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A Structural Approach to Image Segmentation

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***Abstract*—In this work we propose an efficient and polynomial algorithm for the graph segmentation problem based on the coloring problem for graphs. The work here presented extend the algorithm published in [10] making possible the segmentation to any class of graph (not only fuzzy-valued planar graphs) and also improving the computational complexity of the previous work.**

I. Introduction

CLASSIFICATION can be understood as one of the main activities of human brain, perhaps the most characteristic one. According to the tradition of some ancient religions, for example, it looks like the first objective of human being is to name things, i.e., classify objects and animals, as written in the Bible, Genesis 2.19: "And the Lord God formed out of the earth all the wild beasts and all the birds of the sky, and brought them to the man to see what he would call them; and whatever the man called each living creature, that would be its name" (translation taken from The Jewish Study Bible, by A. Berlin, M. Zvu Brettler and M.A. Fishbane, Oxford University Press, 2004). In fact, as proven by modern medicine (see, e.g., [1]), rationality is deeply related to the analysis of the decision making problem (implying the use of concepts and the associated underlying classification problems) rather than to the final decision itself (which is in the sphere of emotion, see also [15]).

Classification use to be a procedure in which individual items are placed into groups based on their quantitative and qualitative description. Classical classification problems assume a statistical approach, connecting to a huge variety of decision making problems within economics, biology, geology, industry, etc. It is then relevant for this paper to stress how quite a number of this applied research on classification take advantage of a nice mathematical properties like independency , a property that can never be proven (see [9] for an interesting critical approach in a related context, but also the shocking experiment shown in [17]). The only thing we can do in Statistics to check independence is to check that dependency seems to be not significative. In fact, things are always connected within a structure that too often seems to be forgotten. And few classification algorithms can be found dealing with connected objects fitting non trivial structures. There is a lot of work ahead for developing classification models that take into account how real objects are related (see, e.g., [6] as an example of classification model without structure and the relevant modification proposed in [14]).

In the case in which the relation between objects can be modeled as a graph, some classification problems can be approached as partition problems in graphs. In a partition problem, a partition of the graph has to be found attending to some desired properties. During the last three decades, many researchers have been developing models, building algorithms and implementing solutions for district-partition problems, for example. Such problems can be viewed as a grouping process of elementary units or atoms of a given territory into larger pieces of land or zones, thus giving rise to a partition, also called a district map. There are many practical questions and applications related to district problems, including defining the electoral districts of a country, establishing different work or delivery zones for a traveling salesperson team; defining areas within metropolitan internet networks in order to install hubs; defining a public transportation network pricing system; designing a school district plan, electrical power zones or a police district map; constructing a district map for salt spreading operations; or defining a district-based health information system, among others. Unfortunately, in order to obtain a partition in a reasonable period of time, it is usually necessary to impose too many constraints to the associated graph.

The algorithm and methodology proposed in this paper, should allow to address classification problems without imposing any a priori constrain to the graph representing its associated structure. As it will be acknowledged below, the classification-coloring algorithm here defined shares key elements of the coloring algorithm proposed in [11], which is here generalized for arbitrary graphs. The original coloring algorithm was developed within the field of image classification, being our main aim to obtain a segmentation of the digital image or the remote sensing image under study. Such an algorithm allowed the classification of the pixels in an image, determining homogeneous regions for a further supervised classification. Later (see [10]), the authors proposed a first extension of such a coloring algorithm to the case in which the graph was fuzzy and planar. This second algorithm allowed to model digital images in a more realistic way, since fuzziness appears naturally when we work with real problems and images, at least in the framework of scenario analysis. It is then important to emphasize that the two algorithms previously developed in [11] and [10] for coloring and classifying images are only valid for a specific class of graphs and had a specific application in the field of classification images, a limitation that we can avoid under the approach of the present paper.

The main contribution of this paper is to develop a

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classification algorithm in which it is supposed that the target or objects that have to be classified are not independent and the relation among this items are given by an arbitrary graph. In order to do that we extend the ideas of the coloring algorithms presented in [11] and [10] to arbitrary graphs.

