

Strategic Assessment of Business

Doña J. M.

Dept. of L. & Computer Science
University of Málaga, 29071
Málaga, Spain
jmdona@lcc.uma.es

Peláez J. I.

Dept. of L. & Computer Science
University of Málaga, 29071
Málaga, Spain
jignacio@lcc.uma.es

Vargas L. G.

Katz Graduate School of Business
University of Pittsburgh, 15260
Pittsburgh, USA
vargas@katz.pitt.edu

Abstract— The stock market volatility and the actual stock exchange activity have increased the need of counting with effective methods on the part of financial analysts to achieve a division in relation to the investment actions, being also growing the demand of methodological instruments that reduce and minimize the risks and uncertainty when valuating financial actives and companies. These systems not only must use quantitative information but the inclusion of qualitative information must also bear heavily on them, as an improvement element in the adjustment of these valuating methods, with the aim of throwing a more well-conceived or less mistaken decision. In this work, we present an alternative strategic assessment of business based in quantitative information.

Keywords— decision making; linguistic information; strategic valuation; discounted cash-flow

I. INTRODUCTION

Nowadays, the success of the stock exchange activity as well as actives valuations into the business market mostly depend on the capacity of anticipating to the stock market trends and the achievement of a quick reply. Managers must assimilate the information and adopt the decisions in a chaotic environment, provided with risk and uncertainty, most of times without counting with experience and an adequate planning, and even without having enough time to carry out an strict and systematic analysis [1, 2].

In the strategic valuation it is pretended to determine an interval of reasonable values in which the definitive value of the considered element will be included. For instance, when valuating a company the aim is to obtain an estimation which may never be a unique or exact number due to the difficulties belongings to the decision process. However this will depend on the company situation, the transaction moment and the method we use. To determine the right valuation it is necessary to establish hypothesis and future uncertainty scenes due to the possibility of event in relation to the risk elements inherent to the event scene. These hypotheses are involved in a risk and uncertainty universe, so that the final result will be an interval or series of values, and not only one of them. Finally, the information derived from the valuation report developed by the experts will mean the base in the

parties' negotiation, from which the definitive transaction price will arise.

The valuation methods use future estimations which, in many cases, are being giving out by experts according to their experience or reality perception, what means an added risk. In these conditions, it is necessary the disposition of several instruments which allow to operate with the uncertainty or risk of the expressed opinions, which normally will be defined in linguistic values in different ways of expression [4]. It is also necessary that these instruments should be able to add the opinions in a representative value of them.

A new strategic valuation model is represented in this work, based on OWA operators and 2-tuples linguistic representation model. The work structure is the following: In the second section the valuating model Discounted Cash-Flow will be introduced; in the third section the LAMA operators with the 2-tuples linguistic model are shown; in the fourth section the new model of strategic valuation is presented, developing a detailed example of application, and finally, conclusions are exposed.

II. STRATEGIC VALUATION USING THE DISCOUNTED CASH-FLOW

Within the last few years, with the stock markets worldwide extension, the technological development of these ones and the appearance of new financial instruments, have promoted new valuation techniques improving the ones already existing [10].

This fact has meant a growth not only in the valuating methods and its possible action setting but also in the need to discriminate against which methods are applicable in certain circumstances and the veracity or credibility of the results.

The need of predicting future scenes in which the own activity is developing, could create the impossibility to determine a specific and certain value, this may origin an interval of possible values within which the most certain and possible value of the enterprise will be found. The definitive value will come by consent and negotiation between the interested parts. As a result, the extent of possible values interval will distinguish the valuation report before the decision.

It is precisely in this point where we want to improve the quality of the available information to the investor, if it is possible to decrease interval extent of the possible values with the methodology proposed, the position of the interested parts will be closer to each other. Being like this, the possibility of agreement to finalize the operation will have increased in a well-balanced consensual price and even minimally negotiated. In this way, a rise in the stock market efficiency and fluidity is produced.

