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Towards a Graph Theoretical Approach to Study Gender Lateralization Effect in Mathematical Thinking

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***Abstract*—Gender differences in mathematical thinking is a common concern of scientists from different research fields. Both parents and teachers report that males seem to perform better in complex mathematics compared to females. This study comes to shed light in the different organization of the underlying functional networks, in order to investigate the aforementioned observation, without supporting or rejecting this statement. In this sense, it is generally accepted that females use their both hemispheres to accomplish a certain task, while males use mostly the hemisphere which is properly suited. For the purposes of the current analysis, electroencephalographic recordings were collected from 11 males and 11 females, during a difficult mathematical task. Then a previously proposed model was used in order to pass from the sensor level to the cortical one, in order to examine the networks formed among the cortical dipoles. Mutual information was employed to form the graphs represeting the functional connectivity among the different dipoles, while the density, the global and the local efficiencies were further examined. The results suggest that females use their both hemisphere to solve the complex mathematical task while males use mostly their left hemisphere which is the responsible one for the mathematical thinking.**

***Index Terms*—Graph Theory, Mathematics, Mathematical Cognition, Mutual Information, Inverse Problem**

I. Introduction

Gender differences in mathematical abilities, still concern the scientists who are trying to investigate the females under-representation at difficult mathematics [1], [2], while stereotypes that males seem to perform better in mathematics are well held by both parents and teachers [3], [4], [5].

Reviewing this difference from a neuroscientific point of view, we observe that males have larger cerebra compared to females of the same age and health status [6]. Although most brain structures are larger in males than females brain, this difference is most prominent in the frontal and the occipital lobes, bilaterally [7]. Also male differentiated brains have a thicker right hemisphere [6]. Given the localization of spatial and mathematical abilities in the right hemisphere [6], this finding may explain, at least to an extend, the pronounced orientation of brain males to spatial and mathematical functionality [6].

Corpus Callosum seems to play a crucial role in the mathematical differentiation between genders. Corpus callosum is a large tract of neural fibers facilitating the inter-hemispheric communication. The larger corpus callosum allows more information to flow between the left and the right hemisphere. Females have larger corpus callosum in contrast to males [8], [9], [10] depicting the fact that when women use their both hemispheres, they tend to create more synapses between them.

Males exhibit significantly increased rightward asymmetry in terms of the corpus callosum thickness, resulting in a more widespread functional organization of callosal projections in the right hemisphere [6]. These findings support the origin of the dimorphic organization of the human brain at the early sexual differentiation. Moreover, it is evident that the gender effects on the brain organization are projected on its functionality and can be studied in the level of inter-hemispheric exchange of information.

Brain functional connectivity is a tool to study arbitrary intra and inter-hemispheric functional relationships, however, with no reference to anatomical connections or an underlying causal model [11]. Many connectivity metrics have been proposed so far [12] in order to quantify the amount of functional information exchanged and its direction as well [13]. Mutual information is preferred as it describes the dependence between the joint distribution of two variables and what the joint distribution would be if these two variables were independent. Following the formation of the brain functional networks, Graph Theory is then employed to reveal their global and local characteristics, such as the density, the global and the local efficiencies, etc. Statistical comparisons on these graph parameters between different brain states have shown significant properties of the brain function, where classic older methods have poorly managed, or even failed, to detect.

There is limited evidence on mathematical thinking investigated in the framework of brain functional connectivity. Multiplication tasks were associated with a widespread pattern of distant signal synchronizations, indicating increased demands for neural networks cooperation during performance of highly demanding cognitive tasks [14]. We hypothesize that the gender lateralization effect, present during mathematical thinking, will be detected by the parameters of the brain functional networks treated as graphs. More specific, our assumption lies with Carter's opinion [15], which supports that there is a tendency for women to bring both sides of their brain to bear on complex mental tasks, while men use mostly the side which is better suited. So for males, we are expecting to see denser graphs in the left hemisphere which is the dominant one for the mathematical interpretation, while the functional networks produced by the female brains to mathematical stimuli are expected to be spreaded in both hemispheres equally.

II. Materials and Methods

A. Participants

Twenty two right handed medical students of the University of Crete were used for the purposes of this study. Half of them (11) are males and the rest half (11) are females. All participants had normal or corrected to normal vision and they were refrained from any alcohol or caffeine consumption the day before the experiment. They were also asked to sleep as adequately and comfortably as possibly achievable the night before the recordings. They also signed an informed consent form, while the experimental protocol was approved by the ethics committee of the medical department.

