# Optimizing the Method for Building an Extended Linguistic Hierarchy

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***Abstract*—In those problems dealing with linguistic information and multiple sources of information may happen that the sources involved have different degree of knowledge about the problem and could be suitable and necessary the use of different linguistic term sets with different granularity defining a multi-granular linguistic context. Different approaches have been presented to deal with this type of context, being the linguistic hierarchies [1] an approach quite interesting due to its accuracy in computational model but with a strong limitation about the term sets that can be used. We presented an extension of the linguistic hierarchies [2] to deal any linguistic term set in a precise way. This new approach presents initially a drawback, it needs a term set with a very high granularity, implying complexity in computing with words processes. Therefore, we propose an optimization to building an extended linguistic hierarchy in order to decrease the granularity of such a term set.**

***Keywords*-Linguistic information, linguistic hierarchies, granularity.**

## I. Introduction

Real world problems can present quantitative or qualitative aspects. Those problems that present quantitative aspects are usually assessed by means of precise numerical values. On the other hand, when the aspects are qualitative or there exists uncertainty related to the quantitative information it is better the use of a qualitative assessments. The use of the fuzzy linguistic approach [3] has obtained successful results in such a type of problems [4], [5], because it provides a direct way to model qualitative and uncertain information by means of linguistic variables.

The concept *granularity of uncertainty* plays a key role when we are dealing with linguistic information, due to the fact that it indicates the level of discrimination that the sources of information can use to express their knowledge, i.e., the cardinality of the term set [6]. Therefore, when multiple sources take part in a problem different ones might have different degree of knowledge about the assessed aspects and, could be suitable that each one can use terms sets with different granularity defining a multigranular linguistic context.

In the literature, different approaches have been developed to deal with Multi-Granular Linguistic Information ($MGLI$) [7], [8], [9], [1], [10]. These approaches manage the $MGLI$ by conducting such an information in an unique linguistic term set in order to accomplish computing with words ($CW$) processes [11], [12], [13]. These approaches present advantages and disadvantages. The main problem is the loss information in the $CW$ processes that was solved by the latter so-called Linguistic Hierarchies ($LH$), that presents a precise computational model but it cannot use any linguistic term sets as: 5, 7 and 9 labels, that it is required in many problems.

In [2], we have proposed an initial Extended Linguistic Hierarchies ($ELH$) that overcomes the disadvantages of the afore mentioned approaches, in other words, this new approach is able to deal with any linguistic term set without loss of information in processes of $CW$.

Even though this initial approach for dealing with $MGLI$ is quite useful, it presents a drawback due to its building process that generates a linguistic term set with a very high granularity that implies a greater complexity in the processes of $CW$.

The aim of this contribution is to present an optimization method of constructing an extended linguistic hierarchy in order to minimize granularity of the term sets that belongs to $ELH$ to simplify the $CW$ processes.

In order to do that, the contribution is structured as follows. Section 2 introduces a linguistic background, the linguistic hierarchies and the extended linguistic hierarchies to understand our proposal. Section 3 presents the optimizing the method for building an extended linguistic hierarchy and its computational model. Finally, we shall point out some concluding remarks in Section 4.

## II. Linguistic Background

In this section, we are going to review some necessary concepts in order to understand our proposal.

### A. Fuzzy Linguistic Approach

Many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case, a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic

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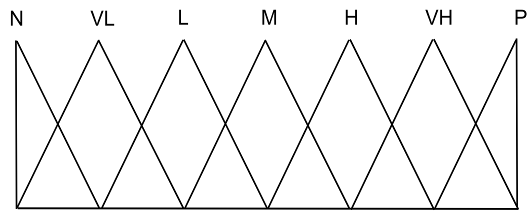


Figure 1. A Set of 7 Terms with its Semantic

approach represents qualitative aspects as linguistic values by means of linguistic variables [3].

