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Consensus with Linguistic Preferences in Web 2.0 Communities

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Abstract

Web 2.0 Communities are a quite recent phenomenon with its own characteristics and particularities (possibility of large amounts of users, real time communication. . .) and so, there is still a necessity of developing tools to help users to reach decisions with a high level of consensus. In this contribution we present a new consensus reaching model with linguistic preferences designed to minimize the main problems that this kind of organization presents (low and intermittent participation rates, difficulty of establishing trust relations and so on) while incorporating the benefits that a Web 2.0 Community offers (rich and diverse knowledge due to a large number of users, real-time communication. . .).

1 Introduction

New Web technologies have allowed the creation of many different services where users from all over the world can join, interact and produce new contents and resources. One of the most recent trends, the so called *Web 2.0* [21], which comprises a set of different web development and design techniques, allows the easy communication, information sharing, interoperatibility and collaboration in this new virtual environment. Web 2.0 Communities, that can take different forms as Internet forums, groups of blogs, social network services and so on, provide a platform in which users can collectively contribute to a Web presence and generate massive content behind their virtual collaboration [18].

Among the different activities that the users of Web Communities usually perform we can cite:

Generate online contents and documents, which is greatly beneficiated with the diversity and knowledge of the involved people. One of the clearest examples of this kind of collaboration success is Wikipedia [3], where millions of articles have been produced by its web community in dozens of different languages.

Provide recommendations about different products and services. Usual recommender systems are increasing their power and accuracy by exploiting their user bases and the explicit and implicit knowledge that they produce [22]. A clear example of recommender systems success, which exploits its users community knowledge to provide personalized recommendations, is the Amazon online store [1].

Make decisions about particular problems. Group Decision Making (GDM) is a typical human activity which consists on selecting the best alternative from a feasible set according to a group of individuals. Thus, the main goal of any GDM process is to identify the best alternative according to some established criteria, and it is normally assumed that the experts have a common interest in obtaining a final solution for the problem. Examples of typical GDM processes are to vote in an election, to choose a place for a meeting or to select the model of laptop that a firm will buy to its employees. Usual simple group decision making schemes, as referendum or voting systems are now widely established in the Web. For example, services like PollDaddy [2] allow to create online surveys and polls where users can vote about the best alternative to choose for a given decision problem.

There have been several efforts in the specialized literature to create different models to correctly address and solve GDM situations. Some of them make use of fuzzy theory as it is a good tool to model and deal with vague or imprecise opinions (which is a quite common situation in any GDM process) [11, 17]. Many of those models are usually focused on solving GDM situations in which a particular issue or difficulty is present. For example, there have been models that allow to use linguistic assessments instead of numerical ones, thus making it easier for the experts to express their preferences about the alternatives [12]. Other models allow experts to use multiple preference structures (and even multi-granular linguistic information) [16, 20] and other different approaches deal with incomplete information situations if experts are not able to provide all their preferences when solving a GDM problem [15] or when a consensus process is carried out [14].

Moreover, usual GDM models have been complemented

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with consensus schemes that allow users to interact until there is a certain degree of agreement on the selected solution [7, 8]. This consensus models allow not only to provide better solutions to decision problems, but also to increase the users satisfaction with the decision process as all the opinions are reconsidered to achieve a high enough level of consensus.

However, those approaches are not usually well suited to be used by Web Communities due to some of their inherent properties. For example, due to the diversity of the users backgrounds, using numerical preferences might be not adequate (and thus, linguistic assessments should be used) or dynamic situations in which some of the parameters of the problem, as the set of experts, the set of alternatives and even the set of criteria to select the solutions change, have not been modeled. This kind of situations are quite common in other environments: in [25] the problem of managing time-dependent preferences (that is preferences expressed at different periods) is presented; the problem of dealing with dynamic real-time information to choose the best routes is shown in [10], and a practical example about resource management where the criteria to make decisions (climate) changes over time can be found in [9]. Thus, it is important to develop new models that take into account this kinds of dynamical situations to solve realistic GDM problems [23].

For the particular case of Web Communities, dynamic situations in which the group of experts vary over time are quite common: a new expert could incorporate to the process, some experts could leave it or a large group of experts could be simplified in order to minimize communications and to ease the computation of solutions. This behaviour is usually found in democratic systems where the individuals delegate into a smaller group of experts to make decisions (it is usually not possible to involve everyone in each decision). There have been some efforts to model this kind of situations. For example, in [5] a recursive procedure to select a qualified subgroups of individuals taking into account their own opinions about the group is presented. However, there is still a big necessity of creating new consensus models that suit Web Communities characteristics appropriately.

