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A A-IFSs Based Image Segmentation Methodology for Gait Analysis

Pedro Couto, Vitor Filipe, Pedro Melo-Pinto
Engineering Dept.
CITAB - UTAD University
Vila Real, Portugal
(pcouto, vfilipe, pmelo)@utad.pt

Humberto Bustince, Edurne Barrenechea
Dept. de Automática y Computación
UPNA University
Pamplona, Spain
(bustince, edurne.barrenechea)@unavarra.es

***Abstract*—In this work, image segmentation is addressed as the starting point within a motion analysis methodology intended for biomechanics behavior characterization.**

First, we propose a general segmentation framework that uses Atanassov's intuitionistic fuzzy sets (A-IFSs) to determine the optimal image threshold value. Atanassov's intuitionistic fuzzy index values are used for representing the unknowledge/ignorance of an expert on determining whether a pixel belongs to the background or the object of the image. Then, we introduce an extension of this methodology that uses a heuristic based multi-threshold approach to determine the optimal threshold.

Experimental results are presented.

***Keywords*-image segmentation; fuzzy logic; Atanassov's intuitionistic fuzzy sets;**

I. Introduction

In the image analysis methodology issued in this work, the segmentation of the image plays a decisive roll towards a good analysis of processed image data since, being the starting point of the process it can critically affect its performance. In this sense, the image is decomposed into meaningful parts for further analysis, resulting in the partition of the set of pixels in the image into a finite set of regions (subsets) according to a certain criterion.

The presented segmentation approach was developed for a specific image analysis application developed within the work presented in [1], [2], [3] where the main goal is to perform kinematic analysis for the left hindlimb in walking rats. The method used for the analysis of the hindlimb movement involve the placing of markers on the skin surface overlying joints under analysis. These markers are to be tracked by a computer vision analysis system in order to characterize the hindlimb movement.

The ultimate goal of this motion analysis methodology is to extract rat gait kinematics parameters by detecting and tracking those markers. Hence, in this image analysis methodology the segmentation purpose is to point out those markers.

Actually, the segmentation of digital images is the process of dividing an image into disjoined parts, regions or subsets so that each one must satisfy a distinct and well-defined property or attribute.

The most commonly used strategy for segmenting images is global thresholding that refers to the process of partitioning the pixels in an image into object and background regions on the basis of the different intensity levels of gray of the pixels in the image. This partition is made by establishing a threshold, in such a way that all the pixels with intensity greater or equal than the threshold belong to the background (or to the object) and all the pixels with intensity lower than the threshold belong to the object (or to the background).

Extensive research has been conducted in this research field over the last years, and many types of segmentation techniques have been proposed in the literature, each one of them based on a certain methodology to classify the regions [4], [5], [6], [7], [8], [9], [10].

The proposed approach is an evolution/extension of the methodology, based on Atanassov's intuitionistic fuzzy sets (A-IFSs), presented in [11] intended for use within the above referred image analysis process and, with the objective of pointing out the rat markers. This proposed methodology uses a multi-level thresholding approach that is the natural extension of the methodology presented in [11] to multi-thresholding. This methodology also makes use of heuristics in order to chose the global optimal threshold among the set of calculated thresholds.

II. General A-IFSs Based Image Segmentation Framework

Being (x, y) the coordinates of each pixel on the image Q, and being $q(x, y)$ the gray level of the pixel (x, y) so that $0 \leq q(x, y) \leq L - 1$ for each $(x, y) \in Q$ where L is the image grayscale, many methods have been proposed for determining the threshold t of an image considering fuzzy set theory as an efficient tool in order to obtain a good segmentation of the image considered. The most commonly algorithm used to obtain the threshold is the one that uses the concept of fuzzy entropy [12], [13], [14] and its main steps are the following:

(a) Assign L fuzzy sets Q_t to each image Q. Each one is associated to a level of intensity t, $(t = 0, 1, \cdots, L - 1)$, of the grayscale L used.

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(b) Calculate the entropy of each one of the L fuzzy sets Q_t associated with Q.

(c) Take, as the *best threshold* gray level t, associated with the fuzzy set corresponding to the lowest entropy.

The main problem of this algorithm is the step (a). In [11] this problem is solved using A-IFSs in the following way: In order to choose/construct the membership function of each pixel of the image to the associated fuzzy set, three numerical values are assigned to each one of them.