In this approach, it is necessary to choose the appropriate linguistic descriptors for the term set and their semantics, there exist different possibilities (further description see [6]). One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a linguistic terms sets on which a total order is defined [14]. For example, a set of seven terms $S$, could be:

$$\{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}$$

Usually, in these cases, it is required that in the linguistic term set there exist:

1) A negation operator: $\mathrm{Neg}(s_i) = s_j$ such that $j = g - i$ ($g + 1$ is the cardinality).
2) An order: $s_i \leq s_j \iff i \leq j$. Therefore, there exists a $\min$ and a $\max$ operator.

The semantics of the terms are given by fuzzy numbers defined in the [0,1] interval, which are usually described by membership functions. For example, we might assign the following semantics to the set of seven terms (graphically, Fig.1):

$$\begin{array}{ll} P = (.83, 1, 1) & VH = (.67, .83, 1) \\ H = (.5, .67, .83) & M = (.33, .5, .67) \\ L = (.17, .33, .5) & VL = (0, .17, .33) \\ N = (0, 0, .17). & \end{array}$$

### B. 2-Tuple Linguistic Representation Model

This representation model was presented in [15] and it is the basis of the computational model for the $LH$. Due to this fact, we review this model in order to understand the $LH$, the $ELH$ and the optimization proposed in this contribution.

This model is based on symbolic methods and takes as the base of its representation the concept of Symbolic Translation.

**Definition 1.** *The Symbolic Translation of a linguistic term $s_i \in S = \{s_0, ..., s_g\}$ is a numerical value assessed in $[-.5, .5)$ that supports the "difference of information" between an amount of information $\beta \in [0, g]$ and the closest value in $\{0, ..., g\}$ that indicates the index of the closest linguistic term $s_i \in S$, being [0,g] the interval of granularity of $S$.*

From this concept the linguistic information is represented by means of 2-tuples $(s_i, \alpha_i)$, $s_i \in S$ and $\alpha_i \in [-.5, .5)$.

This model defines a set of functions between linguistic 2-tuples and numerical values.

**Definition 2.** *Let $S = \{s_0, \ldots, s_g\}$ be a set of linguistic terms. The* 2-tuple set associated with $S$ *is defined as* $\langle S \rangle = S \times [-0.5, 0.5)$. *We define the function* $\Delta : [0, g] \longrightarrow \langle S \rangle$ *given by,*

$$\Delta(\beta) = (s_i, \alpha), \quad \textit{with} \quad \begin{cases} i = \textit{round}\,(\beta), \\ \alpha = \beta - i, \end{cases}$$

*where* round *assigns to $\beta$ the integer number $i \in \{0, 1, \ldots, g\}$ closest to $\beta$.*

We note that $\Delta$ is bijective [15] and $\Delta^{-1} : \langle S \rangle \longrightarrow [0, g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$. In this way, the 2-tuples of $\langle S \rangle$ will be identified with the numerical values in the interval $[0, g]$. This representation model has associated a computational model that was presented in [15].

### C. Linguistic Hierarchies

We have mentioned that our objective in this contribution is to propose an optimization to build an extended linguistic hierarchy in order to deal with any linguistic term set in a precise way with a decrease the granularity of the terms set. The extended linguistic hierarchy is based in the concept of the Linguistic Hierarchies that we are going to review in this section.

In [1] was introduced an approach, so-called Linguistic Hierarchies, that carries out $CW$ processes in a precise way but impose several limitations to the definition context.

A *Linguistic Hierarchy* is a set of levels, where each level is a linguistic term set with different granularity from the remaining of levels of the hierarchy. Each level belonging to a linguistic hierarchy is denoted as *l(t,n(t))*, *t* indicates the level of the hierarchy and *n(t)* indicates the granularity of the linguistic term set of the level *t*.

It is assumed that its levels contain linguistic terms sets with an odd number of terms and whose membership functions are triangular-shaped, symmetrical and uniformly distributed in $[0, 1]$.