In this paper we present a consensus model in which preferences are expressed in a linguistic way and that has been designed taking into account the characteristics of Web 2.0 Communities. In particular, it has been designed considering that the number of users of this kind of communities is usually large [6]. For example, online music communities usually gather hundreds or even thousands of individuals that share an interest about particular bands or music genres. To reach a consensual decision with such a large user base is not an easy task because, for example, not every member of the community is willing to participate and contribute to solve the problem [19]. In addition, this model allows dynamic sets of users, that is, the users set to solve the decision problem may change in time. Moreover, by means of a delegation scheme (based on a particular kind of trust network [24]) we may achieve an important simplification in the obtaining of a proper consensus level. Finally, a trust checking procedure allows to avoid some of the problems that the delegation scheme could introduce in the consensus reaching model.

To do so, the paper is set as follows: in section 2 we present our preliminaries, that is, some of the most important characteristics of Web 2.0 Communities and the basic concepts that we use in our paper. In section 3 we introduce the new consensus model with linguistic preferences that helps to obtain consensual decisions in Web 2.0 Communities. Finally, in section 4 we point out our conclusions.

2 Preliminaries

2.1 Web 2.0 Communities

New Web 2.0 technologies have provided a new framework in which virtual communities can be created in order to collaborate, communicate, share information and resources and so on. This very recent kind of communities allows people from all over the globe to meet other individuals which share some of their interests. Apart from the obvious advantage of meeting new people with similar interests, Web Communities present some characteristics that make them different from other more usual kinds of organizations. In the following we discuss some of those characteristics and how they can affect in the particular case of GDM situations: **Large user base:** Web Communities usually have a large user base [6] (it is easy to find web communities with thousands of users). This can be seen from a double perspective. On the one hand, the total knowledge that a large user base implies is usually greater and more diverse than in a small community. This can be seen as a clear advantage: taking decisions is usually better performed when there is a rich knowledge on the evaluated subject. On the other hand, managing a large and diverse amount of opinions in order to extract and use that knowledge might be a difficult task: for example, some of the users might not find easy to use typical numerical preference representation formats and thus, linguistic ones should be implemented.

Low participation and contribution rates: Although many Web Communities have a quite large user base, many of those users do not directly participate in the community activities. Moreover, encouraging them to do so can be difficult [19]. Many of the users of a web community are mere spectators which make use of the produced resources but that does not (and is not willing to) contribute themselves with additional resources. This can be a serious issue when making decisions if only a small subset of the users contribute to a decision and it does not reflect the overall opinion of the community.

Intermittent contributions: Partially due to the fast communication possibilities and due to a very diverse involvement of the different members, it is a common issue that some of them might not be able to collaborate during a whole decision process, but only in part of it. This phenomenon is well known in web communities: new members are continuously incorporated to the community and existing users leave it or temporarily cease in their contributions.

Real time communication: The technologies that support Web Communities allow near real time communication among its members. This fact let us create models that in traditional scenarios would be quite impractical. For example, in a referendum, it is not easy at all to make a second round if there has been a problem in the first one due to the high amount of resources that it requires.

Difficulty of establishing trust relations: As the main communication schemes in Web Communities use electronic devices and, in the majority of the cases, the members of the community do not know each other personally, it might be difficult to trust in the other members to, for example, delegate votes. This fact implies that it might be necessary to implement control mechanisms to avoid a malicious user taking advantage of others.

2.2 Consensus Models with Fuzzy Linguistic Preferences

Usual GDM models follow a scheme in which two phases are differentiated: the first one consists in a *consensus process* in which the users (that we will call *experts* in the following), discuss about the alternatives and express their preferences about them using a particular preference representation format. A special individual (the moderator) checks the different opinions and confirms if there is enough consensus among all the experts. If there is not enough consensus, the moderator urges the experts to re-discuss about the alternatives and to provide a new set of opinions to improve the consensus level in a new consensus round. Once the desired consensus have been reached (or a maximum number of consensus rounds has been reached) the second phase (the *selection process*) starts and the best solution is obtained by aggregating the last opinions from the experts and applying an exploitation step which identifies the best alternative from the agreggated information.

