the proposed dictionary is depicted in Figure 1. The DCT dictionary has 8x8 patches where the proposed dictionary is 29x29 for the same number of frequency harmonics including the phase information. The identical atoms are removed from the dictionary.

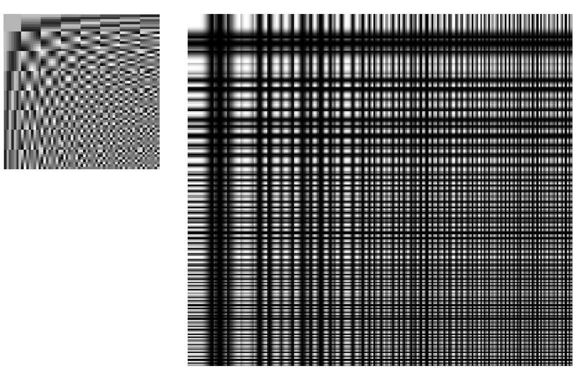


Fig. 1 The dictionary on the left hand side represents the common DCT dictionary and the dictionary on the right side is the proposed DCT dictionary.

B. Sparse Coding Denoising

As mentioned in the introduction, there exist many methods to approximate sparsest solution for equation (1). For our application we consider Orthogonal Matching Pursuit (OMP) algorithm to work out the optimization problem for [7]. This method is one of the most popular sparse coding approaches due to its simplicity and easy implementation. In this paper we have tailored OMP sparse coding method for our denoising purposes. The mathematical definition of the minimization problem is modified in such a way that the influences of the coefficients in the sparse representation vector that correspond to higher frequency atoms in the designed dictionary are reduced. The optimization problem for denoising can be written as:

$$\arg\min_{\alpha}\left\|Y-DW\alpha\right\|_2 \textit{ subject to } \left\|\alpha\right\|_0<L \tag{5}$$

Where L represents the number of non-zero coefficients or equivalently for a certain amount of error tolerance as:

$$\arg\min_{\alpha}\left\|\alpha\right\|_0 \textit{ subject to } \left\|Y-DW\alpha\right\|_2<\varepsilon \tag{6}$$

In which is the amount of error. W is a diagonal matrix where each element, also called weights, in the main diagonal corresponds to a specific atom and controls its contribution in representation of y. Weights that are greater than one, force their corresponding sparse coefficients to shrink, whereas for weights less than one, the sparse coefficients increase. This functionality of W enables us to reduce the impact of atoms that potentially represent the image noise. The very immediate inference is to increase weights related to higher frequency disruptive atoms which results in reduction in their corresponding sparse coefficients, hence the noise factor in the reconstructed signal is weakened. Since we are dealing with two-dimensional images, a simple structure of the W is to increase weights proportional to the vertical and horizontal frequency of corresponding atom,

$$w_x(m_1)=\begin{cases}\dfrac{((K-1)m_1+(N_2-K\times N_s))}{N_1-N_s} & m_1>N_s\\ 1 & m_1\le N_s\end{cases}$$
$$w_y(m_2)=\begin{cases}\dfrac{((K-1)m_2+(N_2-K\times N_s))}{N_2-N_s} & m_2>N_s\\ 1 & m_2\le N_s\end{cases} \tag{7}$$
$$w(m_1,m_2)=w_x(m_1)\cdot w_y(m_2)$$

where w_x and w_y are weights corresponding to vertical and horizontal directions. K is the weighting factor, as explained previously, for Ks greater than one the coefficients is suppressed and for less than one is amplified. N_s refers to the corner frequency of the low-pass filter. The update process in OMP is adjusted accordingly to support the effect of the weight matrix

$$\alpha=((D_IW)^T D_IW)^{-1}(D_IW)^T Y \tag{8}$$

D_I is the set of atoms that are selected in each iteration as in the OMP algorithm and I represents the index of the selected columns. A pseudo algorithm of the proposed algorithm is given below.