In this work it is used one of the methods which is actually the most accepted in the professional and scientific community, the 'Discounted Cash-Flow' model. The following expression distinguishes it:

$$V_E = \sum_{t=1}^n \frac{CFL_t}{\prod_{j=1}^t (1+K_j)^t} \quad (1)$$

where V_E represents the enterprise actual value; CFL is the cash-flow free from the enterprise for the period $-t$ (including the residual value); K_j is the adequate updating valuation and agreed for risk (WACC) to the period j and n is the valuation horizon.

III. LINGUISTIC MODEL AND OWA OPERATORS

In this work we proposed to combine the OWA operator LAMA [6] with the linguistic representation model based on 2-tuple [3].

The 2-tuple linguistic model represents the linguistic information by means of 2-tuples (r_i, α_i) , $r_i \in S$ and $\alpha_i \in [-0.5, 0.5)$. Where S is the set of linguistic labels; r_i represents the linguistics label center of the information and α_i is a numerical value that represents the translation from the original result β to the closest index label in the linguistic term set (r_i) , i.e., the Symbolic Translation.

This linguistic representation model defines a set of functions to make transformations among linguistic terms, 2-tuples and numerical values [3].

Definition. Let $s_i \in S$ be a linguistic term, then its equivalent 2-tuple representation is obtained by means of the function θ as:

$$\theta : S \rightarrow (S \times [-0.5, 0.5)) \quad (2)$$

$$\theta(s_i) = (s_i, 0) / s_i \in S \quad (3)$$

Definition. Let $S = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta : [0, g] \rightarrow S \times ([-0.5, 0.5)), \quad (4)$$

$$\Delta(\beta) = \begin{cases} s_i & i = \text{round}(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5) \end{cases} \quad (5)$$

where round is the usual operation, s_i has the closest index label to β and ∞ is the value of the symbolic translation.

Definition. Let $S = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set and (s_i, α) be a linguistic 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g]$.

$$\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g], \quad (6)$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \quad (7)$$

The LAMA operator is based in majority process [5, 7, 8, 9] and is a mapping function $F: R^n \rightarrow R$ that has associated a weighting vector $W = [w_1, w_2, \dots, w_n]^T$ where $w_i \in [0, 1]$ and

$$\sum_{i=1}^n w_i = 1.$$

$$\phi(a_1, a_2, \dots, a_n) = b_1 \otimes w_1 \oplus \dots \oplus b_n \otimes w_n \quad (8)$$

with b_j ordered using the importance function of the a_i , and \oplus is the sum of labels and \otimes is the product of a label by a positive real.

The weight used in the LAMA operator is usually calculated from the majority process and importance function described in [6].

$$w_i = f_i(b_1, \dots, b_n) = \frac{\gamma_i^{\delta_{\min}}}{\theta_{\delta_{\max}} \cdot \dots \cdot \theta_{\delta_{\min}}} + \dots + \frac{\gamma_i^{\delta_{\max}}}{\theta_{\delta_{\max}}} \quad (9)$$

where

$$\gamma_i^k = \begin{cases} 1 & \text{if } \delta_i \geq k \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

and

$$\theta_i = \begin{cases} (\text{number of item with } \delta \geq i) + 1 & \text{if } i \neq \delta_{\min} \\ \text{number of item with } \delta \geq i & \text{otherwise} \end{cases} \quad (11)$$

The majority operators aggregate in function of δ_i that generally represents the importance of the element i . The calculation method for the value δ_i is independent from the definition of the majority operators.

IV. STRATEGIC ASSESSMENT MODEL

Firstly, we take the estimation of the updating valuation appropriate and agreed to the risk, which is usually the balance of average cost of capital (WACC). We should start

with an analysis which considers every possible section among those we expected a valuated fluctuation to the periods which are considered in the research, in order to be a start point in the decision process between the parts which are taking over such process. In the following example, it has been established an analysis period of three years, and it has been considered the following intervals for the interest rate:

TABLE I. INTERVALS FOR THE UPDATING VALUATIONS

Updating valuations		
Year 1	Year 2	Year 3
[0,04; 0,05]	[0,045; 0,06]	[0,05; 0,06]

Next we ask, for instance, ten experts who express their valuations about the intervals. Three different linguistic domains are obtained with the following semantics:

Set 1: $S_5 = \{S_4^5, S_3^5, S_2^5, S_1^5, S_0^5\}$;

Set 2: $S_7 = \{S_6^7, S_5^7, S_4^7, S_3^7, S_2^7, S_1^7, S_0^7\}$;

Set 3: $S_9 = \{S_8^9, S_7^9, S_6^9, S_5^9, S_4^9, S_3^9, S_2^9, S_1^9, S_0^9\}$.