B. Experimental Design

The EEG recordings were performed in dark electrical and sound attenuated room. They were lying in a comfortable chair and the stimuli were given with a laptop about 80 cm in front of the individual. The EEG signals were recorded with 31 electrodes placed on the scalp according to 10/10 International System (FP2, F4, FC4, C4, CP4, P4, O2, F8, FT8, Fz, FCz, Cc, CPz, Pz, Oz, FP1, F3, FC3, C3, CP3, C3, F3, FC3, C3, CP3, P3, O1, F7, FT7, PO7, AND A1+ A2 as reference). From these 31 electrodes, one was used as the trigger, one for the detection of eye-movements and one for the grounding, resulting to 28 EEG recordings. The signals were amplified and then digitized at 500 Hz and online filtered (1-200 Hz).

In order to examine the cerebral responses to mathematical stimuli, two different tasks were used during the EEG recordings. The first one, which was served as the control one, appears a white cross to a black screen for thirty seconds (CTRL), while the second task included eight trials of two-digit multiplications (MULT) (e.g.31x24). All the trials were the same for all the participants and they were followed by a time frame in order to give the subjects the opportunity to calculate the given multiplication. It has to been noticed that most of the estimations were correct and the herein analysis was based only on them. Each segment's duration was at least 19 seconds but the herein analyzed segments were 10 sec without any visible artifacts, as they were observed by three independent observers .

C. Cortical Activity

EEG records the activity of the cortical dipoles oriented in tangential or radial directions according to the scalp surface. However the variation of the electrical conductivity among the different head compartments leads to the volume conduction problem, which is very serious for the functional connectivity study [16]. To face this problem, the cortical activity was estimated from the 28 EEG signals, by adopting a realistic head model, which describes the different electrical conductivities of the head structures and the sensors' geometry, a cortical dipole model, which predefines the localization of the dipole sources, and the inverse solutions [17] [18]. In the current study an average head model from the reconstruction of 152 normal MRI scans (MNI template http://www.loni.ucla.edu/ICBM/) has been used, while the four different compartments of the head model (scalp, outer and inner skull, cortex) were extracted using the Boundary Element Method (BEM). BEM is implemented in the Brainstorm toolbox which is freely available in http://www.neuroimage.usc.edu/brainstorm. BEM computes the aforementioned compartments by closed triangle meshes with limited number of nodes (in our case we have used 258 nodes). Regarding the regularized solution of the linear inverse problem, we have used the column-norm normalization, resulting to a transition kernel from our 28 scalp signals to 258 cortical signals.

D. Functional Connectivity

The functional connectivity of the cortical sources was computed using the Mutual Information (I) index [12] given by the next formula:

$$I(X;Y) = \sum_{y \in Y} \sum_{x \in X} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

where $p(x,y)$ is the joint probability distribution function of X and Y, and $p(x)$ and $p(y)$ are the marginal probability distribution functions of X and Y respectively. Using I we have computed 22 (2 Tasks x 11 Subjects) adjejency matrices (AM) for males and 22 for females.

E. Graph Analysis

Graph is a mathematical representation of a set of objects where some of them are connected by links. These interconnected objects are called vertices or nodes while the links that connect some pairs of vertices are called edges. In our study, the vertices are the cortical dipoles whereas the edges are defined by the I value of each pair.

In order to pass from the AMs to binary and undirected graphs, having a representation of the population's behaviour, we have employed a method proposed by [19]. According to this methodology, for every (i,j) element of the AMs we form two variables containing 11 values each, and then non-parametric statistical tests, by means of Mann-Whithney U-Test, defines if an element has statistically significant difference among the CTRL and the MULT tasks (Fig.1). If it's true then a link connecting the i^{th} and the j^{th} nodes will be formed, otherwise these nodes will be unconnected. So we have performed 66564 (258x258) U-tests for males and females separately, corrected using the False Discovery Rate [20]. So, with this way one characteristic networks (CN) for