In this paper we center our attention only in the consensus process, where the experts are supposed to narrow their different opinions about the alternatives to obtain a final solution with a high level of consensus. In the consensus model that we propose, the experts $E = \{e^1, \ldots, e^m\}$ will provide their preferences about the set of alternatives $X = \{x_1, \ldots, x_n\}$ in form of fuzzy linguistic preference relations [4]. In particular, we will use the 2-tuple linguistic computational model [13], in which the linguistic information is represented by a 2-tuple (s, α), $s \in S$, where S is a usual term set with odd cardinality and where the terms are uniformly distributed.

Definition 1: Let $\beta \in [0, q]$ be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set $S = \{s_0, \ldots, s_q\}$, i.e., the result of a symbolic aggregation operation. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, q]$ and $\alpha \in [0.5, 0.5)$, then α is called a symbolic translation.

The model also defines two functions Δ^{-1} and Δ to transform 2-tuples to numerical values and vice versa [13].

Definition 2: A 2-tuple linguistic preference relation P^h given by expert e^h on a set of alternatives X is a set of 2-tuples on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_P^h : X \times X \to S \times [-0.5, 0.5)$.

3 A Consensus Model for Web 2.0 Communities

In this section we present a new consensus model that can be applied in Web 2.0 Communities. It takes into account the different characteristics of this kind of communities (see section 2.1) in order to increase the consensus level of the users when making a decision on a set of alternatives. One interesting property of our model is that it does not require the existence of a moderator. Its operation includes several different steps that are repeated in each consensus round: (i) First preferences expression, computation of similar opinions and first global opinion and feedback; (ii) delegation (or change of preferences) and computation of consensus measures; (iii) consensus and trust checks. In figure 1 we have depicted the main steps of the model and in the following we describe them more detail.

3.1 First Step: First Preferences Expression, Computation of Similar Opinions and First Global Opinion and Feedback

In this first step the different alternatives in the problem are presented to the experts (note than in figure 1 we have represented only a small amount of experts, but when applied to a Web 2.0 Community the number of users will usually be larger). Once they know the feasible alternatives, each expert $e^h \in E$ is asked to provide a fuzzy linguistic preference relation P^h that represent his opinions about the alternatives. Although every single member of the community has the opportunity of expressing his preferences about the alternatives, as we have previously mentioned, only a

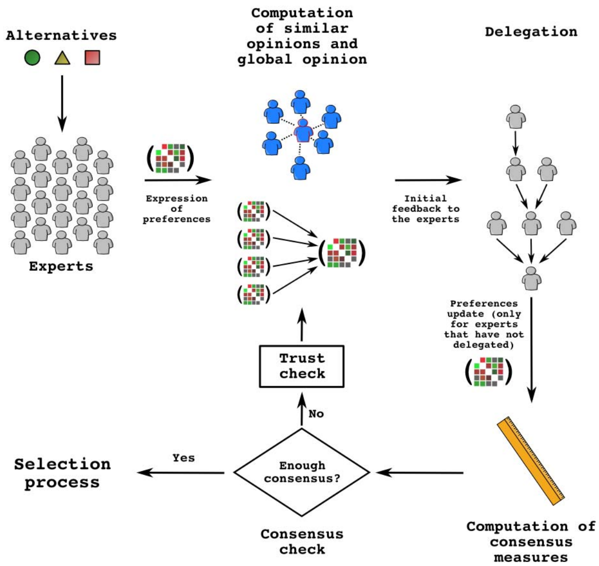

Figure 1. Scheme of the consensus model

subset of those experts $\tilde{E}$ will really provide preference relations. We will note $\tilde{e}^h$ to the experts that have provided a preference relation. It is important to note that if an expert at this stage does not provide a preference relation the model will still allow him to contribute in the consensus process in a later stage. Once a certain amount of time has passed (to allow a sufficient number of preferences to be provided) we compute the distance among each pair of experts $\tilde{e}^h$ and $\tilde{e}^g$ in the following way:

$$d^{hg} = d^{gh} = \sqrt{\sum_{i=1} \sum_{\substack{j=1 \\ j \neq i}} \left(\frac{\Delta^{-1}(p_{ij}^h) - \Delta^{-1}(p_{ij}^g)}{q} \right)^2}$$

This distances will be used to provide information to each expert about the experts that share a similar opinion about of the alternatives. In fact, for each $\tilde{e}^h \in \tilde{E}$ we define his *set of neighbours* as

$$N^h = \{\tilde{e}^{\beta_1}, \ldots, \tilde{e}^{\beta_{nn}}\}$$

where nn is the number of neighbours that each expert will be presented (this parameter is defined prior to the start of the consensus process) and e^{β_i} is the i-th nearest expert to $\tilde{e}^h$ (with lowest $d^{h\beta_i}$).

