

## Searching Common Pattern in Agents Behaviors with Usage of FCA

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### Abstract

*MAS operation brings quite a number of tasks related to behaviors of intelligent agents. More precisely, understanding of agents behaviors and their relationships enables to optimize the whole MAS architecture. The usage of FCA in the area of Multi-agent systems can facilitate solutions of several tasks and/or point to certain data relations, which should be further analyzed in more detail. This paper briefly describes the Triadic Formal Concept Analysis (FCA) which helps us to find some hidden information on MAS and agents interaction. The process of FCA integration within MAS is illustrated on the area of traffic simulation.*

### 1. Introduction

The Formal Concept analysis (FCA) is a data analysis technique that describes the world in terms of objects and the attributes possessed by those objects. The philosophical starting point for FCA was represented by the understanding that a *concept* can be described by its *extension* and *intension*. The mathematical foundations were laid by Birkhoff [4] who demonstrated the correspondence between partial orders and lattices. Birkhoff showed that a lattice can be constructed for every binary relation between a set of objects and a set of attributes with the resulting lattice providing insight into the structure of the original relation.

Formal Concept Analysis arose during the early 1980s from Rudolf Wille pioneer work [14]. It identifies conceptual structures among data sets based on the primary philosophical understanding of a “concept” as a unit of thought comprising its extension and intension as a way of modeling a domain [14, 7]. The extension of a concept is formed by all objects to which the concept applies and the intension consists of all attributes possessed by those objects. These generate a conceptual hierarchy of a domain by finding all possible formal concepts which reflect a certain relationship between attributes and objects.

Till now, so called *biadic* (or *dyadic*) formal context and concepts were introduced. It means, that the context consists of two sets (objects and attributes) and there is one binary relation between them. The triadic approach of Formal Concept Analysis gave rise to a new class of algebraic structures; so-called trilattices [15, 3, 2] which are a triadic generalization of lattices. Since Boolean lattices are fundamental algebraic structures in Lattice Theory and Mathematical Logic, it is natural to ask for the triadic analogue of Boolean lattices, the Boolean trilattices, which play a similar role in the triadic case as Boolean lattices in the dyadic case. The most significant definition are mentioned in following sections. The Triadic approach brings added value in comparison with other data mining techniques. It consist in inclusion of a set of conditions, which says whether there the relation among objects and attributes exists or not.

The usage of FCA in the area of Multi-agent systems can facilitate solutions of several tasks and/or point to certain data relations, which should be further analyzed in more detail. These are the most important task the research focus on:

- Make clusters of agents based on their features,
- observe similar behaviors of agents,
- analyze the behavior of particular agent witch respect to all agents in the same cluster
- predict the behaviors of agents,
- agents substitutions

FCA utilization within MAS is described in this paper. It is illustrated on examples of traffic infrastructure, where the behaviors of agents were analyzed.

### 2. Triadic FCA

This section brings a shot introduction of triadic concepts and lattices.

A triadic context consists of sets of formal objects, formal attributes and formal conditions together with the formalization of the ternary relation saying when an object has an attribute under a certain condition. Triadic contexts provide a natural interpretation for modalities like *necessity* and *possibility*, in particular for the case of the dyadic relationships between formal objects and attributes are considered: a formal object  $g$  has *necessarily* a formal attribute  $m$  if  $g$  has  $m$  under all formal conditions of the context;  $g$  has *possibly*  $m$  if  $g$  has  $m$  under some formal condition. Such necessity and possibility relations give rise to dyadic contexts allowing a modal analysis of triadic data contexts. About ten years ago, triadic contexts were presented by Lehmann and Wille [8] as an extension of Formal Concept Analysis. However, they have rarely been used up to now, which may be due to a rather complex structure of a resulting diagrams.

R. Wille points out in [16] that the theory of multi-contexts is closely connected with the triadic setting of Formal Concept Analysis. Three formal contexts  $\mathbb{K}_1$ ,  $\mathbb{K}_2$  and  $\mathbb{K}_3$  can be regarded as a triadic context. Then a triadic concept lattice can be understood as a natural triadic extension of the three lattices of the concepts  $\mathbb{K}_1$ ,  $\mathbb{K}_2$  and  $\mathbb{K}_3$ . Now, the basic notions of Triadic Concept Analysis [15] are defined.

