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Using Laguerre Expansion within Point-Process Models of Heartbeat Dynamics: A Comparative Study

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Abstract— **Point-process models have been recognized as a distinguished tool for the instantaneous assessment of heartbeat dynamics. Although not thoroughly linked to the physiology, nonlinear models also yield a more accurate quantification of cardiovascular control dynamics. Here, we propose a Laguerre expansion of the linear and nonlinear Wiener-Volterra kernels in order to account for the nonlinear and non-gaussian information contained in the ECG-derived heartbeat series while using a reduced number of parameters. Within an Inverse-Gaussian probability model, up to quadratic nonlinearities were considered to continuously estimate the dynamic spectrum and bispectrum. Results performed on 10 subjects undergoing a stand-up protocol show that this novel methodology improves on the algorithmic performances and, at the same time, more accurately characterizes sympatho-vagal changes to posture.**

I. INTRODUCTION

A point-process is a stochastic process able to continuously characterize the intrinsic probabilistic structure of discrete events. It has been successful applied to study a very wide range of studies, analyzing data such as earthquake occurrences [1], traffic modeling [2], and neural spiking activity [3]. In applying point process models for the assessment of heartbeat dynamics, instantaneous heart rate (HR) and heart rate variability (HRV) measures were defined based on an inverse-gaussian probability distribution, representing the probability to have a new event (i.e. R-wave) given autoregressive history dependence on the previous beat intervals [4], [5]. From a physiological point-of-view, this choice is motivated by a "Wiener process with drift" model of the rising mechanism of the cardiac membrane potential, where a the cardiac contraction is initiated when a threshold is reached [4]. Algorithmic performance, as well as goodness-of-fit improvement, were also considered in defining the best statistical model [6]. Alongside the autoregressive linear combination of the present and past RR intervals, nonlinear terms were also included to define the first moment (mean) of the distribution by using up to second-order nonlinearities of the Wiener-Volterra expansion [7]. Results demonstrated a robust characterization of the inherent heartbeat nonlinear dynamics [7]. The major limitation of these models was that long-term memory and high order nonlinearities were usually discarded due to the complexity of the estimation process (i.e. large number of needed parameters) and to the long time windows that such characterization would require. Here, we illustrate a novel model based on the use of the discrete-time Laguerre expansions of the Wiener-Volterra autoregressive kernels. This choice results in long-term memory and lowest number of parameters [8]. In addition, all previous advantages, such as the high resolution HRV indices without applying any interpolation method, are retained. This approach was first suggested by Wiener in his pivotal monograph [9] and, nowadays, it is widely adopted in system identification [8]. In the next section, the derivation of the novel Laguerre-based point-process linear (ARL) and nonlinear (NARL) models are reported. Both models were compared, in terms of linear (spectral) and nonlinear (bispectral) analysis, with the related current state-of-the-art, represented by the point-process based linear (AR) and the nonlinear (NAR) models.

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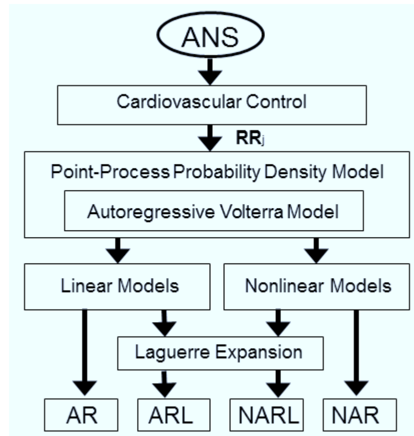