II. Associated graphs

As pointed out previously, we are going to identify the existing relations of objects or items that has to be classify by means of a graph. We will denote by P the set of items and by k the number of characteristic or variables associated to each item $p \in P$. We will identify by $p_i, i \in \{1, \ldots, k\}$, the i^{th}-characteristic or variable of item p.

Let $G = (V, E)$ be the graph that shows the relations among items. Obviously, the set of nodes in the graph coincides with the set of items $P = V$. Now, the problem we want to solve is to classify (finding homogeneous regions) the set P when the a priori information that we have is the characteristic associated to the objects and the graph that show the relations among them.

In order to solve this classification problem, and following the ideas described in [10], [11], from P and G we are going to define a valued graph or a fuzzy graph.

Definition. Given the classification problem previously defined, and given a distance function d on R^k, the *item valued graph* can be defined as the pair

$$G(P) = (P, E)$$

when the value for the edge associated to two adjacent items p and p' is the distance between them, i.e.

$$E_{p,p'} = d\left((p_1, \ldots, p_k), (p'_1, \ldots, p'_k)\right)$$

Unfortunately, there are some situations (see, e.g., [10] for more details) in which the distance between two elements includes some lack of precision or ambiguity. It would be the case, for example, when we consider the aggregation of several measures obtained by different experts: a well known problem in remote sensing is to chose an adequate distance in order to compare opinions from different experts (the analysis of ignorance and overlap measures will be extremely pertinent here, see [2], [3]).

In particular, in order to capture the natural fuzzy uncertainty [21], and in order to give more flexibility to other existing crisp classification procedures, we shall consider that a fuzzy distance (in the sense of [12]) expresses the relation between the measured properties of items,

$$d : P \times P \longrightarrow \widetilde{[0, \infty)}$$

where $\widetilde{[0, \infty)}$ will be here the set of fuzzy numbers with domain in R^+ (see [4] and also [13], [19]).

In this paper we will say that a fuzzy set $\widetilde{A}$ with membership function μ_A and domain in R^+ is considered as a fuzzy number if and only if (see [19]):

- $A_\alpha = \{\ x \in R^+ \ \text{ such that } \mu_A \geq \alpha\}$ is a convex set, denoted by $[\underline{A_\alpha}, \overline{A_\alpha}]$
- μ_A is an upper semicontinuous functions;
- $\widetilde{A}$ is normal, i.e. there exists $x \in R^+$ such that $\mu_A(x) = 1$
- $supp(A) = \{x \ / \ \mu_A(x) > 0\}$ is a bounded set of R^+

We will denote by

$$\widetilde{d_{pp'}} = d(p, p')$$

the fuzzy distance between the items p and p', and its membership function will be

$$\mu_{pp'} : R^+ \to [0, 1]$$

in such a way that

$$\widetilde{D} = \{\widetilde{d_{pp'}} \ / \ (p, p') \in P \times P\}$$

will denote its associated fuzzy distance matrix.

Definition. Given the classification problem previously defined, and given a fuzzy distance function d, the *item fuzzy graph* can be defined as the pair

$$\widetilde{G(P)} = (P, \tilde{E})$$

when the fuzzy value for the edge associated to two adjacent items p and p' is the fuzzy distance between them, i.e.,

$$\widetilde{E_{p,p'}} = \widetilde{d_{pp'}}$$

Finally, it is important to note that in this paper we will used the concept of fuzzy edges greater than a prescribed threshold. The problem of ordering fuzzy numbers has been studied by many authors (see, e.g., [5], [8]) and [19]). An interesting approach is to transform fuzzy numbers into real numbers by means of a ranking function (see [5]).

Definition Let $\aleph$ be the set of fuzzy numbers and let $a, b \in \aleph$. Then $a \ \widetilde{\geq} \ b \longleftrightarrow \ F(a) \geq F(b)$ where F is a ranking function.

III. Segmentation in a graph

With the final aim of obtaining a classification of the set of items P and its associated valued-fuzzy graph, in this section we propose a coloring algorithm of the valued-fuzzy graph based on an iterative binary coloring algorithm.