**Definition 1:** A **triadic context**  $\mathbb{K} := (G, M, B, Y)$  consists of a set of objects  $G$ , a set of attributes  $M$ , a set of conditions  $B$  and a ternary relation  $Y$  between them, i.e.,  $Y \subseteq G \times M \times B$ . A triple  $(g, m, b) \in Y$  is read: The object  $g \in G$  has the attribute  $m \in M$  under the condition  $b \in B$ .

The triadic context can be represented by a three dimensional cross table. In our example (see the figure 1) there are three agents as objects, three behavior as attributes and three common conditions. The ternary relation  $Y$  obviously means which of the agents have a behavior on specific condition.

We need some kinds of derivation operators to introduce the triadic concepts. For their definition, it is useful to write  $K_1, K_2, K_3$  instead of  $G, M, B$ . We define for  $Z \subseteq K_i \times K_j$ ,  $A_i \subseteq K_i$  and  $\{i, j, k\} = \{1, 2, 3\}$  with  $i < j$ :

- $Z^{(k)} := \{a_k \in K_k \mid a_k, a_i, a_j \text{ are related by } Y \text{ for all } (a_i, a_j) \in Z\}$
- $A_j^{(i,j,A_i)} := A_i^{(j,k,A_j)} := \{a_k \in K_k \mid a_k, a_i, a_j \text{ are related by } Y \text{ for all } a_i \in A_i \text{ and } a_j \in A_j\}$
- $A_k^{(k)} := \{(a_i, a_j) \in K_i \times K_j \mid a_i, a_j, a_k \text{ are related by } Y \text{ for all } a_k \in A_k\}$

Thus, triadic concepts can be introduced as a natural generalization of dyadic concepts:

**Definition 2:** A **triadic concept** of a triadic context  $\mathbb{K}$  is defined as a triple  $(A_1, A_2, A_3)$  of subsets  $A_i \subseteq K_i$ , where  $i \in \{1, 2, 3\}$  with  $A_k := (A_i \times A_j)^{(k)}$  for  $\{i, j, k\} = \{1, 2, 3\}$  with  $i < j$ . The sets  $A_1$ ,  $A_2$  and  $A_3$  are called *extent*, *intent* and *modus* of the concept  $c := (A_1, A_2, A_3)$  and are denoted  $\text{Ext}(c)$ ,  $\text{Int}(c)$  and  $\text{Mod}(c)$ . The set of all triadic concepts of a triadic context  $\mathbb{K}$  is denoted by  $\mathfrak{T}(\mathbb{K})$ .

Mathematical structure theory of  $\mathfrak{T}(\mathbb{K})$  is elaborated in [15] and [3]. However, let me state some simple properties of triadic concepts:

Suppose  $\mathbb{K} := (K_1, K_2, K_3)$  is a triadic context. Then for  $X_i \subseteq K_i$  and  $X_k \subseteq K_k$  with  $\{i, j, k\} = \{1, 2, 3\}$  we set

$$\begin{aligned} A_j &:= X^{(i,j,X_k)} \\ A_i &:= A^{(i,j,X_k)} \\ A_k &:= A^{(j,k,A_j)} \end{aligned}$$

Notice that a dyadic context  $\mathbb{K} := (G, M, I)$  can be understood as a triadic context  $\mathbb{K}_t := (G, M, \{b\}, Y)$  with only one condition  $b$  and  $Y := I \times \{b\}$ . Then, the mapping  $(A_1, A_2) \rightarrow (A_1, A_2, \{b\})$  is a bijection from  $\mathfrak{B}(\mathbb{K}) \setminus \{(G, M)\}$  to  $\mathfrak{T}(\mathbb{K}_t) \setminus \{o_3\}$ . We refer to [11]. Thus, the triadic theory can be understood as a generalization of the dyadic theory.

**The triadic diagram:** It is a symmetric structure, for the sets of objects, attributes, and conditions are all treated equally since none of them is preferred to the others. It could be drawn as a triangular graph. A simple example is shown in the figure 2. It consists of three orderings that represent all extents, intents and moduses of all triadic concepts. There is a possibility to create a complete lattice, *side lattice*, for a set of all extents as well as intents and moduses. These lattices are usually drawn along the sides of a triangular graph. Each parallel line from one vertex to the opposite side represents individual extent (intent and modus, respectively). In the vertices of the triangular graph there are bottom concepts of side lattices. The triples, triadic concepts, are represented by the circles in the triadic diagram.

