

Under-sampling Trajectory Design for Compressed Sensing MRI

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Abstract— The under-sampling trajectory design plays a key role in compressed sensing MRI. The traditional design scheme using probability density function (PDF) is based up observation on energy distribution in k-space rather than systematic optimization, which results in non-deterministic trajectory even with a fixed PDF. Guidance-based method like Bayesian inference scheme is always bothered with high computational complexity on entropy. In this paper, we study how to adaptively design an under-sampling trajectory in the context of CS with systematic optimization and small complexity. Simulation results conducted on images from different slices and dynamic sequence demonstrate the effectiveness of the proposed method by comparing the designed trajectory with those by traditional method.

I. INTRODUCTION

Compressed sensing, as a new signal sampling and recovery framework, has been applied to reduce the number of required data in MRI. It can break out the restriction of Nyquist sampling theorem and reconstruct MR images from significantly fewer measurements, provided these images are sparse in transform domain. The quality of image reconstruction in CS-MRI highly depends on the sparsifying transform, under-sampling trajectory and non-linear reconstruction method. A large amount of work has been done for sparsifying transform and reconstruction method. However, there is no too much relevant progress for trajectory design scheme.

Fig.1 shows the importance using under-sampling trajectory design for CS-MRI. We can see apparent ringing artifacts (marked by red arrow) in reconstructed image with uniform sampling (Fig.1 (a)) and no observable artifacts with random sampling (Fig.1 (b)).

An adaptive trajectory design scheme for CS-MRI should be evaluated in the following aspects: (1) the design scheme should be reconstruction-guidance-based not observation-based. (2) in the optimization procedure, criterion could effectively reveal the underlying structure of k-space

and computation complexity should be low. (3) the designed trajectory should possess incoherence property, reasonable distribution in low and high frequency domain and robust transplantation from training set to test set. The variable-density random trajectory [1] based on probability density function is the most common design scheme in CS-MRI. However, this method can not give deterministic design even with a fixed PDF, thus depends on repetitive experiments to obtain the best reconstruction. Most of the modifications on this method focus on reducing coherence artifacts [2-5] and do not provide systematic design guidance. Among the systematic-optimization algorithms, Seeger et al. [6] phrased trajectory optimization as a Bayesian experimental design problem with the minimization of posterior. However, only one trajectory with largest entropy was appended from candidate set in each optimization step and the computation of entropy suffered large matrix decomposition, both leading to low optimization efficiency and high computational complexity. Ravishankar et al. [7] proposed an adaptive under-sampling design to redistribute the phase encodings in each optimization step, based on k-space errors between the reconstructed and full-sampled data. However, this method may not be effective when the initial trajectory included enough central phase encodings.

In this paper, we propose a new trajectory design scheme with deterministic result and low computational complexity. In order to avoid the dependence on initial trajectory like [7], the initial under-sampling trajectory only contain the phase encodings on the central region of k-space. K-space errors used in [7] substitute Bayesian posterior entropy used in [6] as the criterion in each optimization step, due to low computational complexity. Besides, two different optimization strategies considering the characteristics of coarse and fine features are implemented for low-frequency and high-frequency domain, respectively. The simulation results demonstrate the superiority of the trajectory using proposed method comparing to conventional design.

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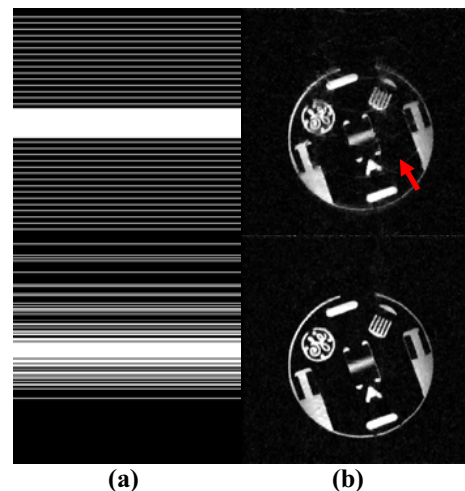


Figure 1. Reconstructed images using two trajectory design scheme. (a) shows the reconstructed result with uniform sampling trajectory (b) shows the reconstructed result with random sampling trajectory.

II. PROPOSED METHOD

We take the framework in [6] and the criterion in [7] for reference. Our algorithm starts with a fully sampled training data and a fixed under-sampling factor to get the designed trajectory. The initial trajectory only contains phase encodings concentrating on the central k-space. All the remainders are in the candidate set. Then the image is reconstructed from the initial under-sampling trajectory using CS and then transformed to k-space by Discrete Fourier transform (DFT), producing reconstructed k-space data.