Fig. 1. Block diagram of the models derivation

II. THE HEARTBEAT INTERVAL POINT-PROCESS NONLINEAR MODEL

The general elements behind the considered model's derivation are shown in Fig. 1. We refer to Barbieri et.al. [4] for the complete derivation of the AR model and to Chen et al. [7] for the NAR model. Starting from the surface ECG signal, sampled at 500 Hz and observed within the interval $t \in (0, T]$, we define $\{u_j\}_{j=1}^{J}$ as the ordered set of R-wave events, and $\mathrm{RR}_j = u_j - u_{j-1} > 0$, as the j^{th} R–R interval. It is also possible to define the counting process $N(t) = \max\{k : u_k \leq t\}$ and its differential, $dN(t)$, which is 1 when there is an event (the ventricular contraction), or 0 otherwise. Similarly, a left continuous function $\widetilde{N}(t)$ can be defined as $\widetilde{N}(t) = \lim_{\tau \to t^-} N(\tau) = \max\{k : u_k < t\}$. Assuming history dependence, we use a physiologically-plausible, continuous Inverse-Gaussian distribution $f(t|\mathcal{H}_t, \xi(t))$ as the probability distribution of the waiting time $t - u_j$ until the next R-wave event appears [4]:

$$f(t|\mathcal{H}_t, \xi(t)) = \left[\frac{\xi_0(t)}{2\pi(t-u_j)^3}\right]^{\frac{1}{2}} \times \exp\left\{-\frac{1}{2}\frac{\xi_0(t)[t-u_j-\mu_{\mathrm{RR}}(t,\mathcal{H}_t,\xi(t))]^2}{\mu_{\mathrm{RR}}(t,\mathcal{H}_t,\xi(t))^2(t-u_j)}\right\} \quad (1)$$

with $j = \widetilde{N}(t)$ the index of the previous R-wave event before time t, $\mathcal{H}_t = (u_j, \mathrm{RR}_j, \mathrm{RR}_{j-1}, ..., \mathrm{RR}_{j-M+1})$, $\xi(t)$ the vector of the time-varing parameters, and $\xi_0(t) > 0$ the shape parameter of the inverse Gaussian distribution. To define the first-moment statistic (mean) of the distribution, $\mu_{\mathrm{RR}}(t, \mathcal{H}_t, \xi(t))$, let us consider the Taylor expansion of a general Nonlinear Autoregressive Model (NAR):

$$y(k) = \gamma_0 + \sum_{i=1}^{M} \gamma_1(i)\, y(k-i) + \sum_{n=2}^{\infty} \sum_{i_1=1}^{M} \cdots \sum_{i_n=1}^{M} \gamma_n(i_1, \ldots, i_n) \prod_{j=1}^{n} y(k-i_j) + \epsilon(k)\,. \quad (2)$$

where $\epsilon(k)$ are independent, identically distributed Gaussian random variables. For further derivation, we also define the extended kernels $\gamma_1'(i)$ and $\gamma_2'(i,j)$ as:

$$\gamma_1'(i) = \begin{cases} 1, & \text{if } i = 0 \\ -\gamma_1(i) & \text{if } 1 \le i \le M \end{cases} \quad (3)$$

$$\gamma_2'(i,j) = \begin{cases} 0, & \text{if } ij = 0 \wedge i+j \le M \\ -\gamma_2(i,j) & \text{if } 1 \le i \le M \wedge 1 \le j \le M \end{cases}\,. \quad (4)$$

We can now consider the Laguerre expansions of the NAR kernels up to the second order, i.e. γ_0, $\gamma_1(i)$, and $\gamma_2(i,j)$ (the quadratic term $\gamma_2(i,j)$ is assumed to be symmetric). This choice of expanding the kernels reduces the number of unknown parameters that need be estimated [8]. In addition, the regression is performed on the derivative RR series, in order to improve the achievement of stationarity within the sliding time window $W = 90$ sec [10]. Thus, the obtained Nonlinear Autoregressive with Laguerre expansion (NARL) model can be written as:

$$\mu_{\mathrm{RR}}(t,\mathcal{H}_t,\xi(t)) = \mathrm{RR}_{\widetilde{N}(t)} + g_0(t) + \sum_{i=0}^{p} g_1(i,t)\, l_i(k) + \sum_{i=0}^{q}\sum_{j=0}^{q} g_2(i,j,t)\, l_i(k)\, l_j(k)\,. \quad (5)$$