- A value for representing the expert knowledge of the membership of the pixel to the background. A membership function, constructed by the expert using dissimilarity functions, is used to obtain this value (see [15]).
- Dissimilarity functions are also used by the expert to construct a membership function to retrieve a value for representing the expert knowledge of the membership of the pixel to the object.
- The expert knowledge/ignorance, in determining the above mentioned membership functions, is represented by a third value obtained trough Atanassov's intuitionistic index (π).

The value represented by Atanassov's intuitionistic index indicates the knowledge/ignorance of the expert when assigning a pixel either to the background or the object, so that, when the expert is absolutely sure that a pixel belongs either to the background or the object, the Atanassov's intuitionistic index associated with that pixel has the value of zero. This value increases with respect to the unknowledge/ignorance of the expert as to whether the pixel belongs to the background or the object. So, if the expert doesn't know if a pixel belongs to the background or the object its membership to both must be represented with the value 0.5 and, in such conditions, it is said that the expert used the greatest unknowledge/ignorance/intuition allowed in the construction of the membership functions of the set associated with that pixel, resulting in a Atanassov's Intuitionistic Fuzzy Index maximum value. For this reason, A-IFSs (Atanassov's Intuitionistic Fuzzy Set [16], [17]) are used.

In a second stage, the entropy values of each one of the L A-IFSs associated with the image are calculated. In this methodology, entropy on A-IFSs is interpreted as a measure of the degree of a A-IFS that a set has with respect to the fuzzyness of the said set (see [18]). Under these conditions the entropy will be null when the set is a FSs and will be maximum when the set is totally intuitionistic.

Finally, the gray level t associated with the fuzzy set with the lowest entropy is selected for the best threshold.

A possible implementation of this methodology [11], and the one used in this work, is now presented.

A. *(Step A1)*

Construct L fuzzy sets Q_{Bt} associated with the background and L fuzzy sets Q_{Ot} associated with the object. Each one of these fuzzy sets is associated with a gray level t of the grayscale L used. The membership functions of these sets are defined by means of restricted dissimilarity functions and the expressions are:

$$\mu_{Q_{Bt}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{\mathrm{B}}(t)}{L-1}\right)\right)$$

$$\mu_{Q_{Ot}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{\mathrm{O}}(t)}{L-1}\right)\right)$$

where

$$m_{\mathrm{B}}(t) = \frac{\sum_{q=0}^{t} q h(q)}{\sum_{q=0}^{t} h(q)} \tag{1}$$

$$m_{\mathrm{O}}(t) = \frac{\sum_{q=t+1}^{L-1} q h(q)}{\sum_{q=t+1}^{L-1} h(q)} \tag{2}$$

being $h(q)$ the number of pixels of the image with the gray level q, $F(x) = 1 - 0.5x$ and, the restricted dissimilarity function $d(x, y) = |x - y|$ (see [11], [19], [20]).

Note that $F(x)$ and $d(x, y)$ are only ones of the set of possibilities that could be used (see [11], [19], [20]).

It is important to acknowledge that, the membership functions constructed are always greater than or equal to 0.5 and, a pixel unequivocally belongs to the background if and only if its intensity q is equal to the mean of intensities of the background $m_B(t)$. When the difference between the pixel's intensity q and the mean of intensities of the background $m_B(t)$ is maximal, then the value of its membership function to the background is minimal. The same interpretation applies to the membership to the object. Hence, the shorter the distance between a pixel's intensity q and the mean of intensities of the background (object), the greater the value of its membership to the background (object).

B. *(Step A2)*

As it has been said before, the unknowledge/ignorance of the expert in the construction of the fuzzy sets (in *Step A1*) is represented by means of Atanassov's intuitionistic fuzzy index (π), meaning that, it is considered that $\mu_{Q_{Bt}}$ ($\mu_{Q_{Ot}}$) indicates the expert's degree of knowledge of the pixel belonging to the background (object).

If the expert is certain of the pixel belonging to the background or the object, then the value of π must be zero. The value of π increases as the unknowledge/ignorance of the expert grows. However, the unknowledge/ignorance must have the lower possible influence on the choice of the membership degree, so, in this implementation, in the worst case, the unknowledge/ignorance will have a maximum influence of 25 percent.

Under these conditions, the following expression is used to calculate π:

$$\pi(q) = \wedge(1 - \mu_{Q_{Bt}}(q), 1 - \mu_{Q_{Ot}}(q)).$$

Again, this expression is only one within the set of all the possible ones (see [11]).