The linguistic valuations are obtained for each expert in his/her domain. These values must be standardized through the 2-tuples linguistic model. In tables 2 the linguistic valuations is shown and in table 3 the translation through the 2-tuples linguistic model is presented.

TABLE II. EXPRESSED VALUES BY THE EXPERTS

	[0,04; 0,05]	[0,045; 0,06]	[0,05; 0,06]
e_1	$S_3^5 - S_4^5$	S_4^5	$S_5^5 - S_2^5$
e_2	$S_4^7 - S_5^7$	$S_4^7 - S_6^7$	S_0^7
e_3	S_6^9	S_8^9	$S_0^9 - S_3^9$
e_4	S_5^9	$S_7^9 - S_8^9$	$S_3^9 - S_4^9$
e_5	$S_3^5 - S_4^5$	$S_3^5 - S_4^5$	$S_3^5 - S_4^5$
e_6	S_4^5	S_4^5	$S_0^5 - S_1^5$
e_7	S_8^9	$S_5^9 - S_7^9$	$S_2^9 - S_4^9$
e_8	$S_5^9 - S_7^9$	$S_5^9 - S_6^9$	$S_5^9 - S_7^9$
e_9	S_6^7	$S_1^7 - S_2^7$	$S_5^7 - S_6^7$
e_{10}	$S_0^9 - S_1^9$	$S_5^9 - S_6^9$	$S_6^9 - S_8^9$

TABLE III. NORMALIZED VALUES IN DOMAIN 3 (9 LABELS)EXPRESSED VALUES BY THE EXPERTS

	[0,04; 0,05]	[0,045; 0,06]	[0,05; 0,06]
e_1	$(S_6,0)-(S_8,0)$	$(S_8,0)$	$(S_2,0)-(S_4,0)$
e_2	$(S_5,-.33)-(S_7,-.37)$	$(S_5,-.33)-(S_8,0)$	$(S_0,0)-(S_4,0)$
e_3	$(S_6,0)$	$(S_8,0)$	$(S_0,0)-(S_3,0)$
e_4	$(S_5,0)$	$(S_7,0)-(S_8,0)$	$(S_2,0)-(S_4,0)$
e_5	$(S_6,0)-(S_8,0)$	$(S_6,0)-(S_8,0)$	$(S_2,0)$
e_6	$(S_8,0)$	$(S_8,0)$	$(S_0,0)-(S_2,0)$
e_7	$(S_8,0)$	$(S_5,0)-(S_7,0)$	$(S_2,0)-(S_4,0)$
e_8	$(S_5,0)-(S_7,0)$	$(S_5,0)-(S_6,0)$	$(S_5,0)-(S_7,0)$
e_9	$(S_8,0)$	$(S_1,-.33)-(S_3,-.33)$	$(S_7,-.37)-(S_8,0)$
e_{10}	$(S_0,0)-(S_1,0)$	$(S_5,0)-(S_6,0)$	$(S_6,0)-(S_8,0)$

Immediately after, we proceed to obtain an agent for each interval. The majority linguistic operator is applied with the aim of obtaining a value which represents the whole collection of opinions made by the different experts in a majority way.

Extreme [0,04]

$$\phi = (S_8,0) \otimes 0.433 \oplus (S_6,0) \otimes 0.433 \oplus (S_5,0) \otimes 0.1 \oplus (S_5,0.33) \otimes 0.017 \oplus (S_0,0) \otimes 0.017 = (S_7,-0.35)$$

Extreme [0,05]

$$\phi = (S_8,0) \otimes 0.947 \oplus (S_7,-0.33) \otimes 0.0106 \oplus (S_7,0) \otimes 0.0106 \oplus (S_6,0) \otimes 0.0106 \oplus (S_5,0) \otimes 0.0106 \oplus (S_1,0) \otimes 0.0106 = (S_8,-0.15)$$

To estimate the interests and future benefits the following expression is applied:

$$[\text{lower limit}] + (\text{upper limit} - \text{lower limit})(\cdot)[L_1^N, L_2^N] \quad (12)$$

where L_1^N, L_2^N are the equivalent numerical values after applying the LAMA operator to the linguistic labels using the symbolic translation.