where

$$l_i(t) = \sum_{n=1}^{\widetilde{N}(t)} \phi_i(n)(\mathrm{RR}_{\widetilde{N}(t)-n} - \mathrm{RR}_{\widetilde{N}(t)-n-1}) \quad (6)$$

is the output of the Laguerre filters and

$$\phi_i(n) = \alpha^{\frac{n-i}{2}}(1-\alpha)^{\frac{1}{2}} \sum_{j=0}^{i} (-1)^j \binom{k}{j}\binom{i}{j} \alpha^{i-j}(1-\alpha)^j$$

is the i^{th}-order discrete time orthonormal Laguerre function, with $(n \ge 0)$ and α the discrete-time Laguerre parameter. When α is chosen in the open interval $(0,1)$, it determines the rate of exponential asymptotic decline of these functions while for $\alpha = 0$ the NARL model corresponds, apart for the sign, to the NAR model. The ARL model is obtained by dropping off the nonlinear term of (5). By substituting (6) in (5), one can obtain for a given NARL model the corresponding NAR model with degree of nonlinearity 2 and long-term memory [8]:

$$\begin{aligned}\mu_{\mathrm{RR}}(t,\mathcal{H}_t,\xi(t)) &= \mathrm{RR}_{\widetilde{N}(t)} + \gamma_0 \\ &+ \sum_{i=1}^{\infty} \gamma_1(i,t)\,(\mathrm{RR}_{\widetilde{N}(t)-i} - \mathrm{RR}_{\widetilde{N}(t)-i-1}) \\ &+ \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \gamma_2(i,j,t)\,(\mathrm{RR}_{\widetilde{N}(t)-i} - \mathrm{RR}_{\widetilde{N}(t)-i-1}) \\ &\times (\mathrm{RR}_{\widetilde{N}(t)-j} - \mathrm{RR}_{\widetilde{N}(t)-j-1})\end{aligned} \quad (7)$$

From (7) it is straightforward to note that, even with an equal degree of nonlinearity, the NARL model retains information about an infinite amount of past samples. Furthermore, the model's number of parameters depends on the number of the Laguerre functions considered rather than on the number of regressions on the past observation. Moreover, $\mu_{\mathrm{RR}}(t, \mathcal{H}_t, \xi(t))$ is still defined in continuous time, so it is possible to obtain an instantaneous R–R mean estimate at a very fine timescale (with an arbitrarily small bin size Δ), without interpolation between the arrival times of two beats. We use the Newton-Raphson procedure to maximize the local log-likelihood defined in [4] in order to estimate the unknown time-varying parameter set $\boldsymbol{\xi}(t) = [\xi_0(t), g_0(t), g_1(0,t), ..., g_1(p,t), g_2(0,0,t), ..., g_2(i,j,t)]$. The optimal order $\{p, q\}$ is estimated by means of the Akaike Information Criterion (AIC) and of the point process model goodness-of-fit applied to a subset of the data [4]. Model goodness-of-fit is based on the Kolmogorov-Smirnov (KS) test and associated KS statistics [4]. Autocorrelation plots are also considered to test the independence of the model-transformed intervals [4]. Once the order $\{p, q\}$ is determined, the initial NARL coefficients are estimated by the method of least squares.