As we know, coarse and fine features are related to the phase encodings on the central and marginal part of k-space, respectively. Besides, most of the image energy tightly concentrates on the low-frequency part, playing a dominating role on image reconstruction. In [6], trajectory was designed through one-by-one appending scheme without considering diverse importance of phase encodings in low and high frequency domain, leading to low optimization efficiency. Therefore, in our method, different strategies are applied for trajectory design in low and high frequency domain respectively.

K-space error, defined as the root-mean-square error (rms) between the reconstructed and reference k-space data, not only possesses low computation complexity but also reveals the importance of every single phase encoding. From Fig.2 (a), we can see that the distribution of k-space errors of candidate phase encodings is similar to Gaussian distribution, where the k-space error of phase encodings in low-frequency is extremely larger than the ones in high-frequency domain. Therefore, in the low-frequency domain, phase encodings in the candidate set can be batch-optimized. Please note, the small region nearby the central phase encoding were fully sampled and the k-space errors are zero. Inspired by [8,9] where multiple non-zero locations are detected in one step, phase encoding lines corresponding to top N k-space errors are simultaneously added into the designed trajectory. This process will be ended up when the difference of two successive parameter α ($\|\alpha_{n+1}-\alpha_n\|$, n : optimization step), which reveals the reconstruction of coarse features is lower than a given threshold. The expression of α is as follows,

$$\left\{ \begin{aligned} \alpha &= \frac{\Gamma(s - \bar{s}, \hat{s} - \bar{\hat{s}})}{\sqrt{\Gamma(s - \bar{s}, s - \bar{s}) \cdot \Gamma(\hat{s} - \bar{\hat{s}}, \hat{s} - \bar{\hat{s}})}} \\ \Gamma(s_1, s_2) &= \sum_{(i,j)} s_1(i, j) \cdot s_2(i, j) \end{aligned} \right. \quad (1)$$

where s and \hat{s} is the original and reconstructed image respectively. \bar{s} is the mean value of s .

Fig.2 (b) demonstrates the k-space error distribution after the optimization in low frequency domain. We can observe that k-space errors of candidate phase encodings prominently decrease, locations close to the central still possess large k-space errors and apparently sectionalized fluctuation appears in the marginal part. If phase encodings corresponding

to next top N k-space errors after low-frequency domain optimization are still appended into designed set, the final trajectory will concentrate on low frequency domain, leading to the loss of fine features and Gibbs ringing artifacts. Consequently, phase encodings out of the low-frequency domain are cell-appended. Similar to [1], the under-sampling density is also different for locations close to and far away from low frequency domain. A threshold is set to distinguish the phase encodings in high frequency domain. Phase encodings with k-space errors larger than the threshold are equally partitioned to cells with sampling density as J_1 (indicated by the black rectangle in Fig.2 (b)) and the reminders are partitioned to cells with sampling density as J_2 (indicated by the red rectangle in Fig.2 (b)) ($J_1 > J_2$). Only one phase encoding corresponding to the highest k-space error from each cell is appended into the designed trajectory. The whole optimization process will be ended up by achieving to a fixed under-sampling factor. Algorithm 1 shows the pseudo-code of our trajectory design scheme.

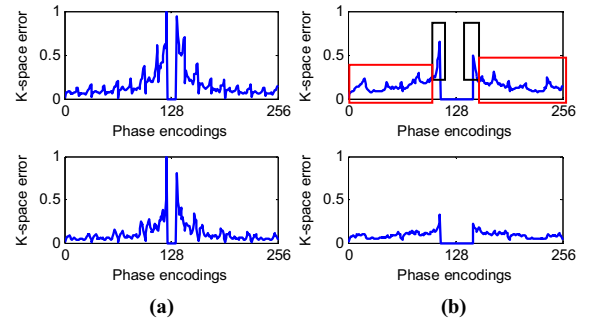


Figure 2. Normalized distribution of k-space error (From top to bottom row: brain and cardiac). The zero and non-zero k-space errors are corresponding to sampled and candidate phase encodings. (a) and (b) are k-space error before and after optimization in low-frequency domain.