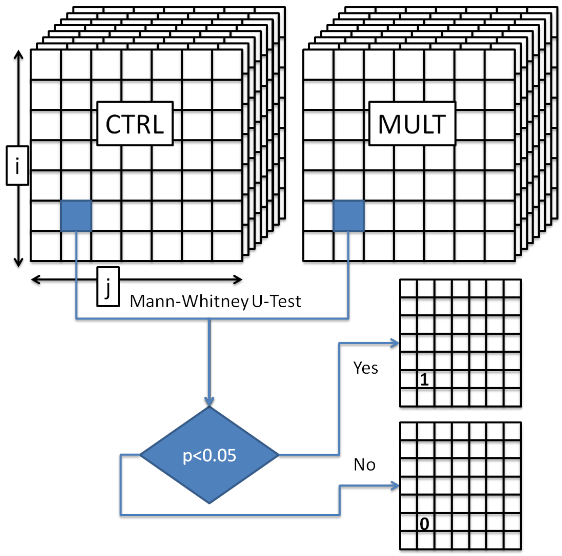

Fig. 1. **Graph Generation**. For the (i, j) element of the AMs we form two variables containing 11 values each (blue squares), and then the non-parametric U-Test defines if the CTRL and the MULT tasks seem to have statistically significant differences in the certain element. If it's true then a link connecting the i^{th} and the j^{th} nodes will be formed (1 in the graph matrix), otherwise these nodes will be unconnected (0 in the graph matrix)

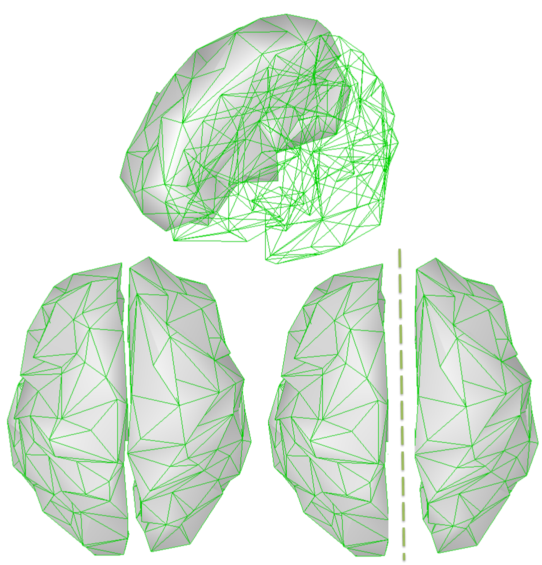

Fig. 2. **Cortex Model**. In the top figure we can see the cortex model, in a 3D format, alongside with the triangular mesh which defines the nodes' and the edges' position. In the left bottom subfigure we have a top view of the same model, while the right one depicts clearly the geometry of the two different networks taken in this study, which corresponds to the left and the right hemisphere respectively.

males and one for females were formed. For the lateralization purposes, we have divided each one of the CNs into two separate networks, which correspond to the two different hemispheres of the cerebral cortex (Fig.2), where the network parameters were further examined.

For the purposes of the current study, three network parameters were employed and they are fully described next.

1) Density: The density (K) of a graph is the ratio of the number of edges across the number of possible edges. The following formula was used for the densitys computation

$$K = \frac{2E}{V(V-1)}$$

where E is the number of edges and V is the number of possible edges. The maximum number of edges is $\frac{1}{2}V(V-1)$, so the maximal density is 1 and the minimal density is 0 [21].

2) Global Efficiency: Latora and Marchiori [22] defined the efficiency of the path between two vertices as the inverse of their shortest distance. In case where there is not any path to connect two vertices, their distance is infinite so their efficiency is 0. So global efficiency (GE) is given by the following typo:

$$GE = \frac{1}{n} \sum_{i \in \mathbf{N}} \frac{\sum_{j \in \mathbf{N}, \mathbf{j} \neq \mathbf{i}} \frac{1}{d_{ij}}}{n-1}$$

where $d_{ij} = 1$ if the i^{th} and the j^{th} nodes are connected, otherwise $d_{ij} = 0$.

3) Local Efficiency: Local efficiency (LE) (E_i) of a graph is the average of the global efficiencies of each subgraph. The subgraphs are formed by removing the i^{th} node, and taking the rest nodes which were connected to the removed one. Local efficiency reflects the tendency of a graph to form clusters. In this sense, it is understandable that a fully connected graph has local efficiency equal to 1, while the local efficiency of an empty graph is 0 [12].

III. Results and Discussion

Results suggest that females use their both hemispheres equally to complete two-digit multiplications while the males make use of their left hemisphere mostly (Fig. 3). Taking into account the figures 3 and 4, it seems also that females produce denser graphs compared to males, as a cerebral response to a complex mathematical task. According to these results our main hypothesis seems to be correct, because graph analysis seems also to support Carter's [15] statement, which mentions that there is a tendency for women to bring both sides of their brain to bear on complex mental tasks, while men use mostly the side which is better suited. This can be further explained by the fact that females have thicker corpus callosum which is the major connection path between the two hemispheres. In this sense, the left hemisphere seems to be dominant in males when they are involved in complex arithmetics, because the left hemisphere, and especially the left fronto-temporal (Fig.3) region is used for logic, math and analytical reasoning.