Table 1 Pseudo-Algorithm of the Proposed Denoising

$\alpha = Denoise(y, D, K, \varepsilon, L, N_1, N_2)$

$r = y$, $i = 0$

Construct the weight matrix W

$while(i < L \quad or \quad \|r\|_2 < \varepsilon)$

$\quad p = D^T r_i$

$\quad v = \arg\max_k |p|$

$\quad I = I \cup v$

$\quad \alpha = ((D_I W)^T D_I W)^{-1} (D_I W)^T y$

$\quad r_i = y - D_I \alpha$

$\quad i = i + 1$

End while

III. Results

For a better evaluation, the results are broken down into two parts: in section 1, we focus on the reconstruction performance of the proposed dictionary on images; and in section 2, the low-dose CT scan denoising using the proposed denoising approach is analyzed. All the implementations were performed in MATLAB 7.10.0 environment using a Mac computer with I3 Processor and 3GB of memory. For our investigation, CT scans of chest and abdominal phantoms, constructed of material that mimics the CT appearance of tissue, were acquired; the scan set are acquired at 120 kVp, 15 mAs of XrayTubeCurrent (low dose), and at 99 mAs (high dose). In our comparisons, the high dose images are used as the ground truth.

A. *Proposed Dictionary Performance*

Images are sparsified into non-overlapping patches and reconstructed using OMP algorithm given the new DCT dictionary. The performance of the proposed dictionary is compared with common DCT dictionary in terms of both visual effect and PSNR value. The PSNR is calculated using

$$PSNR = 20\log_{10}\frac{255}{\sqrt{MSE}} \tag{9}$$

where MSE is the mean square error between the reconstructed image and the original image.

The graph in Figure 2 demonstrates the effects of number of coefficients in sparse coding on the image reconstruction using both dictionaries. It is shown that due to blocking artifacts the PSNR of the reconstructed image using DCT dictionary stays approximately constant using more than 3 coefficients while using the proposed dictionary the quality keeps improving till it becomes steady around 32 coefficients.

Table 2 Reconstruction Accuracy of Proposed Dictionary vs. DCT

Images	DCT PSNR (dB)	Proposed PSNR (dB)
Lena	26.11	36.76
Peppers	24.78	34.46
Cameraman	24.61	35.52

In terms of quality, in Figure 3 it can be observed that use of the proposed dictionary eliminates the blocking artifacts and outperforms common DCT dictionary. All details and edges are preserved and the PSNR is significantly higher. Moreover in Table 2, the PSNR values for the reconstruction results on famous image processing pictures using our dictionary are significantly improved.

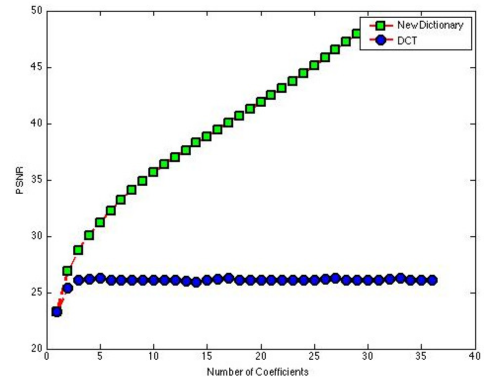


Fig. 2 The influence of number of coefficients using the proposed dictionary and DCT is demonstrated

Fig. 3 The Lena image is sparse coded with 10 coefficients and reconstructed using the proposed dictionary (bottom left) and the common DCT dictionary (bottom right)

B. *Denoising*

CT is one of the most applicable imaging techniques used for medical diagnosis in clinical environments. Concerns for radiation induced cancer, has drawn a lot of attention to reduce the radiation dose given to the patient during each scan. Consequently, the signal to noise ratio (SNR) of scans taken at lower dose is considerably lower than the ones taken at higher dosages, and results in poor diagnostic accuracy. Hence post processing of low dose scans has become a major concern in medical image processing. Noise in CT has various sources with different behaviors. In this work, the reduction of additive noise in the sparse representation framework is targeted.