The size of a triadic diagram is the only disadvantage of such visualization. Usually, the sets of objects, attributes or conditions could contain tens of elements. That is why the number of all triadic concepts grows rapidly. 3D visualization is more suitable in practical applications, programs for a larger data collection. Then X, Y and Z axes

represent the three dimensions, the set of extents, intents and moduses.

*Example 1:* An illustrative example of a triadic diagram. First, there is a three dimensional cross table in the figure 1 that represents a triadic context  $\mathbb{K} := (G, M, B, Y)$ , where:  
 $G := \{1, 2, 3\}$  ... the set of agents  
 $M := \{a, b, c\}$  ... the set of behaviour  
 $B := \{x, y, z\}$  ... the set of conditions  
 All triadic concepts and related diagram is shown in the figure 2.

$x$	a	b	c
1			
2		x	
3		x	

$y$	a	b	c
1			
2		x	x
3		x	

$z$	a	b	c
1			
2	x	x	x
3	x	x	

Figure 1. A 3-dimensional cross table for the triadic graph of the figure 2.

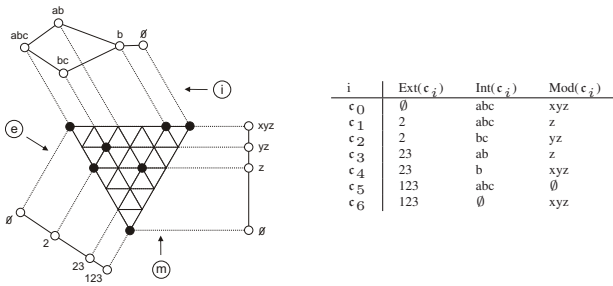


Figure 2. The triadic graph and list of all triadic concepts.

### 3. Triadic FCA within MAS

An application usage of FCA will be demonstrated on particular Situated Multi Agent System [6]. The following text describes possible tasks that could be solved by the SMAS.

#### 3.1. Problem specification

For the sake of simplicity, let us consider *accessible, deterministic, static* and *discrete* environment of SMAS. Let define the main part of such SMAS:

- **O:** the set of objects, namely static elements of the environment, e.g. roads and crossroads.

- **A:** the set of agents. We consider *mobile* agents such as cars that can move along the environment. More precisely they move along the objects, such as roads.
- **R:** the set of relations between agents and objects. It represents a possible movement of some agent along some object. We can find many restrictions of such movements. For example, agents can have features which impose limitations on their movement (a tractor cannot move along a motorway because of its speed, or the trucks cannot be on a village way from 16:00 PM to 6:00 AM next day).

Such an SMAS specification gives rise to several tasks. For example, the 5-ary relation  $R5 \subseteq WHO \times WHERE \times WHEN \times HOW \times WHAT$  can be defined, where:

- **WHO**  $\subseteq A$  (agents)
- **WHERE**  $\subseteq O$  (objects)
- **WHEN** – is a set of time intervals with respect to laws and other rules, e.g. truck movement restriction in villages  $\Rightarrow$  the interval 6:00 – 16:00. Or it can represent another set of conditions, e.g. the scale of truck weights.
- **HOW** – the set of agents activities that can be described by process modeling [13], e.g. slower cars move on the right lane of the highway.
- **WHAT** – the set that represents the outputs of all agent activities.

#### 3.2. Triadic lattice in MAS

Consider the 3-ary relation  $R3 \subseteq WHERE \times WHAT \times WHO$  for the usage of triadic FCA in the above defined SMAS. The concrete usage will be shown on a small instance of an imaginary city that will simulate the reality. The example is focused on the traffic simulation.

The whole geographical area is covered by several roads and crossroads (see the Figure 3). Each crossroad is signed by a small letter and the roads are signed by a pair  $x - y$ , where  $x$  and  $y$  are the letters of border crossroads. Each road type is connected with traffic restrictions, e.g. trucks can not move along the lanes. Such a system of roads represents a static structure of SMAS. Hence in the above defined SMAS, each road or crossroad is represented by an object  $o \in O$ .

The inhabitants of the city make up the dynamic part of the system. Every inhabitant is an agent  $a \in A$  of SMAS.

