On the Use of the Uncertain Induced OWA Operator and the Uncertain Weighted Average and its Application in Tourism Management

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Abstract

We develop a new approach for dealing with uncertain information in a decision making problem about tourism management. We use a new aggregation operator that uses the uncertain weighted average and the uncertain induced ordered weighted averaging (UIOWA) operator in the same formulation. We study some of the main advantages and properties of the new aggregation called uncertain induced ordered weighted averaging - weighted averaging (UIOWAWA) operator. We study its applicability in a decision making problem about the selection of holiday trips. We see that depending on the particular type of UIOWAWA operator used, the results may lead to different decisions.

1. Introduction

The weighted average (WA) is one of the most common aggregation operators found in the literature. It can be used in a wide range of different problems including statistics, economics, engineering, etc. Another interesting aggregation operator is the ordered weighted averaging (OWA) operator [13]. The OWA operator provides a parameterized family of aggregation operators that range from the maximum to the minimum. For further reading on the OWA operator and some of its applications, refer to [1-7,9-17].

Usually, when using these approaches it is considered that the available information is exact numbers. However, this may not be the real situation found in the specific problem considered. Sometimes, the available information is vague or imprecise and it is not possible to analyze it with exact numbers. Then, it is necessary to use another approach that is able to assess the uncertainty such as the use of interval numbers. By using interval numbers we can consider a wide range of possible results included between the maximum and the minimum. Note that in the literature, there are a lot of studies dealing with uncertain information represented in the form of interval numbers such as [4,8,10-12].

Recently, some authors have tried to unify the WA and the OWA in the same formulation. It is worth noting the work developed by Torra [9] with the introduction of the weighted OWA (WOWA) operator and the work of Xu and Da [12] about the hybrid averaging (HA) operator. Both models arrived to a partial unification between the OWA and the WA because both concepts were included in the formulation as particular cases. However, as it has been studied in [4], these models seem to be a partial unification but not a real one because they can unify them but they cannot consider how relevant these concepts are in the specific problem considered. For example, in some problems we may prefer to give more importance to the OWA operator because we believe that it is more relevant and vice versa. This problem is solved with the ordered weighted averaging – weighted averaging (OWAWA) operator [4].

In this paper, we present a new approach to unify the IOWA operator with the WA when the available information is uncertain and can be assessed with interval numbers. We call it the uncertain induced ordered weighted averaging – weighted averaging (UIOWAWA) operator. The main advantage of this approach is that it unifies the OWA and the WA taking into account the degree of importance of each case in the formulation and considering that the information is given with interval numbers. Thus, we are able to consider situations where we give more or less

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importance to the UOWA and the UWA depending on our interests and the problem analysed. Furthermore, by using the UIOWAWA, we are able to use a complex reordering process in the OWA operator in order to represent complex attitudinal characters. We also study different properties of the UIOWAWA operator and different particular cases.

We also analyze the applicability of the new approach and we see that it is possible to develop an astonishingly wide range of applications. For example, we can apply it in a lot of problems about statistics, economics, engineering and decision theory. In this paper, we focus on a decision making problem about tourism management. We develop a decision making problem where a decision maker want to select an optimal holiday trip. The main advantage of the UIOWAWA in these problems is that it is possible to consider the subjective probability (or the degree of importance) and the attitudinal character of the decision maker at the same time.

This paper is organized as follows. In Section 2 we revise some basic concepts. In Section 3 we present the new aggregation operator. Section 4 analyzes different families of UIOWAWA operators. In Section 5 we develop an application of the new approach and in Section 7 we summarize the main results of the paper.

2. Preliminaries

In this Section we briefly review the interval numbers, the OWAWA operator, the IOWA operator and the UOWA operator.

2.1. Interval Numbers

The interval numbers [8] are a very useful and simple technique for representing the uncertainty. They have been used in a wide range of applications.