We applied the proposed denoising method on low dose CT phantoms. The patch sizes are selected to be 8x8. The graph in Figure 4 shows how parameters K and N_S affect the peak signal to noise ratio on the tested images. Except for the case $N_s = 1$ as K is increased the PSNR increases, however blocking artifacts start to appear after a specific value. Accordingly, to achieve the best qualitative and quantitative results the value of K is set to 3 and N_S is set to 1 for our experiments. At fixed mentioned weighting factor the experiment was performed on 17 scans of the chest and abdominal regions and an improvement of roughly 3.22dB in PSNR was achieved. Figure 5 demonstrates the results obtained using the proposed method on one of the abdominal phantom CT scans. By applying the proposed method, the PSNR of the low dose scan with respect to high dose image is improved while the edges and details are preserved. The common K-SVD denoising [3] method with 20 iterations was also applied on the same images. To obtain the same PSNR, the time consumed is approximately 9 times more. These numbers are presented in the table below.

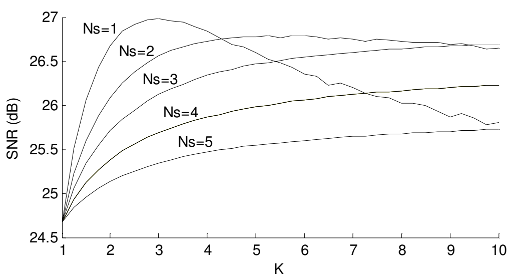
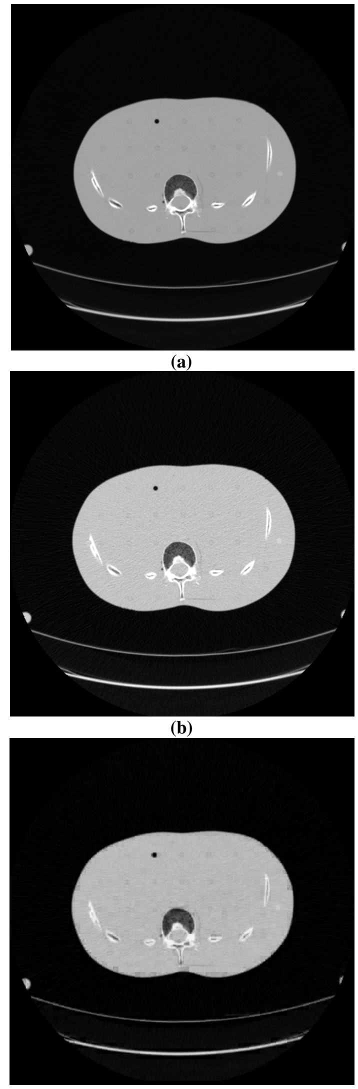


Fig 4. Displaying the relation between SNR versus parameters K and N_s.

Table 3 Comparison of New Approach vs. K-SVD in Terms of Accuracy and Process Time

#	Method	Average PSNR(dB)	Average Process Time (s)
1	K-SVD Denoising	26.95	111.78
2	Proposed Method	27.12	13.60

The time calculated for both processes includes the dictionary construction.



(a)

(b)

(c)

Fig. 4 A) Typical high dose CT phantom. **B)** Typical low dose CT phantom 24.69dB **C)** Denoised Low-Dose CT phantom 27.40dB using our proposed method.

IV. Conclusion

The sparse representation method brings the advantage of non-distorting noise reduction by trying to reconstruct image patches while minimizing the noise representation. In common usage of the sparse coding and dictionary learning in denoising, overlapping patches are selected and averaged out to reduce the noise. This process requires a large amount of memory and is computationally complex. However, our results show that through utilizing our proposed denoising technique along with the designed dictionary, a more efficient and much faster system due to fewer calculations is achieved. Also, our results show that the weighting matrix can be adjusted depending on the application. It is to our great interest to investigate to find a possible relation between the K and the noise variance that brings a further adaptation to the driven approach. In addition to our preliminary evaluations, a more clinically relevant evaluation of our denoising method by trained clinicians to determine filter performance and its potentials is planned.

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