C. (Step A3)

Construct an A-IFS, using π, with each one of the fuzzy sets Q_{Bt} and Q_{Ot}.

$$\tilde{Q}_{Bt} = \{(q, \mu_{\tilde{Q}_{Bt}}(q), \nu_{\tilde{Q}_{Bt}}(q)) | q = 0, 1, \cdots, L-1\},$$

given by

$$\mu_{\tilde{Q}_{Bt}}(q) = \mu_{Q_{Bt}}(q)$$
$$\nu_{\tilde{Q}_{Bt}}(q) = 1 - \mu_{\tilde{Q}_{Bt}}(q) - \pi(q)$$

and

$$\tilde{Q}_{Ot} = \{(q, \mu_{\tilde{Q}_{Ot}}(q), \nu_{\tilde{Q}_{Ot}}(q)) | q = 0, 1, \cdots, L-1\},$$

given by

$$\mu_{\tilde{Q}_{Ot}}(q) = \mu_{Q_{Ot}}(q)$$
$$\nu_{\tilde{Q}_{Ot}}(q) = 1 - \mu_{\tilde{Q}_{Ot}}(q) - \pi(q)$$

D. (Step B)

Calculate the entropy (IE) of each one of the L Atanassov's intuitionistic fuzzy sets, using the following expression, so that $0 \leq IE(\tilde{Q}_{Bt}) \leq 0.25$.

$$IE(\tilde{Q}_{Bt}) = \frac{1}{N \times M} \sum_{q=0}^{L-1} h(q) \cdot \pi(q) \tag{3}$$

where $N \times M$ are the image dimensions in pixels.

E. (Step C)

Finally, the gray level associated with the Atanassov's intuitionistic fuzzy set $\tilde{Q}_{Bt}$ of lowest entropy (IE) is chosen as the best threshold.

III. Materials and methods

In the image processing system boarded in this work, the main goal is to perform kinematic analysis for the left hindlimb in treadmill walking rats. The method used for the analysis of the hindlimb movement involved the placing of markers on the skin surface overlying joints under analysis. These markers are to be tracked by the system in order to characterize the hindlimb movement [1], [2], [3].

Image sequences acquired at the usual rate of 25 images per second are insufficient to characterize the rat's hindlimb movement, particularly due to aliasing phenomena's. In order to avoid this aliasing problem, a high-speed digital image camera (Redlake PCI 1000S, San Diego, USA) was used to record the rat gait at 125 frames per second, resulting in images of 480×420 pixels coded in 8 bits (256 gray levels).

Due to the high speed acquisition, other problems arise in contrast, noise, illumination, resolution, etc., resulting in noisy images with imprecision on the gray levels that conducts to fuzzy boundaries and ill defined regions, which makes the current approach to the segmentation of such images the natural approach.

IV. Proposed approach

In this section we propose an extension of the above presented methodology intended to be used in images from the walking rats' sequences.

We will present this approach for the calculation of two threshold levels $t1$ and $t2$ such that $0 \leqslant t1 \leqslant t2 \leqslant L-1$. The same reasoning can be done to extend the methodology to a larger number of required thresholds.

Under the same conditions described in section II we will consider an image Q and two intensity thresholds $t1$ and $t2$.

In this section we will discard the concept of background and object and will refer to the three considered regions as object 1, object 2 and, object 3. Thus, for each image Q we will construct L fuzzy sets $\tilde{Q}_{O1t}$ associated with the object 1, L fuzzy sets $\tilde{Q}_{O2t}$ associated with the object 2, and another L fuzzy sets $\tilde{Q}_{O3t}$ associated with the object 3.

A. Step (A1)

Like in section II, the membership function of each element to the sets $\tilde{Q}_{O1t}$, $\tilde{Q}_{O2t}$ and $\tilde{Q}_{O3t}$ must express the relationship between the intensity q of the pixel and its membership to the object 1, object 2 or object 3 respectively.

For each possible combinations of $t1, t2 \in \{0, 1, \cdots, L-1\}$, such that $t1 < t2$, the mean of the intensities of gray of the pixels that belong to the object 1 (m_{O1t}), the mean of the intensities of gray of the pixels that belong to the object 2 (m_{O2t}), and the mean of the intensities of gray of the pixels that belong to the object 3 (m_{O3t}) are given by the following expressions:

$$m_{O1}(t) = \frac{\sum_{q=0}^{t1} qh(q)}{\sum_{q=0}^{t1} h(q)},$$

$$m_{O2}(t) = \frac{\sum_{q=t1+1}^{t2} qh(q)}{\sum_{q=t1+1}^{t2} h(q)},$$

$$m_{O3}(t) = \frac{\sum_{q=t2+1}^{L-1} qh(q)}{\sum_{q=t2+1}^{L-1} h(q)}.$$

With $h(q)$ being the number of pixels of the image with intensity q.