Future valuation for year 1

$$i_1 = [0,04] + (0,01)(\cdot)[0,738; 0,872] = [0,04738; 0,04872]$$

Future valuation for year 2

$$i_2 = [0,045] + (0,015)(\cdot)[0,6172; 0,8577] = [0,05426; 0,05787]$$

Future valuation for year 3

$$i_3 = [0,05] + (0,01)(\cdot)[0,1831; 0,4362] = [0,05183; 0,05436]$$

The following step needs to establish some values which customers and sellers are agree with according to the possible Cash Flows free to obtain in the considered periods. In order to get it, firstly we start with intervals to qualify the CFL which will be useful as a reference to apply for the opinion of the experts at such content. These must be established not only for the customers' part, but also for the seller's one. To operative effects of the practical decision it has been established the following intervals indicating the possible CFL in financial units for the three analysis periods: year 1 [4.000; 6.000]; year 2 [3.000; 6.000]; year 3 [2.000; 5.000]

From the previous valuations, it is possible to apply for the cooperation of experts when expressing their opinions through linguistic valuations taking customers and seller positions.

TABLE IV. LINGUISTIC VALUATIONS

	[4.000; 6000]	[3.000; 6000]	[2.000; 5.000]
<i>Customer</i>			
e_1	$(S_4,0)-(S_6,0)$	$(S_2,0)-(S_4,0)$	$(S_2,0)-(S_4,0)$
e_2	$(S_3,-.33)-(S_4,0)$	$(S_4,0)-(S_7,-.33)$	$(S_0,0)-(S_4,0)$
e_3	$(S_5,0)$	$(S_1,0)-(S_2,0)$	$(S_0,0)-(S_3,0)$
e_4	$(S_5,0)$	$(S_5,0)-(S_6,0)$	$(S_2,0)-(S_4,0)$
e_5	$(S_2,0)-(S_4,0)$	$(S_4,0)-(S_6,0)$	$(S_2,0)$
<i>Seller</i>			
e_1	$(S_7,-.33)-(S_8,0)$	$(S_4,0)-(S_5,-.33)$	$(S_5,-.33)-(S_7,-.33)$
e_2	$(S_5,0)-(S_6,0)$	$(S_7,0)-(S_8,0)$	$(S_3,0)-(S_4,0)$
e_3	$(S_4,0)-(S_6,0)$	$(S_6,0)$	$(S_2,0)-(S_4,0)$
e_4	$(S_2,0)-(S_6,0)$	$(S_2,0)-(S_4,0)$	$(S_2,0)-(S_6,0)$
e_5	$(S_5,0)-(S_6,0)$	$(S_7,0)-(S_8,0)$	$(S_6,0)-(S_7,0)$

Then, the unified information will be aggregated being used again the last OWA operator. In order not to reaffirming the calculus, we only develop the operations bellowing to the first period for the customers.

Extreme [4.000]
 $\phi = (S_5, 0) \otimes 0.625 \oplus (S_4, 0) \otimes 0.125 \oplus$
 $\oplus (S_2, 0) \otimes 0.125 \oplus (S_1, 0.33) \otimes 0.125 = (S_4, 0.04)$

Extreme [6.000]
 $\phi = (S_5, 0) \otimes 0.625 \oplus (S_4, 0) \otimes 0.125 \oplus$
 $\oplus (S_2, 0) \otimes 0.125 \oplus (S_1, 0.33) \otimes 0.125 = (S_4, 0.04)$

CFL Seller
 $CFL_1^C = [4.000] + (2.000)(\cdot)[0,4490; 0,5185] =$
 $= [4.898; 5.037]$

To the customers we obtain:

CFL Seller
 $CFL_1^V = [4.000] + (2.000)(\cdot)[0,4768; 0,6805] =$
 $= [4.953; 5.361]$