The levels belonging to a $LH$ are ordered according to their granularity. A linguistic hierarchy, $LH$, is defined as the union of all levels $t$: $LH = \bigcup_t l(t, n(t))$. We are going to review the methodology to build a linguistic hierarchy and its computational model.

*1) Building Linguistic Hierarchies:* In the construction of a linguistic hierarchy the order is given by increasing the granularity of the linguistic term sets in each level.

Given a $LH$, being $S$ a linguistic term set in the level t: $S = \{s_0, ..., s_{n(t)-1}\}$, $s_k \in S, (k = 0, ..., n(t) - 1)$ a linguistic term of $S$. It is then denoted as, $S^{n(t)} =$

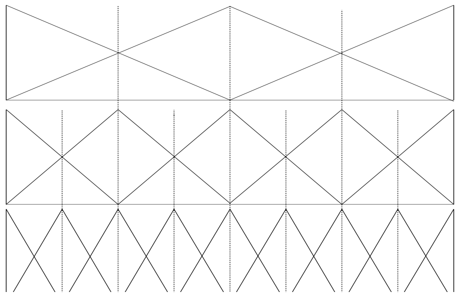

Figure 2. LH of 3, 5 and 9

$\{s_0^{n(t)}, ..., s_{n(t)-1}^{n(t)}\}$, because it belongs to level $t$ and its granularity of uncertainty is $n(t)$.

A methodology to construct a $LH$ was presented in [1] that imposed the following rules, so-called *linguistic hierarchy basic rules*:

1) To preserve all *former modal points* of the membership functions of each linguistic term from one level to the following one.
2) To make *smooth transitions between successive levels*. The aim is to build a new linguistic term set, $S^{n(t+1)}$. A new linguistic term will be added between each pair of terms belonging to the term set of the previous level $t$. To carry out this insertion, we shall reduce the support of the linguistic labels in order to keep place for the new one located in the middle of them.

Generally, a linguistic term set of level $t+1$ is obtained from its predecessor as:

$$l(t, n(t)) \rightarrow l(t+1, 2 \cdot n(t) - 1).$$

*2) Computational Model:* In order to carry out $CW$ processes with $MGLI$ in a $LH$ without loss of information, in [1] was presented a transformation function, $TF_{t'}^{t}$ that permits to transform labels between levels without loss of information in order to conduct the $MGLI$ in one expression domain:

$$TF_{t'}^{t} : l(t, n(t)) \longrightarrow l(t', n(t')).$$

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta\left(\frac{\Delta^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \cdot (n(t') - 1)}{n(t) - 1}\right). \tag{1}$$

$TF_{t'}^{t}$ is a one-to-one function between levels of the $LH$ [1].

### D. Building Extended Linguistic Hierarchies

It is clear that the hierarchy basic rules has some limitations to deal with $MGLI$ without loss of information. Due to those assumptions the model does not allow to deal with contexts using term sets with 5, 7 and 9 labels that are quite common and necessary in many problems.

In this section, we review the methodology to build an extended linguistic hierarchies [2] to deal any linguistic term set in a precise way.

The reason that the $LH$ keeps the information in $CW$ processes is due to the basic rule 1 that keeps all former modal points from one level to another. The rule 2 just proposes the easiest way to keep these former modal points among all the level being possible to transform the information between any two levels without loss of information in the multigranular context.

In order to extend the $LH$, the following concepts and tools are clarified.

**Lemma 1.** *Let $S^{n(t_j)}$ be a linguistic term set. Then the former modal points set of the level $t_j$ is $FP_{t_j} = \{fp_{t_j}^i, ..., fp_{t_j}^i, ..., fp_{t_j}^i\}, i = 0, ..., 2*\delta_j$, where each former modal point $fp_{t_j}^i$ is located at: $\frac{i}{(2*\delta_j)} \in [0,1]$, being $\delta_j = n(t_j) - 1$.*

Due to the fact that ELH wants to deal with any linguistic terms set, this replaces the basic rules that obligates to keep the former modal points from one level, $t$, to the next one, t+1, by the *extended hierarchical rules*:

- *Extended Rule 1:* in order to build an $ELH$ first, it should be included a finite number of the levels, $l(t, n(t))$, with $t = 1, ..., m$ that defines the multigranular linguistic context, required by the sources to express their knowledge. It is not necessary to keep the former modal points among each other.
- *Extended Rule 2:* To obtain a $ELH$ a new level $l(t^*, n(t^*))$ with $t^* = m + 1$ should be added such that keeps all the former modal points of all the levels included previously $l(t, n(t))$, $t = 1, ..., m$.