We construct the membership functions of each possible combinations of intensities $t1, t2$ in the above mentioned conditions, to the sets $\tilde{Q}_{O1t}$, $\tilde{Q}_{O2t}$ and $\tilde{Q}_{O3t}$ in the following way:

$$\mu_{\tilde{Q}_{O1t}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{O1}(t)}{L-1}\right)\right),$$

$$\mu_{\tilde{Q}_{O2t}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{O2}(t)}{L-1}\right)\right),$$

$$\mu_{\tilde{Q}_{O3t}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{O3}(t)}{L-1}\right)\right).$$

In this approach we use the function $F(x) = 1 - 0.5x$ along with the restricted dissimilarity function $d(x, y) = |x - y|$ which conduct us to the fuzzy sets $\tilde{Q}_{O1t}$, $\tilde{Q}_{O2t}$ and $\tilde{Q}_{O3t}$ represented by the following membership functions:

$$\begin{cases} \mu_{\tilde{Q}_{O1t}}(q) = 1 - 0.5\left|\frac{q}{L-1} - \frac{m_{O1}(t)}{L-1}\right| \\ \mu_{\tilde{Q}_{O2t}}(q) = 1 - 0.5\left|\frac{q}{L-1} - \frac{m_{O2}(t)}{L-1}\right| \\ \mu_{\tilde{Q}_{O3t}}(q) = 1 - 0.5\left|\frac{q}{L-1} - \frac{m_{O3}(t)}{L-1}\right| \end{cases}$$

Like in section II, the constructed membership functions are always greater than or equal to 0.5 and, the smaller the distance between a pixel's intensity q and the mean of intensities of the object considered (object 1, 2 or 3), the greater the value of its membership to that object.

B. *Step* $(A2)$

In this approach we interpret Atanassov's intuitionistic index π as the *unknowledge/ignorance* of the expert in assigning the membership value of a certain pixel to the objects 1, 2 or 3 of the image. Under this interpretation of π, we will consider that $\mu_{\tilde{Q}_{O1t}}$, $\mu_{\tilde{Q}_{O2t}}$ and $\mu_{\tilde{Q}_{O3t}}$ indicates the expert's degree of knowledge of the pixel belonging to the object 1, 2 or 3 respectively.

In any case the following conditions must be fulfilled:

1) The unknowledge that the expert uses in the choice of the membership of a pixel must be zero if he is certain that the pixel belongs to one of the considered objects.
2) The unknowledge/ignorance must decrease with respect to the certainty of the expert as to the pixel belonging to one of the objects.
3) The unknowledge/ignorance must have the lower possible influence on the choice of the membership degree. In the worst of cases, the unknowledge will have a maximum influence of 50 percent.

In this context, $\pi(q)$ is the quantification of the unknowledge/ignorance of the expert in the selection of the membership functions $\mu_{\tilde{Q}_{O1t}}(q)$, $\mu_{\tilde{Q}_{O2t}}(q)$ and $\mu_{\tilde{Q}_{O3t}}(q)$.

We used the following expression for $\pi(q)$:

$$\pi(q) = \wedge(1 - \mu_{\tilde{Q}_{O1t}}(q), 1 - \mu_{\tilde{Q}_{O2t}}(q), 1 - \mu_{\tilde{Q}_{O3t}}(q)) \quad (4)$$

The expression 4 fulfils the above mentioned conditions since, $0.5 \leqslant \mu_{\tilde{Q}_{O1t}}(q) \leqslant 1$, $0.5 \leqslant \mu_{\tilde{Q}_{O2t}}(q) \leqslant 1$ and $0.5 \leqslant \mu_{\tilde{Q}_{O3t}}(q) \leqslant 1$ then:

$$\pi(q) = 0, \qquad \text{if and only if}$$

$$\mu_{\tilde{Q}_{O1t}}(q) = 1 \text{ or } \mu_{\tilde{Q}_{O2t}}(q) = 1 \text{ or } \mu_{\tilde{Q}_{O3t}}(q) = 1$$

meaning that the expert is positively sure that the pixel belongs to one of the objects.

$$\pi(q) = 0.5, \qquad \text{if and only if}$$

$$\mu_{\tilde{Q}_{O1t}}(q) = 0.5 \text{ and } \mu_{\tilde{Q}_{O2t}}(q) = 0.5 \text{ and } \mu_{\tilde{Q}_{O3t}}(q) = 0.5$$

meaning that the expert has the greatest unknowledge/ignorance in determining to which object the pixel belongs to.