Algorithm 1 Trajectory design optimization algorithm

Require: Candidate set C of phase encodings. Initial design X , Reconstruct image y corresponding to X

Repeat

- (1) Transform y to k-space using DFT
- (2) Compute k-space error for all candidate phase encodings

if $\|\alpha_{n+1}-\alpha_n\|$ is larger than a given threshold

- (3) Batch-optimization

In the low-frequency domain, append the winning candidate phase encodings with top N k-space errors into X and remove them from C . Reconstruct image y corresponding to new design X^* .

else

- (4) Cell-optimization

In the high-frequency domain, the candidate locations close to and far away from low-frequency are partitioned into cells with sampling density as J_1 and J_2 ($J_1 > J_2$), respectively. One winning candidate phase encoding corresponding to the highest k-space error from each cell is appended into X . Reconstruct image y corresponding to new design X^* .

Until fixed under-sampling ratio is achieved

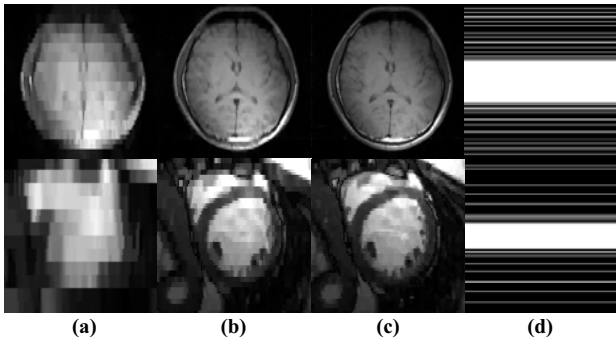


Figure 3. (a), (b) and (c) are reconstructions from initial, after low-frequency optimization and final trajectory. (cardiac image is the zoomed region from original image) (d) is the final designed trajectory. Reduction factor of 2.5 and 4 are used from top to bottom.

III. SIMULATION RESULTS

The proposed method was validated on brain and cardiac images, all of size 256×256 . The initial trajectory contains 5 phase encodings in the central region of k-space and the remainder 251 ones are in the candidate set. The net under-sampling factor is set as 2.5 for brain and 4 for cardiac, respectively. Split Bregman [10] is used to reconstruct images. The difference image between original and reconstructed image is used to evaluate reconstruction quality.

All methods were implemented in MATLAB (Mathworks, Natick, MA) with 4GB RAM and 2.66 GHZ CPU. Fig.3 shows the reconstructions using initial (Fig.3(a)), after low-frequency optimization (Fig.3(b)) and final designed trajectory (Fig.3(c)). We can see the final reconstructions only show negligible artifacts with 2/5 k-space data for brain and 1/4 for cardiac using designed trajectory. And the whole optimization process only costs 2 minutes.

For each kind of image, the transplanted property of the designed trajectory is verified with two k-space data from different slices, showing modest difference in fine features. These two k-space data constitute cross validation set and act as the training and test data crossly. It means we can reconstruct image from one k-space data using the trajectory designed from another slice. The variation reflected in reconstructed image and corresponding difference images (Fig.4) is subtle using two different designed trajectory from the training and testing data, respectively. Therefore, the optimized trajectory can be effectively transplanted from training to test image crossly, which is very crucial for dynamic imaging.

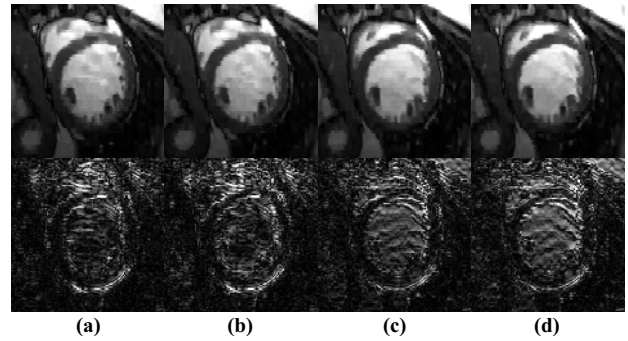
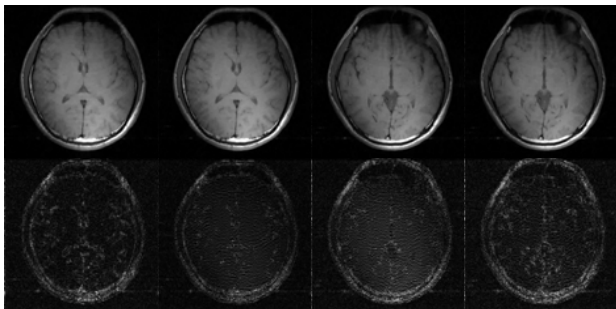


Figure 4. Reconstructions with the optimized trajectory by cross validation. (a) and (c) are images reconstructed by the designed trajectory from their own k-space. (b) and (d) are images reconstructed by the designed trajectory from another slice.