The interval numbers can be expressed in different forms. For example, if we assume a 4-tuple (c_1, c_2, c_3, c_4), that is to say, a quadruplet; we could consider that c_1 and c_4 represents the minimum and the maximum of the interval number, and c_2 and c_3, the interval with the highest probability or possibility, depending on the use we want to give to the interval numbers. Note that $c_1 \leq c_2 \leq c_3 \leq c_4$. If $c_1 = c_2 = c_3 = c_4$, then, the interval number is an exact number; if $c_2 = c_3$, it is a 3-tuple known as triplet; and if $c_1 = c_2$ and $c_3 = c_4$, it is a simple 2-tuple interval number.

In the following, we are going to review some basic interval number operations. Let C and B be two triplets, where $C = (c_1, c_2, c_3)$ and $B = (b_1, b_2, b_3)$. Then:

1. $C + B = (c_1 + b_1, c_2 + b_2, c_3 + b_3)$
2. $C - B = (c_1 - b_3, c_2 - b_2, c_3 - b_1)$
3. $C \times k = (k \times c_1, k \times c_2, k \times c_3)$; for $k > 0$.
4. $C \times B = (c_1 \times b_1, c_2 \times b_2, c_3 \times b_3)$; for R^+.
5. $C \div B = (c_1 \div b_3, c_2 \div b_2, c_3 \div b_1)$; for R^+.

Note that other operations could be studied [8] but in this paper we will focus on these ones.

2.2. OWAWA Operator

The ordered weighted averaging – weighted averaging (OWAWA) operator is an aggregation operator that unifies the WA and the OWA operator in the same formulation [4]. It can be defined as follows.

Definition 1. An OWAWA operator of dimension n is a mapping OWAWA: $R^n \to R$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$OWAWA\,(a_1, \ldots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j \tag{1}$$

where b_j is the jth largest of the a_i, each argument a_i has an associated weight (WA) v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$, being β a parameter such that $\beta \in [0, 1]$ and v_j is the weight (WA) v_i ordered according to b_j, that is, according to the jth largest of the a_i.

By choosing a different manifestation in the weighting vector, we are able to obtain a wide range of particular types of OWAWA operators [4]. Especially, when $\beta = 0$, we get the WA, and if $\beta = 1$, we get the OWA operator.

2.3. IOWA Operator

The IOWA operator [16] is an extension of the OWA operator that uses order inducing variables in the reordering step. It can be defined as follows:

Definition 2. An IOWA operator is a mapping IOWA: $R^n \to R$ that has an associated weighting vector W with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$\text{IOWA}(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} w_j b_j \tag{2}$$

where b_j is the a_i value of the IOWA pair $\langle u_i, a_i \rangle$ having the jth largest u_i, u_i is the order inducing variable and a_i is the argument variable.

2.4. UOWA Operator

The uncertain OWA (UOWA) operator was introduced by [11]. It is an extension of the OWA operator for uncertain situations where the available information can be assessed with interval numbers. It can be defined as follows:

Definition 3. Let Ω be the set of interval numbers. An UOWA operator of dimension n is a mapping UOWA: $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$\text{UOWA}(\tilde{a}_1, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j b_j \tag{3}$$

where b_j is the jth largest of the $\tilde{a}_i$ and $\tilde{a}_i$ is the argument variable represented in the form of interval numbers.

Note that in the reordering step of the interval numbers it is necessary to establish a criterion for its comparison. We will use the following one. First, we will analyze if there is an order between the interval numbers. That is, if all the values of the interval a are higher than the corresponding values in the interval c. If not, we will calculate an average of the interval number. For example, if $n = 2$, $(a_1 + a_2) / 2$; if $n = 3$, $(a_1 + 2a_2 + a_3) / 4$; etc. If there is still a tie, then, we will follow a subjective criterion such as considering only the minimum, etc.

3. Uncertain Induced OWAWA Operator

The uncertain induced OWAWA (UIOWAWA) operator is an aggregation operator that uses the WA and OWAs in the same formulation. It also uses order inducing variables in order to represent the reordering process, from a general point of view. Moreover, the UIOWAWA also deals with an uncertain environment that cannot be assessed with exact numbers but it is possible to use interval numbers. It can be defined as follows.