Hence,

$$0 \leqslant \pi(q) \leqslant 0.5$$

C. *Step* $(A3)$

In this section, for all possible combinations of $t1, t2 \in \{0, 1, \cdots, L-1\}$, such that $t1 < t2$, we will associate an A-IFS (using the index π described in the subsection above) with each one of the fuzzy sets $\tilde{Q}_{O1t}$, $\tilde{Q}_{O2t}$ and $\tilde{Q}_{O3t}$, in the following way:

$$Q_{O1t} = \{(q, \mu_{Q_{O1t}}(q), \nu_{Q_{O1t}}(q)) | q = 0, 1, \cdots, L-1\},$$

given by

$$\mu_{Q_{O1t}}(q) = \mu_{\tilde{Q}_{O1t}}(q)$$
$$\nu_{Q_{O1t}}(q) = 1 - \mu_{Q_{O1t}}(q) - \pi(q)$$

and

$$Q_{O2t} = \{(q, \mu_{Q_{O2t}}(q), \nu_{Q_{O2t}}(q)) | q = 0, 1, \cdots, L-1\},$$

given by

$$\mu_{Q_{O2t}}(q) = \mu_{\tilde{Q}_{O2t}}(q)$$
$$\nu_{Q_{O2t}}(q) = 1 - \mu_{Q_{O2t}}(q) - \pi(q)$$

and

$$Q_{O3t} = \{(q, \mu_{Q_{O3t}}(q), \nu_{Q_{O3t}}(q)) | q = 0, 1, \cdots, L-1\},$$

given by

$$\mu_{Q_{O3t}}(q) = \mu_{\tilde{Q}_{O3t}}(q)$$
$$\nu_{Q_{O3t}}(q) = 1 - \mu_{Q_{O3t}}(q) - \pi(q)$$

D. Step (B)

At this step we are going to calculate, for each possible combinations of $t1, t2 \in \{0, 1, \cdots, L-1\}$, such that $t1 < t2$, the entropy ε_T of each one of the L intuitionistic fuzzy sets of Atanassov Q_{O1t}, Q_{O2t} and Q_{O3t}. In this approach we use the Type 2 entropy defined by Burillo and Bustince by means of the following expression:

$$\varepsilon_{T_2}(Q_{O1t}) = \frac{1}{N \times M} \sum_{q=0}^{L-1} h(q) \cdot \pi(q) \qquad (5)$$

where π is obtained with equation 4.

E. Step (C)

Again, the gray level associated with the A-IFS of lowest entropy is chosen as the best threshold.

V. Experimental results

In order to test the performance of the proposed approach, four images, presenting contrast problems (more prone to difficulties), from the walking rats' sequences were selected and used as test images (see Fig. 1). Each one of these images is part of a sequence of images where the rat gait is recorded and is analyzed by tracking the markers placed on the rats in each image. The purpose of the segmentation step in this process is to point out those markers.

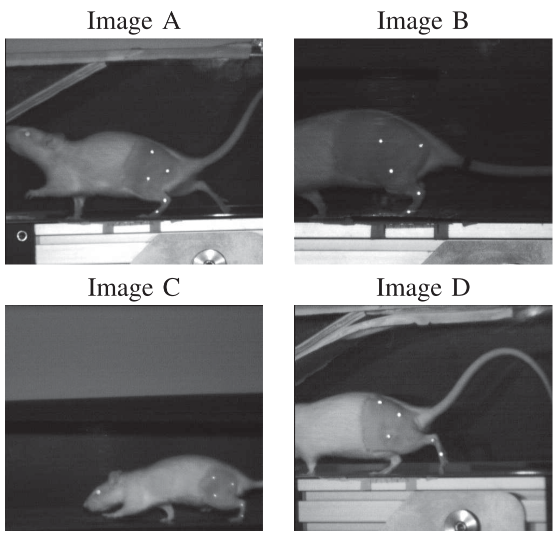

Figure 1. Original images.

In Fig. 2 the results obtained with the general image segmentation framework presented in section II are shown and, the results obtained with the proposed methodology are presented in Fig. 3.