Our trajectory is compared with design scheme in [1]: the variable-density trajectory (vd) design based on PDF. Please note, the final result of “vd” shown here is the average result with four repetitive experiments and the used trajectory in our method is not designed from its own design-space but from another slice. Fig.5 demonstrates that the reconstruction results have apparent difference in different experimental steps with fixed PDF. From Fig.6, the reconstructions with designed trajectory using our method not only provide stable result than those with variable-density trajectory but also exhibit better image quality.

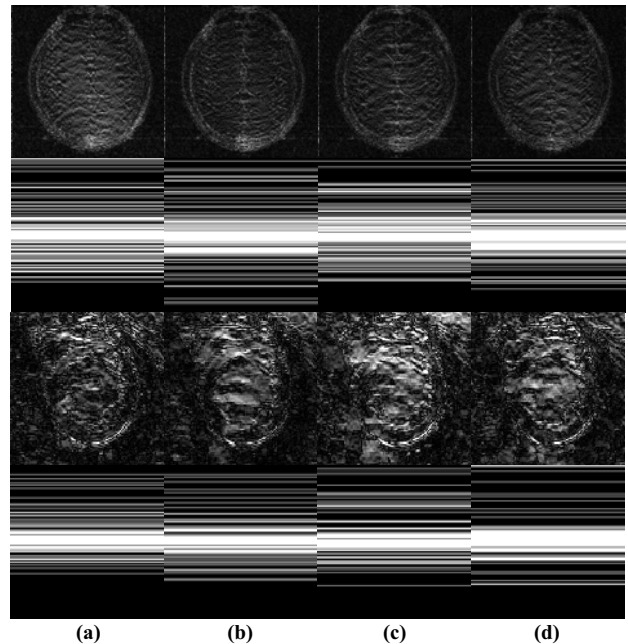


Figure 5. Evaluation of the designed trajectory with variable-density method in four repetitive experiments (1-4 indicates every single experiment). The first and third row are difference images. The second and four rows are corresponding trajectory design generated by VD design scheme in every experimental step.

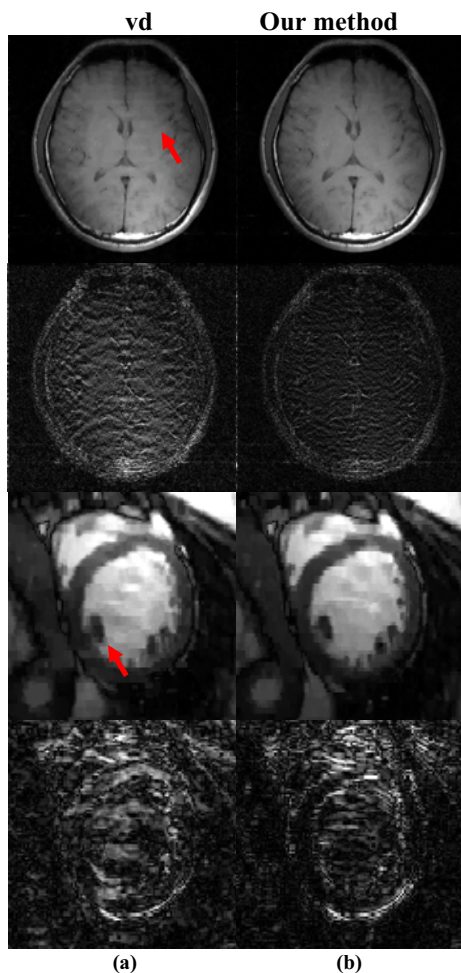


Figure 6. Reconstructed images using two trajectory design scheme. (a) shows the reconstructed result with variable-density trajectory (b) shows the reconstructed result with our trajectory

IV. CONCLUSION

In this paper, an under-sampling trajectory design scheme is proposed for CS-MRI. As the criterion, k-space error can effectively reveals the underlying k-space structure with low calculation complexity. The optimization process is partitioned into two parts with the consideration of distinct characteristics of phase encodings in low and high frequency domain. In low frequency domain, multiple-appending scheme greatly accelerates the optimization speed. In high frequency domain, cell-optimization scheme preserves the incoherence property and fine details. The simulation results in images from different slices and dynamic sequence show that the proposed trajectory design scheme is able to suppress more noise and preserve more details than the variable-density trajectory design and robust in transplanting from training set to test set. Besides, this design scheme can also be extended to non-Cartesian sampling design.

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