Regarding the GE we observe that both males and females seem to have more efficient graphs in the left hemisphere compared to the right one. However the difference between the GE among the two hemispheres is lower in females rather than in males. This gives us a further evidence to support that females use their both hemispheres to bear on two-digit multiplications, that's why both hemispheres' CNs have similar efficiency. Samely, we can conclude for males that

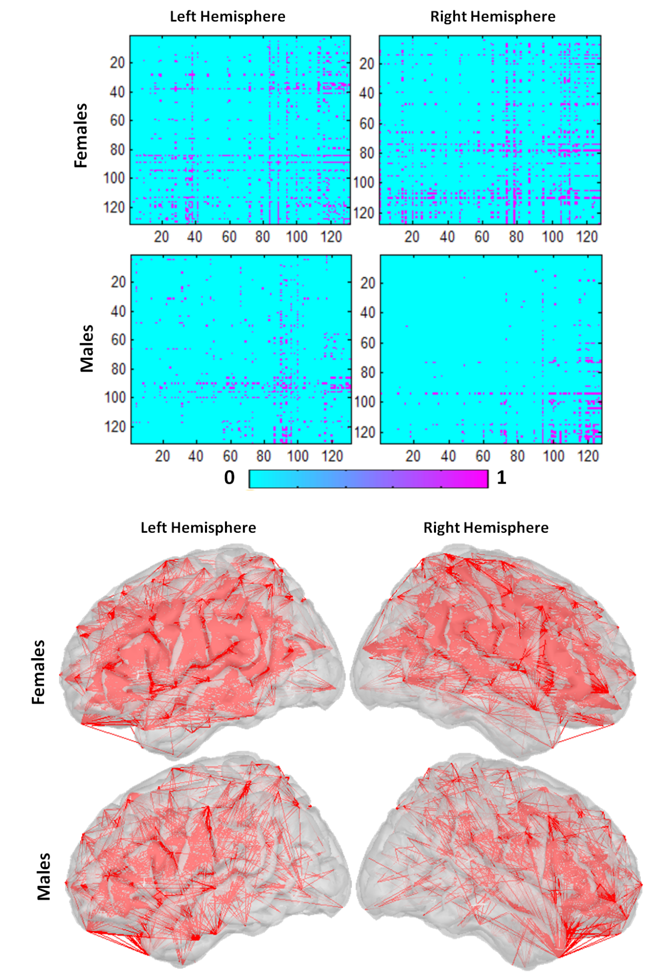

Fig. 3. **Gender Differences among the Left and the Right Hemispheres**. In this figure it is clearly observable that females are engaging both hemispheres in order to accomplish a mathematical task while males are mostly based to their left hemisphere. In the first four plots, the graphs are depicted in the form of their AMs, while in the last four images the CNs are depicted inside the cerebral cortex.

their right hemisphere's global efficiency is too low, compared to the left's one, because they are not based on their right hemisphere to solve difficult arithmetics.

Higher values of local efficiency suggest a larger level of internal organization and fault tolerance. So it seems that for both males and females right hemisphere is more faults tolerant in contrast to the left one, because the right hemisphere is not the dominant hemisphere for the complex arithmetics. We also observe that males seem to have greater difference of their local efficiency among the two hemispheres compared to females, which is explained by the fact that males use more their left hemisphere, so the right one is much more faults tolerant, while the females use both hemispheres, so the right hemisphere is less faults tolerant compared to the right one.

IV. Conclusion

This study shed more light, in the differences between males and females during difficult mental arithmetics, using

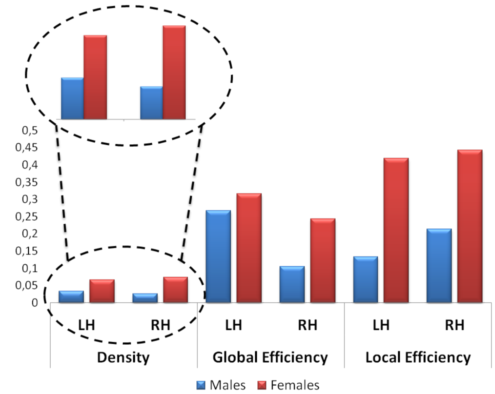

Fig. 4. **Graph Parameters**.

the graph theory. All of the aforementioned results suggest that females use their both hemisphere to solve two-digit multiplications while males use mostly their left hemisphere which is the responsible one for the mathematical thinking.

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