The results obtained with the general image segmentation framework methodology (Fig. 2) show that it does not perform well in identifying the rat´s markers for all images. Only in one situation (Image B) the markers are

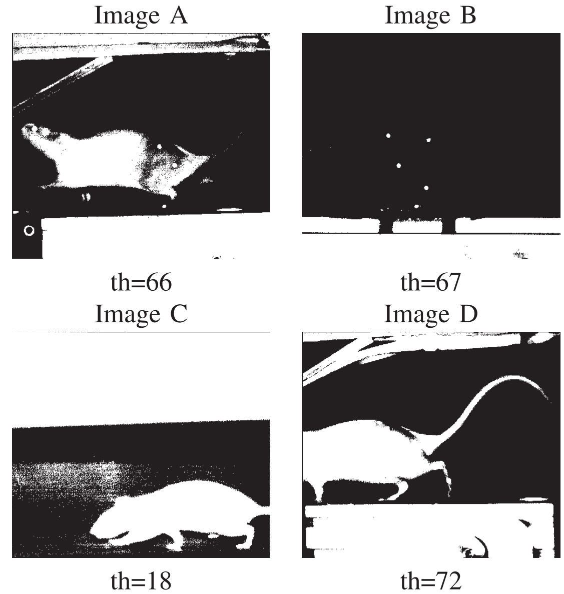

Figure 2. Binary images obtained with the general image segmentation framework.

clearly identified for further processing. On the contrary, the proposed methodology (Fig. 3) succeeds in identifying the markers for all the images and, thus, is more reliable for the necessary further processing in order to extract the markers position in the image.

VI. Conclusion

The problem of segmentation in spite of all the work over the last decades, is still an important research field in image processing and it is still suitable to develop new threshold techniques, or new extensions to the existing ones, that can effectively lead us to an optimal threshold within the specificities of one's application.

The proposed segmentation approach is applied to walking rats' image sequences and its purpose is to point out the rat markers for further processing.

Although the general image segmentation framework methodology presented in section II give good results under experimental conditions, it does not take into account the specificities of the image analysis process in which it is going to be applied. The new approach presented, successfully intended to endow the algorithm with the capability of adapting itself to a particular image analysis process.

The preliminary results show that all of the tested images can be properly segmented since, the proposed approach succeeds in identifying the markers for all the images and, thus, is more reliable for the necessary further processing in order to extract the markers position in the image and ultimately access the biomechanics behavior characterization.

Further work is intended, focusing on the adaptation of the proposed algorithm to color image segmentation.

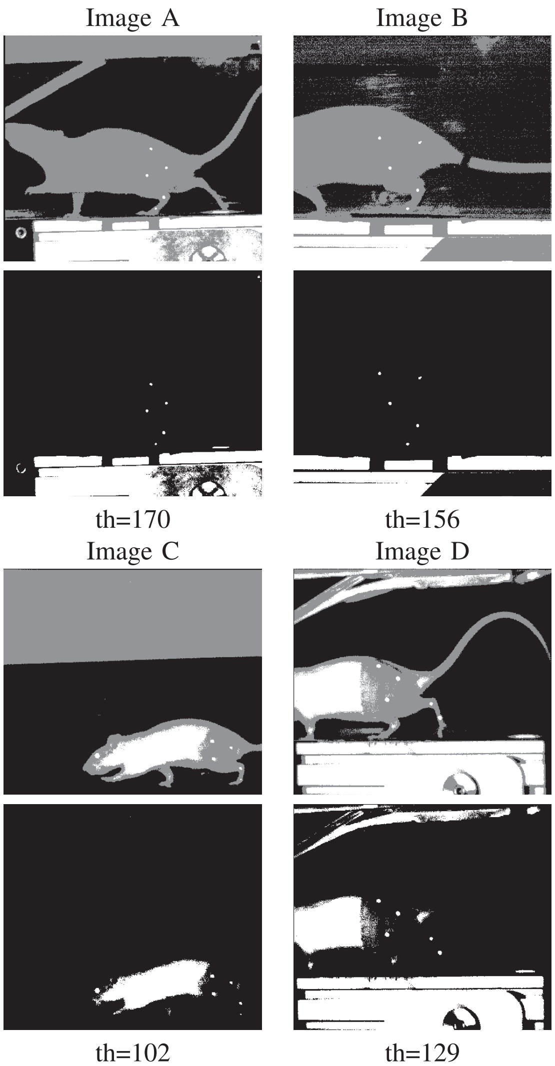

Figure 3. Binary images obtained with the proposed methodology.

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