In the crisp framework, a c-coloring of a graph $G = (V, E)$, see [18], is a mapping

$$C : V \longrightarrow \{0, \ldots, c-1\}$$

verifying $C(v) \neq C(v')$ if $\{v, v'\} \in E$. Any c-coloring induces a crisp classification of the nodes set V, being each class associated to one color:

$$V_C(k) = \{v \in V \ / \ C(v) = k, k \in \{0, \ldots, c-1\}$$

As a particular case a binary coloring will be a mapping

$$col : V \to \{0, 1\}$$

Our objective is to allow a classification of items through a c-coloring C of the valued or fuzzy graph, $G(P)$ or $\widetilde{G(P)}$: the item $p \in P$ will be classified as $k \in \{0, \ldots, c-1\}$ if its color is $C(p) = k$ (several aspects of the coloring problem for valued-fuzzy graphs has been studied by the authors in [16], [10], [11]).

In order to obtain a partition through a c-coloring C of the the graph $G(P)$ (or a classification of the set P) we propose to successively apply a basic binary coloring process, leading to a hierarchical coloring of the image. A binary coloring of the graph $G(P) = (P, E)$ is a 2-coloring, given by a mapping

$$col \;\; P \;\longrightarrow\; \{0, 1\}$$

The first binary coloring analyzes the items set P, assigning to each item p either the value "0" or the value "1". A second binary coloring can be then applied, separately, to both the subgraph generated by those items previously colored as "0" (obtaining color classes "00" and "01"), and the subgraph generated by those items previously colored as "1" (obtaining color classes "10" and "11"). Repeating this process t times, a c-coloring C will be defined on $G(P)$, where $c = 2^t - 1$; for instance, if some item $p \in P$ has been colored three times as "1", "0" and "1", then, taking into account that 5 is the decimal representation of the binary number 101, the color of item p is $C(p) = 5$.

A description of the basic binary coloring procedure can be found in [11], together with a detailed analysis of the inconsistent assignments, i.e., when there is a cycle in the graph $G(P) = (P, E)$ that allows us to color depending on the path that is chosen (these cycles will be called *inconsistent cycles*): if

$$col \;:\; P \longrightarrow \{0, 1\}$$

is a binary coloring of $G(P)$, the first binary coloring can be then obtained assigning an arbitrary color ("0" or "1") to an arbitrary item, and fixing the order in which item will be colored. Once a initial item has to be colored the remainder of the items that are in the same connected component of the graph $G(P)$ are colored following this rule. If two items p and q are adjacent then

$$col(q) = \begin{cases} col(p) & if\ E_{(p,q)}\ or\ d(p,q) < \alpha \\ 1 - col(p) & if\ E_{(p,q)} \geq \alpha \end{cases}$$

for all (p, q) $adjacent\ in\ P$.

In case we have modeled the graph of relations among items by means of a fuzzy graph, the coloring function is defined as follows: given p and q two adjacent items in the graph

$$\widetilde{G(P)} = (P, \widetilde{(E)})$$

then

$$col(q) = \begin{cases} col(p) & if\ \widetilde{E_{(p,q)}} \widetilde{\leq} \alpha \\ 1 - col(p) & if\ \widetilde{E_{(p,q)}} \widetilde{\geq} \alpha \end{cases}$$

for all (p, q) $adjacent\ in\ P$.

Let us observe that from previous formulas the α value changes with the iteration.

If the graph is acyclic, we can obtain a consistent coloring by choosing, randomly, any initial item from every connected component, and assigning to each one of these items either color 0 or color 1, arbitrarily. In this case, once a value α has been fixed, coloring of each adjacent item is unique. Otherwise, if the connected graph is not acyclic (this is the case for most of the real situations) this coloring rule could produce some problems. In general, given an item p already colored and a fixed valued of α, it could be exist a cycle that produces an *inconsistent* coloring, i.e. for a given item on that cycle, you can used two different colors depending on the path that you use. In order to deal with these inconsistencies, in [11] the authors defined an arbitrary spanning tree for valued graphs. Once a spanning tree $T(G)$ of the graph has been defined, we can produce a binary coloring of the valued-fuzzy graph without inconsistencies. Now the question is to decide for a given valued of α what is the best spanning tree of the graph for our proposes (a related problem with this can be seen in [20]). Taking into account that our idea is to classify the items of P in homogeneous groups we are going to choose the spanning tree of the valued $G(P)$ or fuzzy graph $\widetilde{G(P)}$ as follows:

- **1 Step**. We build the graph $G(P)^* = (P, E^*)$, when now the associated valued for two adjacent items p and q is $E^*_{p,q} = E_{(p,q)}$ if $E_{(p,q)} \leq \alpha$ and $-E_{(p,q)}$ otherwise. In the fuzzy framework we build the fuzzy graph $\widetilde{G(P)} = (P, \tilde{E}^*)$, when the fuzzy value for the edge associated to two adjacent items p and q is $\widetilde{E^*_{p,q}} = \widetilde{E_{p,q}}$ if $\widetilde{E_{p,q}} \;\widetilde{\leq} \alpha$ and $\widetilde{-E_{p,q}}$ otherwise.
- **2 Step**. We obtain the minimum spanning tree of the graph $G(P)^*$. Let us denote by MST the minimum spanning tree maintaining the original values of the edges, i.e. (MST, E) instead of (MST, E^*).
  In the fuzzy framework we obtain the minimum spanning tree of the graph $\widetilde{G(P)}^*$. Let us denote by $\widetilde{MST}$ the minimum spanning tree maintaining the original values of the edges, i.e. $(\widetilde{MST}, E)$ instead of $(\widetilde{MST}, E^*)$.

The segmentation process induced by the previous binary coloring can be repeated so that a more refined classification or partition of the graph can be obtained. For instance, color classes "00" and "01" will be obtained applying the binary coloring process to the subset

$$P' = \{p \in P \;/\; col(p) = 0\}$$

Analogously, color classes "10" and "11" will be obtained applying the binary coloring process to the subset

$$P' = \{p \in P \;/\; col(p) = 1\}$$

In this way, four color classes are being obtained: "00", "01", "10" and "11", which are identified by the mapping

$$C \;:\; P \;\longrightarrow\; \{0, 1, 2, 3\}$$

where $C(p)$ is the integer number associated to the binary number of the color class of p. The binary coloring procedure can be successively applied to each family of items belonging to the same color class, meanwhile there are adjacent items being different.

However, and in order to avoid the exponential growth of the binary coloring processes, this process (that we will denote as $bincol$) will be successively applied only t times,

being t previously fixed, obtaining in this way 2^t color classes.

It is important to note that the color classes are not necessary connected. We will define the term of homogeneous region or group in the graph as a set of connected items with the same color class. This homogeneous regions will be the final partition or classification of the items in the graph.

Definition. Given a set P with associate graph $G(P)$ and let C be the previous coloring C process, a connected component of each color class induced by C will be said a *region*.

A. Computational complexity analysis

The coloring algorithm here proposed tries to classify the nodes in the graph obtaining a result similar to a dendogram in which in the first iteration all nodes are in the same homogeneous region and in the last iteration each nodes represent a homogeneous region. Obviously, from a computational point of view is not necessary to reach to this final iteration since is not relevant and not informative.

So, once the number of iterations is fixed (and that we can denote as it) we realize that the basic coloring procedures are polynomial, so the final algorithm is polynomial.

Hence, to choose an appropriate decreasing scheme of parameter α is the key issue.

B. About the election of α

First at all, let us observe that the value of α is bounded by the following two extreme cases:

- $\overline{\alpha} = \max_{p,q \in P}\{E_{(p,q)}\}$: if we fix a threshold $\alpha > \overline{\alpha}$, then the whole graph is considered as a unique color class ($col(p) = col(q), \forall (p,q) \in P$).
- $\underline{\alpha} = \min_{p,q \in P}\{E_{(p,q)}\}$: in case $\alpha \leq \underline{\alpha}$, all differences are considered in the graph but the result is the same choosing $\alpha = \underline{\alpha}$.