A. Quantitative Tools

Once the model's parameters are derived, a few additional steps are required to calculate the quantitative tools, i.e. the instantaneous autospectrum and bispectrum. The general scheme is shown in Fig. 2. For instance, starting from the NARL model's coefficients, the following transformations are needed:

- from the fitted coefficients $g_n(...)$ of the NARL model (5), use the Laguerre deconvolution [8] to obtain the NAR kernels $\gamma_n(...)$;

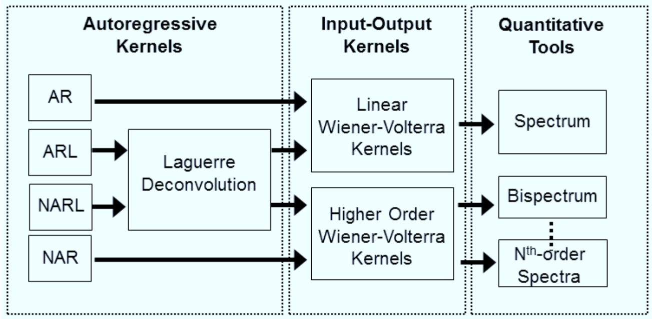

Fig. 2. Block diagram of the quantitative tools derivation

- from $\gamma_n(...)$ find $\gamma'_n(...)$;
- compute the Fourier transforms $\Gamma'_n(...)$ of the kernels $\gamma'_n(...)$;
- compute the Wiener-Volterra Input-Output kernels.

Starting from the Fourier transforms of the extended NAR kernels, $\Gamma'_1(f_1)$ and $\Gamma'_2(f_1, f_2)$, the required Wiener-Volterra Input-Output kernels of order p, $H_p(f_1, \ldots, f_n)$, can be estimated by using the following recursive relationships [11]:

$$H_1(f) = \frac{1}{\Gamma'_1(f)} \tag{8}$$

$$H_2(f_1, f_2) = -\frac{\Gamma'_2(f_1, f_2)}{\Gamma'_1(f_1)\Gamma'_1(f_2)} H_1(f_1 + f_2) \tag{9}$$

A more general formulation of these relationship can also be found in [11]. Given the $\Gamma'_1(f_1)$ term, we can compute the time-varying parametric (linear) autospectrum [12] of the derivative series:

$$\mathcal{Q}(f, t) = S_{xx}(f, t)H_1(f, t)H_1(-f, t) \tag{10}$$

where $S_{xx}(f, t) = \sigma^2_{\mathrm{RR}}$. Referring to the mentioned derivative regression, the time-varying parametric autospectrum of the RR intervals is given by multiplying its derivative spectrum $\mathcal{Q}(f, t)$ by the quantity $2(1 - cos(\omega))$ [10]. By integrating (10) in each frequency band, we can compute the index within the very low frequency (VLF = 0.01-0.05 Hz), low frequency (LF = 0.05-0.15 Hz), and high frequency (HF = 0.15-0.5 Hz) ranges.

Moreover, given the $\Gamma'_2(f_1, f_2)$ term, it is possible to define the instantaneous bispectrum as follows [13]:

$$\begin{aligned} Bis(f_1, f_2, t) &= 2H_2(f_1 + f_2, -f_2, t)H_1(-f_1 - f_2, t) \\ &\times H_1(f_2, t)\mathcal{S}_{xx}(f_1 + f_2, t)\mathcal{S}_{xx}(f_2, t) + 2H_2(f_1 + f_2, -f_1, t) \\ &\times H_1(-f_1 - f_2, t)H_1(f_1, t)\mathcal{S}_{xx}(f_1 + f_2, t)\mathcal{S}_{xx}(f_1, t) \\ &+ 2H_2(-f_1, -f_2, t)H_1(f_1, t)H_1(f_2, t) \times \mathcal{S}_{xx}(f_1, t)\mathcal{S}_{xx}(f_2, t) \end{aligned}$$

The dynamic bispectrum is an important tool for evaluating the instantaneous presence of non-linearity in time series [14]. Since the bispectrum presents several symmetry properties that divide the (f_1, f_2) plane in eight symmetric zones, for a real signal the bispectrum is uniquely defined by its values in the triangular region of computation Ω, $0 \leq f_1 \leq f_2 \leq f_1 + f_2 \leq 1$. Through bispectral analysis it is possible to further evaluate the nonlinerar sympatho-vagal interactions

TABLE I

RESULTS OF THE COMPARISON AMONG THE AR, ARL, NAR AND NARL MODELS ON THE STAND-UP DATA.