Now we are able to create a three dimensional cross table (see the Table 4 on the right) that will represent a context  $\mathbb{K} := (G, M, B, Y)$ , where:

- $G$  is a set of places (roads and crossroads), thus  $O \in G$
- $M$  is a set of traffic violations
- $B$  is a set of agents, thus  $A \in B$
- $Y$  is a ternary relations that represents the above defined kind of the relation  $WHERE \times WHAT \times WHO$ .  
( $g, m, b$ )  $\in Y$  if and only if an agent  $b \in B$  has committed a traffic violation  $m \in M$  on a place  $g \in G$ .

Te Table 4 shows the formal context  $\mathbb{K} := (G, M, B, Y)$ .

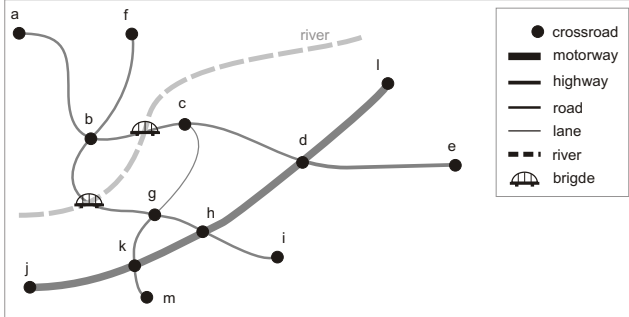


Figure 3. The map of all places in our system.

After having constructed the three dimensional incidence matrix, a concept list (see the Figure 4) and a triadic lattice are computed. The process of such a computation as well as its integration into MAS will be described in the following section. However, the resulting concept list gives rise to several questions. For example:

- Concept 6: Almost every agent exceeded the speed on the road  $d-e$ . Is the the speed limitation on this road reasonably settled?
- Concept 11: The agent  $A$  does not follow the driving directives on the crossroad  $b$ . Why does he do that?
- Concept 34: Are there any common features of places  $m, a-b, d-e$  that cause the agents to exceed the speed?

The previous illustrative example described the usage of formal concept analysis and its triadic approach in the area of traffic simulation within SMAS.

#### 4. AgentStudio Simulator

It is quite difficult to apply and verify mentioned approach in the real world. However, a simulation tool **AgentStudio Simulator** (see the Figure 5) was developed as the framework for modeling, operating, controlling and analyzing of the agents. This is a part of the more complex application called **AgentStudio**, which contains a tool

$\mathbb{K} := (G, M, B, Y)$	1 - car accident	2 - car fatality	3 - illegal parking	4 - speeding	5 - driving ban violation	6 - contra-flow-lane driving	7 - right-of-way violation
a - crossroad	F		G,I	B	A		
b - crossroad							
c - crossroad							
d - crossroad		G		H	I		
e - crossroad		J					
f - crossroad							
g - crossroad	I,K	B					H
h - crossroad	F				A,H		
i - crossroad							H
j - crossroad		A,H			B	J	
k - crossroad		K	A				
l - crossroad							
m - crossroad			B	D,J			
a-b - road				J			
b-c - road							
c-d - road							K
d-e - highway	J	A	A	H			I
f-b - road	I	C					F,J
b-g - road							
c-g - lane		J	I				
g-h - road							C
h-i - road			B				
g-k - road							
k-m - road	B				G		
j-k - motorway	K	F					
k-h - motorway		D					
h-d - motorway		D					
d-l - motorway	E				D		J

Figure 4. Formal context  $\mathbb{K} := (G, M, B, Y)$ , where  $G$  is a set of places,  $M$  is a set of traffic violations, and  $B$  is a set of inhabitants (agents).

for agent behavior modeling based on process approach as well.

Architecture of implemented MAS is extensible thanks to JADE framework [1] [5]. This enables connection of extra modules (set of agents) to search and analyze agents behavior patterns based on FCA.

For illustration purposes, the problem domain of traffic management was chosen as primary simulation background. Our **AgentStudio Simulator** enables to define a model of traffic infrastructure, including roads, lanes, crossroads, obstacles, parking, traffic signs, etc. Traffic simulations try to reflect real situations taking place on roads.