In this step we also compute the current global preference as an aggregation of all the provided preference relations. To do so, we will apply a simple arithmetic average to compute it, as at this point the preferences expressed by all the experts are considered to have the same weight:

$$p_{ij}^c = \Delta\left(\frac{\sum_{\tilde{e}^h \in \tilde{E}} \Delta^{-1}(p_{ij}^h)}{\#\tilde{E}}\right)$$

Once the distances among experts, the neighbours of each expert and the global preference relation have been computed, this information will be presented to the experts. After receiving this feedback, an expert will know if his opinions are very different to the current global preferences and he will also know which are the experts that share similar opinions. Apart from just his neighbour list, an expert is also able to check the particular preference relations that his neighbours have introduced in order to really check the preferences expressed by his neighbourhood.

3.2 Second Step: Delegation (or Change of Preferences) and Computation of Consensus Measures

In this second step the model incorporates a delegation scheme in which experts may choose to delegate into other experts (typically experts from their neighbourhood, with similar opinions). To allow that, we define $t^h \in \{1, \ldots, m\} \cup \emptyset$ as the expert in which $\tilde{e}^h$ delegates. Note that as experts may choose not to delegate, it is possible to have $t^h = \emptyset$. Thus, in this phase each expert that thinks that he will not be able to continue in the consensus process, instead of just leaving the process, can choose another expert and delegate on him. When an expert delegates on another expert, he will not be required to update his preferences to improve the consensus level. As experts may delegate in other experts that have already delegated, this scheme produces a tree structure among the set of experts. This tree structure conforms a kind of trust network in which some transitivity conditions are applied: if an expert $\tilde{e}^h$ delegates in an expert $\tilde{e}^k$ and $\tilde{e}^k$ delegates in $\tilde{e}^j$ the situation would be similar as if both $\tilde{e}^h$ and $\tilde{e}^k$ would have delegated in $\tilde{e}^j$. Note that the model should avoid cicles in the trust network. If an expert ask to delegate in another one and this delegation would produce a cicle in the trust network the system should alert him about this situation and ask him to reconsider his decision by delegating over a different expert or simply by not delegating.

Once a certain amount of time have passed (enough time for the experts to decide if they wanted to delegate or not), the system will compute a trust weight τ^h for every expert according to the trust network. Initially all the experts in $\tilde{E}$ have a $\tau^h = 1$. The system should then check every t^h and if $t^h \neq \emptyset$ it will follow the chain of delegations until it founds an expert $\tilde{e}^k$ which has not delegated. Then, the trust weights will be updated: $\tau^k = \tau^k + 1$ and $\tau^h = 0$.

This delegation mechanism provides several advantages to the model: first of all, it allows experts not to provide their preferences in every consensus round. If an expert delegates in another one, he will not have to update his preferences but, in a certain way (through the delegate), his opinion will still influence the consensus state. Thus, the consensus rounds may be carried out faster as only a subset of experts will have to change their preferences. Moreover, the computations will also be reduced as the system will not have to deal with a large amount of preference relations.

Once the trust weights have been computed the system will ask the remaining experts to update their linguistic preference relations P^h in order to achieve a greater level of consensus. This experts will conform the new $\tilde{E}$ subset. Once the updated preferences have been given we can compute some consensus degrees. To do so, we firstly define for each pair of experts $(\tilde{e}^h, \tilde{e}^l)$ $(h < l)$ of the new $\tilde{E}$ a similarity matrix $SM^{hl} = \left(sm_{ik}^{hl}\right)$ where

$$sm_{ik}^{hl} = \tau^h \cdot \tau^l \cdot \left(1 - \left|\frac{\Delta^{-1}(p_{ik}^h) - \Delta^{-1}(p_{ik}^l)}{q}\right|\right)$$

Then, a collective similarity matrix, $SM = (sm_{ik})$ is obtained by aggregating all the $(\#\tilde{E} - 1) \times (\#\tilde{E} - 2)$ similarity matrices using following expression:

$$sm_{ik} = \frac{\sum\limits_{h,l \in \tilde{E} | h<l} sm_{ik}^{hl}}{T \cdot (T-1)/2}$$

where $T = \sum_{i=1}^{m} \tau^i$.