How to determine an appropriate intermediate α level is not a trivial task. But it is clear that only the interval

$$[\underline{\alpha}, \overline{\alpha}]$$

should be considered. Once the number of iterations (or divisions) has been fixed, we have decided to choose the decreasing scheme of parameter α in the following way.

Taking into account that in the first iteration we want to consider only the big differences between items, and doing that progressively, if it is the number of iterations, then α_1 will be the value that leaves to the right the $\frac{100}{it+1}\%$ of the distances (i.e., considering only the big differences to partitioning the graph), α_2 will be the value that leaves the $2\frac{100}{it+1}\%$ of the distances and so on. Finally, α_{it} will be the value that leaves only the $it\frac{100}{it+1}\%$ of the distances in the graph. Obviously, $\alpha_1 \geq \alpha_2 \ldots \geq \alpha_{it}$.

IV. Application to a remote sensing image

Let us consider an image as a bidimensional map of pixels, each one of them being characterized by a fixed number of measurable attributes. These attributes can be, for example, the values of the three bands of the visible spectrum (red, green and blue), the whole family of spectrum band intensities, or any other family of physical measurements. Hence,

$$P = \{(i,j) \ / \ 1 \leq i \leq r \ , \ 1 \leq j \leq s\}$$

will denote the set of pixel positions of an $r \times s$ image.

If each pixel is characterized by b numerical measures, the whole image I can be characterized as

$$I = \left\{(x^1_{i,j}, \ldots, x^b_{i,j}) \ / \ (i,j) \in P\right\}$$

For a given image I, the information of the b measures of any pixel

$$(x^1_{i,j}, \ldots, x^b_{i,j})$$

can be represented by its position, $p = (i,j) \in P$, without confusion.

Given such an image I, a standard crisp classification problem pursues a partition in crisp regions, being each one a subset of pixels, to be considered a candidate for a class (in case such a region is homogeneous enough). In this way, a crisp classification approach looks for a family of subsets of pixels

$$\{A_1, \ldots, A_c\}$$

such that

$$P = \cup_{k=1}^{c} A_k$$

but

$$A_i \cap A_j = \emptyset, \forall i \neq j$$

where $A_1, \ldots A_c$ are the family of crisp classes explaining the image.

A certain uncertainty arrives when we consider a dissimilarity measure between pixels in order to identify possible homogeneous regions in the image. In crisp image classification problems, the selection of an adequate distance is a difficult issue that has been studied by many authors. Obviously, any classification process will be strongly dependent on the selection of the appropriate distance, to be chosen taking into account all features of the image under consideration, together with our particular classification objectives. Furthermore, in many instances the distance between two elements includes some lack of precision or ambiguity (it would be the case, for example, when we consider the aggregation of several measures obtained by different experts: a well known problem in remote sensing is to chose an adequate distance in order to compare opinions from different experts).

In order to capture the natural fuzzy uncertainty [21], and in order to give more flexibility to other already existing crisp classification procedures, we shall consider that a fuzzy distance (see [12] or section 2) expresses the relation between the measured properties of pixels.

Given a $r \times s$ image, a planar graph (P, E) can be defined considering P as the set of nodes and E the set of edges linking any couple of adjacent pixels. Two pixels

$$p = (i,j), p' = (i',j') \in P$$

are adjacent if

$$|i - i'| + |j - j'| = 1$$

that is, if they share one coordinate being the other one contiguous.

Let us now denote by

$$\widetilde{G(I)} = (P, \tilde{E})$$

the graph associated to our image I, where

$$\tilde{E} = \{\widetilde{d_{pp'}} \quad / \ (p, p') \quad adjacents\}$$

and

$$\widetilde{d_{pp'}}$$

are fuzzy numbers with domain in R^+.

Definition. Given the image I and a fuzzy distance d, the *pixels fuzzy graph* is defined as the pair

$$\widetilde{G(I)} = (P, \tilde{E})$$

Notice that our pixels fuzzy graph

$$\widetilde{G(I)}$$

can be also characterized by the set P plus two $r \times s$ fuzzy matrices, $\widetilde{D^1}$ and $\widetilde{D^2}$, where

$$\widetilde{D^1_{i,j}} = d((i,j),(i+1,j))$$

for all $(i,j) \in \{1,\ldots,r-1\} \times \{1,\ldots,s\}$, and

$$\widetilde{D^2_{i,j}} = d((i,j),(i,j+1))$$

for all $\in \{1,\ldots,r\} \times \{1,\ldots,s-1\}$.