And for the remaining intervals:
Interval CFL Customer - Seller [3000, 6000]

$CFL_2^C = [3.000] + (3.000)(\cdot)[0,388; 0,6018] =$
 $= [4.164; 4.805]$

$CFL_2^V = [3.000] + (3.000)(\cdot)[0,6527; 0,7688] =$
 $= [4.958; 5.306]$

Interval CFL Customer - Seller [2000, 5000]

$CFL_3^C = [2.000] + (3.000)(\cdot)[0,1666; 0,4166] =$
 $= [2.500; 3.249]$

$CFL_3^V = [2.000] + (3.000)(\cdot)[0,3379; 0,5601] =$
 $= [3.013; 3.680]$

In the table IV is presented a calculus summary.

TABLE V. CALCULUS SUMMARY

Updating valuations	Year-1	
Interval-K	0,040	0,050
Interval- ϕ	0,738	0,872
Interval-K _{adjusted}	0,04738	0,04872
Cash-Flow Free	Year-1	
Interval-CFL	4.000	6.000
Interval- ϕ	0,4490	0,6805
Interval-CFL _{adjusted}	4.898,00	5.361,00
Updating valuations	Year-2	
Interval-K	0,045	0,060
Interval- ϕ	0,617	0,858
Interval-K _{adjusted}	0,05426	0,05787
Cash-Flow Free	Year-2	
Interval-CFL	3.000	6.000
Interval- ϕ	0,3880	0,7688
Interval-CFL _{adjusted}	4.164,00	5.306,40
Updating valuations	Year-3	
Interval-K	0,050	0,060
Interval- ϕ	0,183	0,436
Interval-K _{adjusted}	0,05183	0,05436
Cash-Flow Free	Year-3	
Interval-CFL	2.000	5.000
Interval- ϕ	0,1666	0,5601
Interval-CFL _{adjusted}	2.499,80	3.680,30

We notice how using the majority operator we get to reduce the interval of variable values considered in the valuation (table 5), which leads us to consider that the enterprise value derived from them will equally present a more reduced interval than if we do not use such operators.

TABLE VI. RANGE OF THE INTERVAL

Updating valuation	Year-1	Year-2	Year-3
Range of Interval-K	0,010	0,015	0,010
Range of Interval-K _{adjusted}	0,00134	0,00361	0,00253
Cash-Flow Free	Year-1	Year-2	Year-3
Interval-CFL	2.000,00	3.000,00	3.000,00
Range of Interval-CFL _{adjusted}	463,00	1.142,40	1.180,50

In fact, it is shown in the table 6 the comparative calculus of the two related versions, proving that the use of the majority operators OWA reduces the interval of the positive estimated values in a considerable form, reaching like this our targets. In our example, if we apply directly to the first information the classic expression of Discounted Cash-Flow, we will obtain the following interval [8.971; 17.122] with a breadth of 8.151, whereas if we consider the information in the form proposed by the majority operators OWA, we will obtain a more appreciably narrow interval, that is [11.514; 14.379] of breadth 2.864. The reduction of the range interval is due to the increase of the inferior extreme and the decrease of the higher one.

TABLE VII. COMPARATIVE RESULTS

Valuation	Interval of Values		Range
V _{E(CFL)}	8.971	17.122	8.151
V _{E(adjusted)}	11.514	14.379	2.864

V. CONCLUSIONS

In this work it has been presented a new strategic valuation system based on the Discounted Cash-Flow model, OWA operators and linguistic information. Due to the importance that valuation process is representative of the main part of the estimation done by the experts, it has been used the majority operator LAMA extended to the linguistic representation of 2-tuple, which allows to work with a manifold information in the attaching process.

This paper shows that the use of the majority operators OWA reduces the interval of the positive estimated values in a considerable form, reaching like this our targets.

Finally, The strategic proposed in this work is absolutely flexible and adaptable to whichever decision stage both on business and stock market, then allowing having the valuation weights by means of a previous calculus, being used again in other valuation processes, making the method application almost immediate. This is not possible with the traditional methods.

ACKNOWLEDGMENT

This work is supported by the Ministry of Education and Science of Spain. Project TIN2006-14285.

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