Therefore to construct an $ELH$, first, it fixed $m$ linguistic terms sets that use the sources to express their information. And the term set, $l(t^*, n(t^*))$, will be added according to the following theorem.

**Theorem 1.** Let $\{S_1^{n(t)}, ..., S_m^{n(t)}\}$ be the sets of linguistic terms sets, whose granularity $n(t)$ is odd. A new term set $l(t^*, n(t^*))$ that keeps all the former modal points of the $m$ term sets will have the following granularity:

$$n(t^*) = (\prod_{t=m}^{t=1} \delta_t) + 1.$$

**Proof.**

According to Lemma 1:

$$fp_t^i = \frac{i}{(2 \cdot \delta_t)} \in [0,1]$$

Then, $\forall fp_t^i \in FP_t,\ t \in \{1, ..., m\} \rightarrow \exists fp_{t^*}^j \in FP_{t^*}, t^* = m + 1,\ fp_t^i = fp_{t^*}^i$

$$\frac{i}{(2 \cdot \delta_t)} = \frac{j}{(2 \cdot \delta_t^*)} \Rightarrow j = \frac{i \cdot \delta_t^*}{\delta_t}$$

Given that $\delta_{t^*}$ is multiple of $\delta_t$ it is then proved that $FP_t \subset FP_{t^*}\ \forall t \in \{1, ..., m\}$.

**Remark:** Every the $fp_t^i$ corresponds to $fp_{t^*}^i$ whose membership value is 1.

Therefore an $ELH$ is the union of the $m$ levels required by the sources and the term set $l(t^*, n(t^*))$ that keeps all the former points in order to provide accuracy in the processes of $CW$.

$$ELH = \bigcup_{t=1}^{t=m+1} (l(t, n(t))).$$

Fig 3 shows the granularity needed in the level $t^*$ according to the $m$ previous levels included in the framework. We can observe that the last level ($t^*$) contains all the former modal points of the membership functions of each linguistic term set in the previous levels, $t = 1, ..., m$.

## III. New Methodology to Build and Extended Linguistic Hierarchies

In this section, we present the optimizing the building Extended Linguistic Hierarchies and its computational model, such that allow to deal with any linguistic term set in a precise way with a decrease the granularity of the linguistic terms set in the last level.

### A. Optimizing the Building of the $ELH$

In the previous section, we reviewed an initial approach to build an $ELH$ where the level $t^*$ keeps all the former modal points of the previous $t$ levels. However, Theorem 1 produces that the granularity of $t^*$ would be too high and might then make confuse the computational model. In order to make simpler the use and construction of the $ELH$, we propose an alternative way to minimize the granularity of $t^*$ that still keeps all the former modal points of the previous levels. To achieve this aim we will use the least common multiple ($LCM$).

**Definition 3.** The least common multiple of $m$ nonzero integer $a_1, ..., a_m$ is defined as $LCM(a_1, ..., a_m) = min\ \{n \in \mathbb{N}\ : n/a_i \in \mathbb{N}\ for\ i = 1, ..., m\}$.

By using the $LCM$ a new theorem to compute the granularity of the level $t^*$ is proposed.