Since our coloring procedure will be based upon this alternative representation, from now on we shall denote our pixels fuzzy graph

$$\widetilde{G(I)}$$

by

$$(r, s, \widetilde{D^1}, \widetilde{D^2})$$

The above algorithm has been applied to an orthoimage of Sevilla Province (south Spain) that was taken on August 18, 1987, by the LANDSAT 5 satellite (Worldwide Reference System Spain (WRS) image 202-34-4) see [7] for a detailed description of this image and Fig.1). In this case we have consider two crisp distances (Euclidean and Manhattan), normalized in order to build a fuzzy distance. In this example we consider tetrahedral fuzzy numbers, that can be characterized by four numbers (a_1, a_2, a_3, a_4) (see [13]). The chosen ranking function for this example has been

$$F(a) = \sqrt{a_x^2 + a_y^2}$$

where a_x and a_y are respectively, the horizontal and the vertical coordinates of the centroid of a. The coloring algorithm with $it = 4$ in the second iteration is shown in Fig. 2. Let us observe that the results here obtained coincides with the results given in [10] where main features seem to be captured. It is important to note that the new algorithm

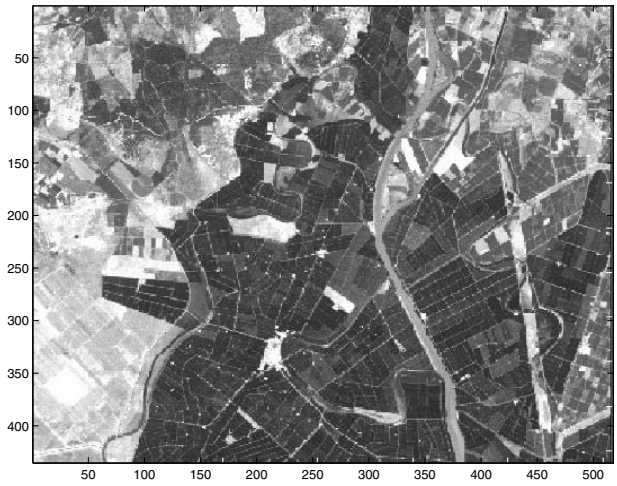

Fig. 1. Orthoimage of Sevilla Province.

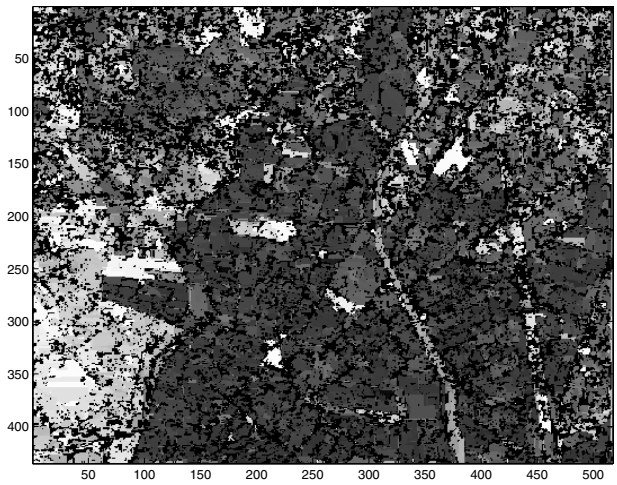

Fig. 2. Visualization of the Sevilla image segmentation in the second iteration.

required less computational effort and can be extended to any class of graph.

The above algorithm has been also applied to several standard images, and with few iterations we obtain similar results to those analyzed in [11]. But the approach presented in this paper is ready to be implemented in other problems where the distance is not deduced from crisp measurements (for example, in order to get an aggregated classification from several proposals obtained with different classification methods).