Statistical Indices	Model	Rest	Stand-Up	P-Value	AUC
μ_{RR}(ms)	AR	906.17±116.21	774.48±80.41	<0.02	0.744
	ARL	914.94±122.70	773.46±80.67	<0.02	0.754
	NAR	908.05±117.03	771.94±75.23	<0.03	0.744
	NARL	**917.39±124.72**	**770.04±75.29**	**<0.02**	**0.761**
σ_{RR}(ms)	AR	19.69±9.37	15.84±5.06	>0.05	0.588
	ARL	19.72±9.37	16.57±4.89	>0.05	0.568
	NAR	20.05±9.39	16.19±6.59	>0.05	0.550
	NARL	20.62±9.45	15.66±5.20	>0.05	0.560
LF (ms^2)	AR	328.54±260.34	410.03±305.24	>0.05	0.503
	ARL	569.52±347.75	581.42±422.77	>0.05	0.509
	NAR	392.38±258.09	310.99±307.15	>0.05	0.487
	NARL	**345.76±261.93**	**415.40±338.17**	>0.05	**0.566**
HF (ms^2)	AR	179.39±149.43	76.13±51.63	>0.05	0.656
	ARL	194.05±159.26	120.44±65.97	>0.05	0.625
	NAR	162.61±107.35	59.07±37.05	>0.05	0.678
	NARL	204.19±169.47	91.28±58.65	>0.05	0.658
$LF/HF.$	AR	1.37±0.78	2.58±2.41	>0.05	0.625
	ARL	1.64±1.09	2.77±2.07	>0.05	0.612
	NAR	1.23±0.97	3.39±3.13	>0.05	0.567
	NARL	**0.83±0.73**	**2.52 ±2.32**	**>0.05**	**0.628**
$LL(10^6)$	NAR	118.80±383.58	138.03±98.67	>0.05	0.735
	NARL	163.89±146.24	162.21±135.41	>0.05	0.641
$LH(10^6)$	NAR	313.77±219.27	403.60±308.62	>0.05	0.465
	NARL	**326.78±272.38**	**137.66±88.89**	**>0.05**	**0.641**
$HH(10^6)$	NAR	163.62±149.16	61.36±52.40	>0.05	0.687
	NARL	**768.73±663.26**	**267.62±220.08**	**<0.03**	**0.714**

P-values are obtained by rank-sum test between the Rest and Tilt epochs. Lines in bold indicate an improvement in terms of AUC results given by the NARL model.

by integrating $|B(f_1, f_2)|$ in the appropriate frequency bands. Specifically, we can evaluate:

$$LL(t) = \int_{f_1=0^+}^{0.15} \int_{f_2=0^+}^{0.15} Bis(f_1, f_2, t) df_1 df_2 \tag{11}$$

$$LH(t) = \int_{f_1=0^+}^{0.15} \int_{f_2=0.15^+}^{0.4} Bis(f_1, f_2, t) df_1 df_2 \tag{12}$$

$$HH(t) = \int_{f_1=0.15^+}^{0.4} \int_{f_2=0.15^+}^{0.4} Bis(f_1, f_2, t) df_1 df_2 \tag{13}$$

B. Experimental Results

We compared the proposed Laguerre-based models in a study of the RR series of 10 healthy subjects when performing a stand-up protocol. The study, fully described in [4], was conducted at the Massachusetts Institute of Technology (MIT) General Clinical Research Center (GCRC). The AIC analysis indicated $p \in \{8, 9, 10\}$ as the optimal AR order and $p \in \{6, 7, 8\}$ and $q \in \{1, 2\}$ as optimal NAR orders in almost all cases. Likewise, we obtained $p \in \{4, ..., 8\}$ and $p \in \{3, 4\}$ and $q \in \{2, 3\}$ for the ARL and the NARL models, respectively. A representative tracking result is shown in Fig. 3. We evaluated the statistical differences between the supine and upright epochs of the rest and the stand-up protocol. The difference was expressed in terms of p-values from a