**Theorem 2.** Let $\{S_1^{n(t)}, ..., S_m^{n(t)}\}$ be the set of linguistic terms sets with any odd value of granularity. A new level, $t^* = m + 1$, that keeps the former modal points of the $m$ term sets will have the following granularity:

$$n(t^*) = (LCM(\delta_1, ..., \delta_m)) + 1, t = 1, ..., m.$$

**Proof.**

In this case, we still keep that $\delta_{t^*}$ is multiple of $\delta_t$. Therefore,

$j = \frac{(i \cdot \delta_t^*)}{\delta_t} \Rightarrow FP_t \subset FP_{t^*}, \forall t \in \{1, ..., m\}$.

**Remark:** In this case, the $fp_t^i$ can correspond to $fp_{t^*}^j$ with different membership values.

Figure 4 summarizes and shows graphically the values that optimizes the construction of the extension linguistic hierarchies. We can observe that this optimization reduces drastically the number of labels in the last level, $t^*$ in comparison with the methodology revised in the section II-D.

### B. Computational Model

Due to the fact that in the $LH$ all the levels have to keep the former points of the predecessor. The transformations between any level can be carried out without loss of information. Nevertheless, in the $ELH$ that feet does not happen, therefore to keep the information in the transformations, $TF_{t'}^t$ (see Equation 1), one of the levels ($t$ or $t'$) must be $t^*$ that is $t_{m+1}$. This way guarantees the transformation between any level and the level $t^*$ (and vice versa) of an extended linguistic hierarchy is carried out without loss of information.

A computational process with $MGLI$ in an $ELH$ is defined as follows:

- First, the labels $s_i^{n(t_j)}$ are transformed into the labels in the level $t_{m+1}$.

  $$(s_j^{n(t_j)}, \alpha) \Rightarrow TF_{t_{m+1}}^{t_j}(s_j^{n(t_j)}, \alpha) = (s_k^{n(t_{m+1})}, \alpha').$$

  Here, we show how the transformation functions act over the extended linguistic hierarchy, $ELH = \bigcup l(1, 3), l(2, 5), l(3, 7), l(4, 13)$, whose term sets are: The transformations between terms of the different levels are carried out as:

  $$TF_4^1(s_1^3, 0) = \Delta^{-1}(\frac{\Delta(s_1^3, 0) \cdot (13 - 1)}{3 - 1}) = (s_6^{13}, 0).$$

  $$TF_4^2(s_1^5, 0) = \Delta^{-1}(\frac{\Delta(s_1^5, 0) \cdot (13 - 1)}{5 - 1}) = (s_3^{13}, 0).$$

  $$TF_4^3(s_2^7, 0) = \Delta^{-1}(\frac{\Delta(s_2^7, 0) \cdot (13 - 1)}{7 - 1}) = (s_2^{13}, 0).$$

  The 2-tuple computational model is used to make the computations with the linguistic 2-tuples expressed in the term set, $S^{n(t_{m+1})}$. Obtaining results expressed by means of linguistic 2-tuples assessed in the same level, $t_{m+1}$.

  For example, using the 2-tuple mean operator [15] whose expression is:

  $$\overline{x} = \Delta(\frac{\sum_{i=1}^{n} \Delta^{-1}(s_i, \alpha_i)}{n}). \quad (2)$$

  The collective value to aggregate the 2-tuples obtained in the previous transformations is:

  $$\overline{x} = \Delta(\frac{\Delta^{-1}(s_6^{13}, 0) + \Delta^{-1}(s_3^{13}, 0) + \Delta^{-1}(s_4^{13}, 0)}{3}) =$$

  $$\Delta(\frac{6 + 3 + 4}{3}) = \Delta(4, 33) = (s_4^{13}, 0.33).$$

| $ELH$ | $\delta_t$ | $Number of FP$ |
|---|---|---|
| $l(t,n(t))$ | $n(t)-1$ | $(2\cdot\delta_t)+1$ |
| $l(1,3)$ | 2 | 5 |
| $l(2,5)$ | 4 | 9 |
| $l(3,7)$ | 6 | 13 |
| $l(4,49)$ | 48 | 97 |