Above this "model of reality", there are the agents (mainly intelligent agents with brain facility) which represent autonomous cars [10] [9]. These agents have predefined behavior as well as possibility of own decision making based on logic approaches [12]. This is the list of situations which the agents are able to realize:

- Cars overtake each other and they will recognize traffic

**Table 1. The list of chosen triadic concepts of the context  $\mathbb{K}$**

$c_i$	extent	intent	modus
0		car accident, car fatality, illegal parking, speeding, driving ban violation, contra-flow-lane driving, right-of-way violation	A, B, C, D, E, F, G, H, I, J, K
⋮		⋮	
6	d-e	speeding	B, C, D, H, I, J
⋮		⋮	
11	b	car accident, car fatality, speeding, driving ban violation	A
⋮		⋮	
34	m, a-b, d-e	speeding	J
⋮		⋮	
42	a, b, c, d, e, f, g, h, i, j, k, l, m, a-b, b-c, c-d, d-e, f-b, b-g, c-g, g-h, h-i, g-k, k-m, j-k, k-h, h-d, d-l	car accident, car fatality, illegal parking, speeding, driving ban violation, contra-flow-lane driving, right-of-way violation	
43	a, b, c, d, e, f, g, h, i, j, k, l, m, a-b, b-c, c-d, d-e, f-b, b-g, c-g, g-h, h-i, g-k, k-m, j-k, k-h, h-d, d-l		A, B, C, D, E, F, G, H, I, J, K

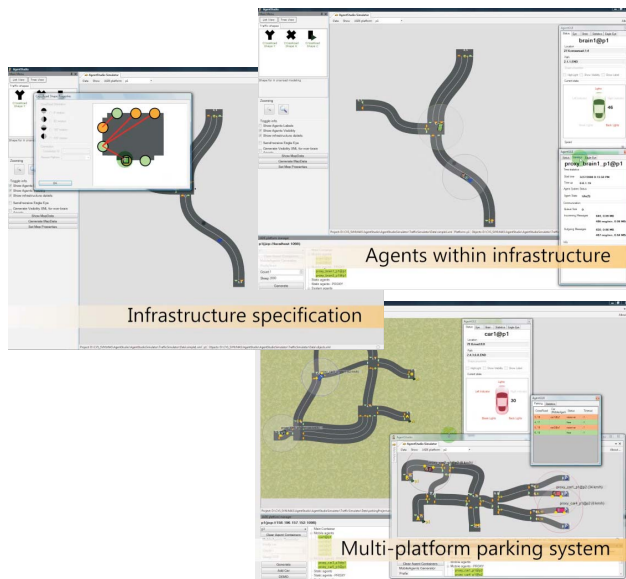
obstacles,

- they safety pass through crossroads,
- they keep safety distance from other agents (cars),
- they keep basic rules defined in Highway Code.

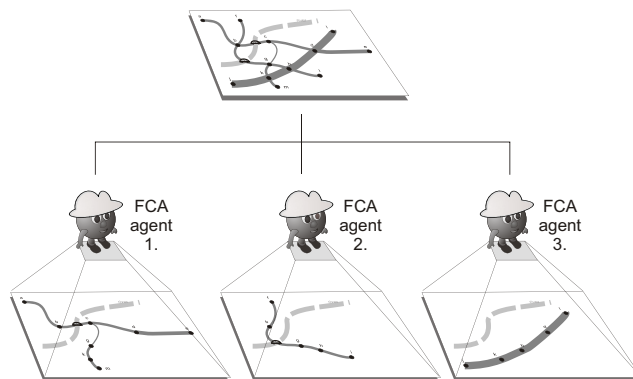
Thanks to this decision making approach, some other situations resp. behaviors can appear during the agent live. These extra situations represent inputs for the process of FCA behavior recognition. The process is executed by FCA agents, which observe the simulated world (see the Figure 6). FCA agents collect information on every agent with respect to its state, time and location. FCA tool provides outputs in form of formal triadic concept and diagrams, which are used in further decisions making process.

## 5. Conclusion

We presented the data mining technique based on Triadic Formal Concept Analysis in the area of Multi-agent system. It was illustrated on the example of traffic simulation. Thanks to this connection, we are able to identify common features of agent behaviors which is necessary for



**Figure 5. AgentStudio Simulator**



**Figure 6. FCA agents and their fields of view**

further evaluation of data of MAS. The outputs do not provide final solutions of all task related to MAS operation. However, they offer a concept view of agents behaviors in time and place. This can make MAS more flexible, stable and intelligent.

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