V. Conclusions

The main contribution of this paper is to obtain a polynomial algorithm that allows us to classify step by step (by means of a dendogram) a set of items that are related by means of a graph. This problem was partially addressed in [11], [10] for a special class of graphs and in this paper it

has been extended to arbitrary graphs.

REFERENCES

[1] A. Bechara, D. Tranel and H. Damasio, "Characterization of the decision-making deficit of patients with ventromedial prefrontal cortex lesions." *Brain* 123, 2189-2202 (2000).

[2] H. Bustince, M. Pagola, E. Barrenechea, J. Fernndez, P. Melo-Pinto, P. Couto, H.R. Tizhoosh and J. Montero: "Ignorance functions, an application to the calculation of the threshold in prostate ultrasound images." *Fuzzy Sets and Systems* (to appear).

[3] H. Bustince, J. Fernández, R. Mesiar and J. Montero: "Overlap functions." Submitted.

[4] D. Dubois and H. Prade: *Fuzzy sets and Systems, Theory and Applications* (Academic Press, Orlando, Florida, 1997).

[5] D. Dubois, H. Prade: "Ranking fuzzy numbers in the setting of possibility theory." *Information Sciences* 30, 183-224 (1983).

[6] A. del Amo, J. Montero, G. Biging and V. Cutello, "Fuzzy classification systems." *European Journal of Operational Research* 156, 459-507 (2004).

[7] A. del Amo, J. Montero, A. Fernández, M. López, J. Tordesillas and G. Biging: "Spectral fuzzy classification: an application." IEEE Transactions on Systems Man and Cybernetics (C) 32, 42-48 (2002).

[8] G. Facchinetti, R.G. Ricci: A characterization of a general class of ranking functions on triangular fuzzy numbers. *Fuzzy Sets and Systems* 146, 297-312 (2004).

[9] D. Gómez and J. Montero, "A discussion on aggregation operators." *Kybernetika* 40, 107-120 (2004).

[10] D. Gómez, J. Montero and J. Yá nez: "A coloring algorithm for image classification." *Information Sciences* 176, 3645-3657 (2006).

[11] D. Gómez, J. Montero, J. Yá nez and C. Poidomani, "A graph coloring algorithm approach for image segmentation". *Omega* 35, 173-183, 2007.

[12] O. Kabva and S. Seikkala: "On fuzzy metric spaces." *Fuzzy Sets and Systems* 12, 215-229 (1984).

[13] A. Kaufman, M. Gupta: *Introduction to Fuzzy Arithmetic* (Van Nostrand Reinhold, New York, 1985).

[14] J. Montero, D. Gómez and H. Bustince, "On the relevance of some families of fuzzy sets. " *Fuzzy Sets and Systems* 158, 2429-2442 (2007).

[15] J. Montero, V. López and D. Gómez, "The role of fuzziness in decision making." In: D. Ruan *et al.*, *Fuzzy Logic: an spectrum of applied and theoretical issues*, pp. 337-349 (Springer, Berlin, 2007).

[16] S. Muñoz, M.T. Ortuño, J. Ramírez, J. Yáñez: "Coloring fuzzy graphs." *Omega* 33, 211-221 (2005).

[17] M. Nachtegael, T. Mlange and E.E. Kerre, "The possibilities of fuzzy logic in image processing." *Lecture Notes in Computer Science* 4815, 198-208 (2007).

[18] P.M. Pardalos, T. Mavridou and J. Xue: "The Graph coloring problem: A bibliographic survey." In: D.Z. Du and P.M. Pardalos (Eds.), *Handbook of Combinatorial Optimization*, pp. 331-395 (Kluwer Academic Publishers, Boston, 1998).

[19] X. Wang and E.E. Kerre: "Reasonable properties for the ordering of fuzzy quantities (I)." *Fuzzy Sets and Systems* 118, 375-385 (2001).

[20] J. Yáñez, S. Muñoz and J. Montero: "Graph coloring inconsistencies in image segmentation." *Computer Engineering and Information Sciences* 1, 435-440 (2008).

[21] L.A. Zadeh: "Fuzzy sets." *Information and Control* 8, 338-353 (1965).