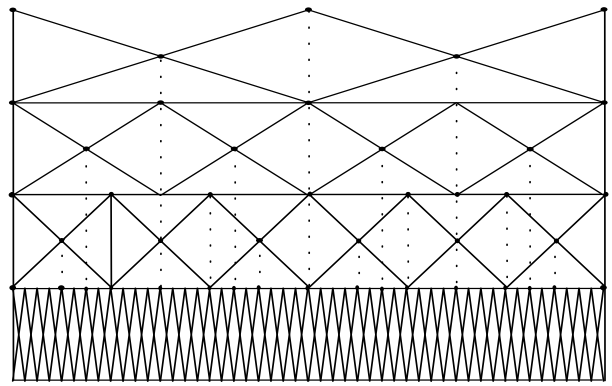

Figure 3. $ELH$ of 3, 5, 7 and 49 labels using the Theorem 2

| $ELH$ | $\delta_t$ | $Number of FP$ |
|---|---|---|
| $l(t,n(t))$ | $n(t)-1$ | $(2\cdot\delta_t)+1$ |
| $l(1,3)$ | 2 | 5 |
| $l(2,5)$ | 4 | 9 |
| $l(3,7)$ | 6 | 13 |
| $l(4,13)$ | 12 | 25 |

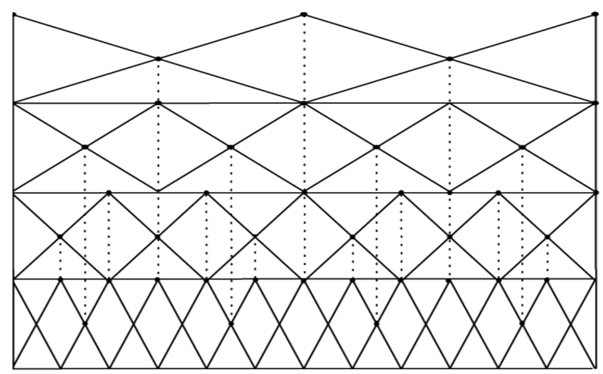

Figure 4. $ELH$ of 3, 5, 7, and 13 labels using the Theorem 2

- Once the results have been obtained in the level $t_{m+1}$ by means of linguistic 2-tuples, we can express them in the initial expression levels of the $ELH$ by means of the transformation:

$$TF_{t_j}^{t_{m+1}}(s_f^{n(t_{m+1})},\alpha_f)=(s_k^{n(t_j)},\alpha).$$

The collective value, $(s_4^{13},0.33)$, can be expressed in any linguistic term of the linguistic hierarchy:

$$TF_1^4(s_4^{13},0.33)=\Delta^{-1}(\frac{\Delta(s_4^{13},0.33)\cdot(3-1)}{13-1})=$$
$$\Delta^{-1}(0.72)=(s_1^3,-0.27).$$
$$TF_2^4(s_7^{13},0.33)=\Delta^{-1}(\frac{\Delta(s_4^{13},0.33)\cdot(5-1)}{13-1})=$$
$$\Delta^{-1}(1.44)=(s_1^5,0.44).$$
$$TF_3^4(s_7^{13},0.33)=\Delta^{-1}(\frac{\Delta(s_4^{13},0.33)\cdot(7-1)}{13-1})=$$
$$\Delta^{-1}(2.165)=(s_2^7,0.165).$$

## IV. Concluding Remarks

The use of linguistic information is common in problems dealing with qualitative and/or uncertain information. In problems with multiple sources of information it may happen that different sources have different degree of knowledge so they might need different term sets. In the literature, there exist different proposals to deal with this type of information so-called Multi-Granular Linguistic Information. However, these proposals have some drawbacks as loss of information, limitations to deal with $MGLI$, a linguistic term set with a very high granularity is necessary, etc. In this contribution, we have presented an optimization of the methodology to built an Extended Linguistic Hierarchies in order to deal with any linguistic terms set in a precise way with a decrease the granularity of the terms set.


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