Definition 4. Let Ω be the set of interval numbers. An UIOWAWA operator of dimension n is a mapping UIOWAWA: $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$\text{UIOWAWA}(\langle u_1, \tilde{a}_1 \rangle, \ldots, \langle u_n, \tilde{a}_n \rangle) = \sum_{j=1}^{n} \hat{v}_j b_j \tag{4}$$

where b_j is the $\tilde{a}_i$ value of the UIOWAWA pair $\langle u_i, \tilde{a}_i \rangle$ having the jth largest u_i, u_i is the order inducing variable, each argument $\tilde{a}_i$ is an interval number and it has an associated weight v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, $\hat{v}_j = \beta w_j + (1 - \beta) v_j$ with $\beta \in [0, 1]$ and v_j is the weight v_i ordered according to b_j, that is, according to the jth largest of the $\tilde{a}_i$.

Note that it is also possible to formulate the UIOWAWA operator separating the part that strictly affects the OWA operator and the WAs.

Definition 5. Let Ω be the set of interval numbers. An UIOWAWA operator is a mapping UIOWAWA: $\Omega^n \rightarrow \Omega$ of dimension n, if it has an associated weighting vector W, with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$ and a weighting vector V, with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$, such that:

$$\text{UIOWAWA}(\langle u_1, \tilde{a}_1 \rangle, \ldots, \langle u_n, \tilde{a}_n \rangle) = \beta \sum_{j=1}^{n} w_j b_j + (1 - \beta) \sum_{i=1}^{n} v_i \tilde{a}_i \tag{5}$$

where b_j is the $\tilde{a}_i$ value of the UIOWAWA pair $\langle u_i, \tilde{a}_i \rangle$ having the jth largest u_i, u_i is the inducing variable, each argument $\tilde{a}_i$ is an interval number and $\beta \in [0, 1]$.

Note that if $\beta = 1$, we get the UIOWA operator and if $\beta = 0$, the uncertain weighted average.

In the following, we are going to give a simple example of how to aggregate with the UIOWAWA operator. We consider the aggregation with both definitions.

Example 1. Assume the following arguments in an aggregation process: ([20, 30], [60, 70], [40, 50], [30, 40]). Assume the following weighting vector $W = (0.2, 0.2, 0.3, 0.3)$ and the following subjective weighting vector $V = (0.4, 0.3, 0.2, 0.1)$. And assume the following order inducing variables: $U = (8, 4, 7, 3)$. In this example, we propose that the subjective information has a degree of importance of 60% while the weighting vector W a degree of 40% ($\beta = 0.4$). If we want to aggregate this information by using the UIOWAWA operator, we will get the following. The aggregation can be solved either with the Eq. (4) or (5). With Eq. (5) we get the following.

UIOWAWA = 0.4 × (0.2 × [20, 30] + 0.2 × [40, 50] + 0.3 × [60, 70] + 0.3 × [30, 40]) + 0.6 × (0.4 × [20, 30] + 0.3 × [60, 70] + 0.2 × [40, 50] + 0.1 × [30, 40]) = [37.8, 47.8].

Note that different types of interval numbers could be used in the aggregation such as 2-tuples, triplets, quadruplets, etc.

When using interval numbers we have the additional problem of how to reorder the arguments because now we are using interval numbers. For simplicity, we recommend the criteria explained in Section 2.4. Note that in the reordering of the arguments of the UIOWAWA operator, this is not a problem because they are reordered according to the order inducing variables.

Note that it is possible to distinguish between the descending UIOWAWA (DUIOWAWA) and the ascending UIOWAWA (AUIOWAWA) operator by using $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DUIOWAWA and w^*_{n-j+1} the jth weight of the AUIOWAWA operator.

Note that if the weighting vector is not normalized, i.e., $W = \sum_{j=1}^{n} w_j \neq 1$, or $V = \sum_{j=1}^{n} v_i \neq 1$, then, the UIOWAWA operator can be expressed as:

$$\text{UIOWAWA}(\langle u_1, \tilde{a}_1\rangle, \ldots, \langle u_n, \tilde{a}_n\rangle) = = \frac{\beta}{W}\sum_{j=1}^{n} w_j b_j + \frac{(1-\beta)}{V}\sum_{i=1}^{n} v_i \tilde{a}_i \quad (6)$$

Note that it is possible to consider that the weights of the UIOWAWA operator are also interval numbers. In future research, we will analyze this issue.