Once the similarity matrices are computed we proceed to obtain the consensus degrees at the three different levels:

L. 1. *Consensus degree on pairs of alternatives*. The consensus degree on a pair of alternatives (x_i, x_k), denoted cop_{ik}, is defined to measure the consensus degree amongst all the experts on that pair of alternatives:

$$cop_{ik} = sm_{ik}$$

L. 2. *Consensus degree on alternatives*. The consensus degree on alternative x_i, denoted ca_i, is defined to measure the consensus degree amongst all the experts on that alternative:

$$ca_i = \frac{\sum_{k=1; k \neq i}^{n} (cop_{ik} + cop_{ki})}{2(n-1)}$$

L. 3. *Consensus degree on the relation*. The consensus degree on the relation, denoted CR, is defined to measure the global consensus degree amongst all the experts' opinions:

$$CR = \frac{\sum_{i=1}^{n} ca_i}{n}$$

3.3 Third Step: Consensus and Trust Checks

In the end of each consensus round we must check the current consensus state. If it is considered a high enough consensus value the consensus process would finish and a selection process would be applied to obtain the final solution for the decision problem. To do so, we check if $CR > \gamma$, being γ a threshold value fixed prior to the beginning of the GDM process. In the case that the level of consensus is not high enough we would continue with the trust check that is described in the following. Note that in real applications it might be desirable to include a $maximumRounds$ parameter to control the maximum consensus rounds that can be executed in order to avoid stagnation.

The trust check is introduced to avoid some of the problems that can be derived to one of the characteristics of Web Communities: the difficulty of establishing real trust relations. It is not difficult to imagine an scenario where some experts delegate into another that shares a common point of view on the decision that has to be made and in a certain consensus round, this expert decides to drastically change his preferences, probably not reflecting the other experts opinions anymore. To avoid this kind of situations the trust check will compare the last preference relation expressed by expert $\tilde{e}^h$ with the last preference relations of the experts that delegated in him (direct or indirectly). This comparison can be made by applying a distance operator (as the euclidean or cosine distances) over the preference relations. If this distance is greater than a certain stablished threshold, the expert that delegated in $\tilde{e}^h$ would be informed with a special message to warn him about this problematic situation and thus allowing him to take a different course of action in the next consensus round if appropriate.

At this point a new consensus round begins. In this new round the current global preference will not be computed as a simple arithmetic mean but as a weighted mean of the preferences expressed by the experts in $\tilde{E}$. The weights to be used in this aggregation operation are the trust weights τ^h:

$$p_{ij}^c = \Delta\left(\frac{\sum\limits_{\tilde{e}^h \in \tilde{E}} \tau^h \cdot \Delta^{-1}(p_{ij}^h)}{T}\right)$$

We would like to note that in each new consensus round all the members of the Web Community can participate, independently of what they did in the previous rounds. For example, an expert that delegated in a previous consensus round may decide not to continue delegating (maybe because the trust check mechanism has warned him that the expert in which he delegated has drastically changed his preferences) and thus to provide again a new fuzzy linguistic preference relation or to delegate in a different individual; an expert which had not delegated in any of the previous rounds might decide to delegate in the current consensus round or even an expert which has not participated until this moment in the consensus process (he did not provide any preference relation in the first step of the model) could join the process by providing his initial preferences.

4 Conclusions

In this contribution we have presented a novel consensus model which has been specially designed to be applied in Web 2.0 Communities. Particularly, it uses fuzzy linguistic preference relations for the expression and management of experts' preferences and it has been designed to manage a large users base by means of a delegation scheme. This delegation scheme is based in a particular kind of trust network that simplifies the computations and the time needed to obtain the users preferences. Moreover, this delegation scheme also solves the intermittent contributions problem which is present in almost any online community (that is, many of the users will not continuously collaborate but will do it from time to time). In addition, the model allows to incorporate new experts to the consensus process, that is, the model is able to handle some of the dynamic properties that real Web Communities have.

Finally, the model incorporates a trust check mechanism that allow to detect some abnormal situations in which an expert may try to take advantage of others by drastically changing his opinion and benefiting from the trust that the other experts might have deposited in him in previous consensus rounds.

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