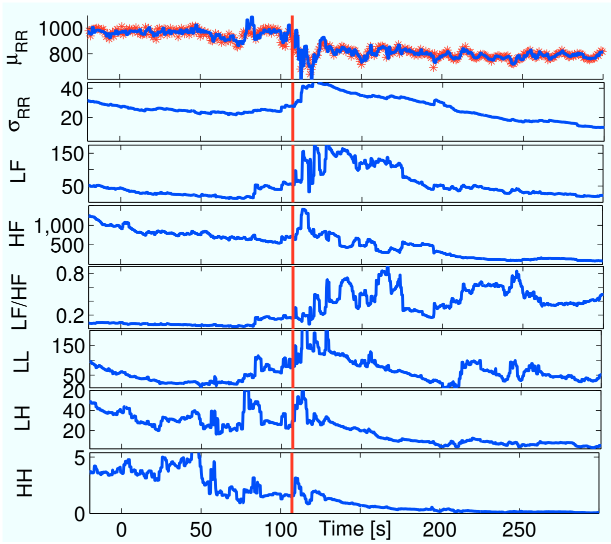

Fig. 3. Instantaneous heartbeat statistics computed from a representative subject (N. 1) of the stand-up protocol using the NARL model. In the first panel, the estimated $\mu_{\mathrm{RR}}(t)$ is superimposed on the recorded R-R series. Alongside, the instantaneous heartbeat Power spectra evaluated in Low frequency (LF), High frequency (HF), their ratio (LF/HF) and three bispectral measures are reported.

non-parametric rank-sum test, under the null hypothesis that the medians of the two sample groups are equal. Given the rank-sum statistics, we also calculated the Area Under the receiver operating characteristic Curve (AUC). The results are shown in Table I. The LF and HF power as well as their ratio were calculated from all of the models, whereas the nonlinear frequency interactions LL, LH and HH were estimated from the two nonlinear models (i.e. NAR and NARL). All features were calculated instantaneously with a 5 ms temporal resolution. For the statistical analysis we considered the average time-varying values over each of the computed indices along the considered protocol. The confidence interval of all of the linear features is quite similar among the different models, thus demonstrating that the inclusion of nonlinear terms in the model does not affect the linear part of the signal. It is worth to notice that even when there is no statistical difference between the rest and stand-up conditions, the NARL model almost always provides the best results in term of AUC. Moreover, the high inter-subject variability leads to no statistical difference between the two considered states (Rest vs. Stand-up) for all considered indices, with the exception of the HH parameter evaluated by means of the NARL model ($p<0.03$).

III. CONCLUSIONS

We presented a comparison among four point-process models used for instantaneous heartbeat assessment. The previous developed linear and nonlinear autoregressive models (namely AR and NAR) along with respective novel Laguerre-expanded versions (namely ARL and NARL) were tested to assess sympatho-vagal dynamics in ten healthy volunteers performing a stand-up protocol. According to the AIC evaluations, the models using Laguerre expansion required a reduced number of regressors. In addition, they gave better results in terms of AUC in six of the eight considered features. Among them, the HH measure coming from the nonlinear coefficients of the NARL model was the only one able to discriminate the rest (supine) phases with respect to relative stand-up. It is possible to conclude that assessment of heartbeat dynamics is more accurately performed when a nonlinear model is adopted. Furthermore, the inclusion of the orthonormal basis of Laguerre functions improves the performance and reduces the number of regressors. Future works will focus on the comparison of the performance with other time-variant algorithms, e.g. the Swami's algorithm for bispectral analysis.

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