The UIOWAWA is monotonic, commutative, bounded and idempotent. It is monotonic because if $\tilde{a}_i \geq e_i$, for all $\tilde{a}_i$, then, UIOWAWA($\langle u_1, \tilde{a}_1\rangle, \ldots, \langle u_n, \tilde{a}_n\rangle$) ≥ UIOWAWA($\langle u_1, e_1\rangle, \ldots, \langle u_n, e_n\rangle$). It is commutative because any permutation of the arguments has the same evaluation. That is, UIOWAWA($\langle u_1, \tilde{a}_1\rangle, \ldots, \langle u_n, \tilde{a}_n\rangle$) = UIOWAWA($\langle u_1, e_1\rangle, \ldots, \langle u_n, e_n\rangle$), where ($\langle u_1, e_1\rangle, \ldots, \langle u_n, e_n\rangle$) is any permutation of the arguments ($\langle u_1, e_1\rangle, \ldots, \langle u_n, e_n\rangle$). It is bounded because the UIOWAWA aggregation is delimitated by the uncertain minimum and the uncertain maximum. That is, Min$\{\tilde{a}_i\}$ ≤ UIOWAWA($\langle u_1, \tilde{a}_1\rangle, \ldots, \langle u_n, \tilde{a}_n\rangle$) ≤ Max$\{\tilde{a}_i\}$. It is idempotent because if $\tilde{a}_i = a$, for all $\tilde{a}_i$, then, UIOWAWA($\langle u_1, \tilde{a}_1\rangle, \ldots, \langle u_n, \tilde{a}_n\rangle$) = a.

4 Families of UIOWAWA Operators

A further interesting issue to consider is the different families of UIOWAWA operators that are found by analysing the weighting vector W and the coefficient β. If we look to the coefficient β, we get the following particular cases.

- If β = 1, we get the UIOWA operator.
- If β = 0, we get the uncertain weighted average (UWA).

And if we look to the weighting vector W, we get, for example, the following ones.

- The uncertain weighted maximum ($w_p = 1$ and $w_j = 0$, for all $j \neq p$, and $u_p = \text{Max}\{\tilde{a}_i\}$).
- The uncertain weighted minimum ($w_p = 1$ and $w_j = 0$, for all $j \neq p$, and $u_p = \text{Min}\{\tilde{a}_i\}$).
- The uncertain average (UA) ($w_j = 1/n$, and $v_j = 1/n$, for all $\tilde{a}_i$).
- The UOWA average ($v_i = 1/n$, for all $\tilde{a}_i$).
- The uncertain average with the UWA ($w_j = 1/n$, for all $\tilde{a}_i$)
- The step-UIOWAWA ($w_k = 1$ and $w_j = 0$, for all $j \neq k$).
- The general olympic-UIOWAWA operator ($w_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$; and for all others $w_{j^*} = 1/(n-2k)$, where $k < n/2$).
- The centered-UIOWAWA (if it is symmetric, strongly decaying from the center to the maximum and the minimum, and inclusive).

Theorem 1. If the interval numbers are reduced to the usual exact numbers, then, the UIOWAWA operator becomes the IOWAWA operator [4].

Proof. Assume a triplet = (a_1, a_2, a_3). If $a_1 = a_2 = a_3$, then $(a_1, a_2, a_3) = a$. Thus, we get the IOWAWA operator.

Note that other families of UIOWAWA operators could be found following a similar methodology as it has been developed in a wide range of papers for the OWA operator and its extensions [1-7,9-17].

5. Application in Tourism Management

In the following, we present a numerical example of the new approach in a decision making problem about tourism management. We analyze the decision of selecting an optimal holiday trip.

Note that similar problems could be developed in the selection of other problems in tourism management such as the selection of hotels, selection of human resources for touristic purposes and the selection of touristic strategies. Assume a decision maker is planning his holidays and he considers 5 possible trips to do during his holidays:

- A_1 = Trip to North America.
- A_2 = Trip to Asia.
- A_3 = Trip to Australia.
- A_4 = Trip to South America.
- A_5 = Trip to Africa.

In order to evaluate these trips, the decision maker considers 5 key characteristics that the optimal trip should have:

- C_1 = Price of the trip.
- C_2 = Environment of the country.
- C_3 = Climate of the country.
- C_4 = Touristic attractions of the country.
- C_5 = Other variables.

The results of the available trips, depending on the characteristic C_i and the alternative A_k that the decision maker chooses, are shown in Table 1.

In this problem, the decision maker assumes the following degrees of importance (UWA) of the characteristics: $V = (0.1, 0.2, 0.2, 0.2, 0.3)$. He assumes that the UIOWA weight is: $W = (0.1, 0.1, 0.2, 0.2, 0.4)$; with the following order inducing variables: $U = (8, 4, 9, 3, 2)$. Note that UWA has an importance of 70% and the UIOWA an importance of 30% ($\beta = 0.3$) because he believes that the UWA is more relevant in the problem. For doing so, we will use Eq. (4) and Eq. (5) to calculate the UIOWAWA aggregation. The results are shown in Table 2.

Table 2: UIOWAWA weights

	$\hat{v}_1$	$\hat{v}_2$	$\hat{v}_3$	$\hat{v}_4$	$\hat{v}_5$
V*	0.17	0.1	0.2	0.2	0.33

With this information, we can aggregate the expected results for each state of nature in order to make a decision. In Table 3, we present different results obtained by using different types of UIOWAWA operators.

As we can see, in this example, the optimal choice is A_2, excepting when using the UA where we get A_1 and A_4 as optimal choices. Note that the decision maker will use the method that it is closest to his interests.

6. Conclusions

We have presented the UIOWAWA operator. It is a new aggregation operator that unifies the UIOWA operator with the UWA when the available information is uncertain and can be assessed with interval numbers. The main advantage of this operator is that it provides more complete information because it represents the information in a more complete way considering the maximum and the minimum results.

We have analysed the applicability of the new approach and we have seen that it is very broad because it can be applied in a lot of problems where previously were studied with the WA or the OWA. In this paper, we have focussed on an application in tourism management where a decision maker is looking for an optimal holiday trip. We have seen that depending on the aggregation operator used the results may lead to different decisions.

Table 1. Uncertain payoff matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	(70,80,90)	(60,70,80)	(40,50,60)	(50,60,70)	(30,40,50)
A_2	(30,40,50)	(30,40,50)	(30,40,50)	(70,80,90)	(70,80,90)
A_3	(60,70,80)	(80,90,100)	(30,40,50)	(20,30,40)	(50,60,70)
A_4	(40,50,60)	(70,80,90)	(20,30,40)	(90,100,110)	(30,40,50)
A_5	(60,70,80)	(50,60,70)	(30,40,50)	(40,50,60)	(60,70,80)

Table 3. Uncertain aggregated results.

	UA	UWA.	UIOWA	UIOWAWA	IOWAWA
A_1	(50,60,70)	(46,56,66)	(45,55,65)	(45.7,55.7,65.7)	55.7
A_2	(46,56,66)	(50,60,70)	(54,64,74)	(51.2,61.2,71.2)	61.2
A_3	(48,58,68)	(47,57,67)	(49,59,69)	(47.6,57.6,67.6)	57.6
A_4	(50,60,70)	(49,59,69)	(50,60,70)	(49.3,59.3,69.3)	59.3
A_5	(48,58,68)	(48,58,68)	(51,61,71)	(48.9,58.9,68.9)	58.9

In future research, we expect to develop further developments by using other types of information such as fuzzy numbers, linguistic variables and expertons (probabilistic sets with intervals). We will also add other characteristics in order to obtain a more complete formulation such as generalized and quasi-arithmetic means, distance measures, t-norms and t-conorms, etc. Finally, we will also develop different types of applications especially in decision theory but also in other fields such as statistics, engineering, business and economics.

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