

Adaptive Paralleled DMC-PID Controller Design on System with Uncertainties

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Abstract

This paper presents an adaptive controller design method for a class of system with modeling uncertainties or environment disturbance. The controller has a paralleled structure of Dynamic Matrix Control and PID Control. The weight for each of the controller can be adaptively tuned through iteratively learning. It can make full use of the model information, meanwhile resisting disturbance and overcoming the un-modeled uncertainties in a certain degree. The simulation and comparison with other control method show that this method has better tracking performance, disturbance resistance, robustness and great feasibility to be implemented in engineering application.

Key words : Model Uncertainty; Dynamic Matrix Control; PID control; Adaptive Control

1. Introduction

As we know, the actual controlled system always has inevitable modeling uncertainties. For example, we only consider the situation when the system works near the operating point; or the components of the system have tolerance and wearing out, or the unmeasurable disturbance existing in the operation environment. Hence the analysis results or the designed controller based on the mathematical model can be quite sensitive to un-modeled uncertainties; some even can cause instabilities and divergence.

From the point of view of the system identification, researchers usually take the combination of model and uncertainty bound to describe the system, see literature review by Ljung [1] and Laskey [2].

From the point of view of the controller design, the robust control theories H_2/H_∞ [3], μ theory [4], LMI (Linear Matrix Inequality) [4] and so on are developed to overcome the system uncertainty. However those theories are to find a way to guarantee the stability for the nominal system within the bound of uncertainties and have strong conservations and unsatisfactory

tracking abilities. The multi-model adaptive control proposed by Narendra[5], Li and Wang [6] is to approach the system dynamics by using a multi-model scheme, according to which design a multi-model adaptive controller. This method can have better tracking ability by switching among different models and the corresponding controllers. Yet the scale of model set and the switching scheme is of importance to the performance and stability.

Besides the methods mentioned above, the iterative learning control [7], non-parameter model control [8] can overcome some uncertainties in a certain degree. But the lack of using information of the system implies that more energy will be cost during the parameter choosing and stability guarantee.

All in all, the robust of the controller design and the tracking performance of the system is anything but complimentary. When we specify a big phase margin, the controlled system can be stable when there are big model uncertainties, meanwhile the control precision will become worse. How to meanwhile guarantee the robustness and tracking performance is the biggest problem we confront now.

Based on the model predictive control, we proposed a paralleled structure of controller design consisting of Dynamic Matrix Control (DMC) [9] and PID. The weight of each controller can be tuned adaptively through iterative learning during repetitions. This controller design method can make full use of the plant model, meanwhile overcoming the disturbance and the unmolded dynamics of the system. The simulation results show that this method has better tracking performance and robustness than DMC or PID.

2. Controller Design

2.1. Overview of the Controller Design

The block diagram of the control system is shown in Fig 1, where the weighted DMC output and PID control is paralleled to compose the controller (shown in the dashed box), $P_0+\Delta P$ is the actual controlled plant,

ΔP is the modeling error, P_0 is the identified or simplified plant. DMC is designed based on the step response of the plant. $\omega_1(0 < \omega_1 < 1)$ is the DMC weight and be called in this paper as model believable degree. The design of DMC depends on the step response of the system which can be easily gotten in the engineering practice. Its algorithm is pretty simple yet has great robustness and suitable for system with time delay, asymptotically stable or non-minimum phase system. The weight of each controller can be adaptively adjusted through the iterative learning of the performance monitoring part. This kind of controller design not only use the model-based DMC control which can make full use of the information of the system, but also has model-free PID controller which can overcome the un-modeled dynamics and the unmeasured disturbance.

$$\begin{aligned} u(k+1) &= \omega_1 u_{PID}(k+1) + \omega_2 u_{DMC}(k+1) \quad (1) \\ u_{PID}(k+1) &= K_p e(k) + K_i \sum_{i=1}^k e(i) + K_d [e(k) - e(k-1)] \quad (2) \end{aligned}$$

2.2. The Design of Dynamic Matrix Controller

The dynamic matrix control, brought forward by Cutler [9], is one of the predictive control theories. It adopts the non-parameter model based the step response of the controlled object which can be easily get in the engineering practice, and adopts strategies of multi-step prediction, Rolling Time optimization and feedback correction; thus increase the algorithm robustness for the parameter variance and the disturbance. Yet the length of sampling period and early truncation and nonlinear part of the system has great influence for the design of DMC. While taking the method proposed in this paper are less sensitive to the sampling periods and thus decrease the dimension of calculation. The algorithm used in this paper is described as following.

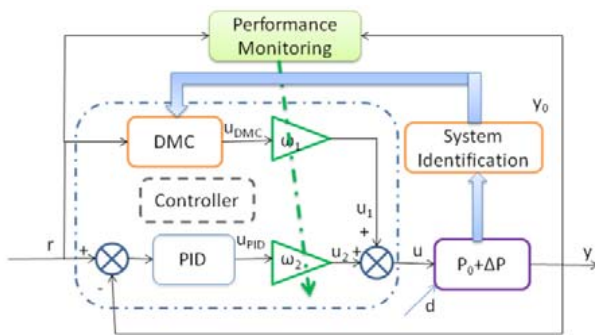


Fig 1. Block Diagram of the Control System

2.2.1. Prediction Model

At first, we need get the coefficients of the dynamic matrix from sampling the step response of the system. For the plant with asymptotic stability, the step response will be stable at finite sampling periods, i.e. $a_N = a(\infty)$. The dynamic characteristic of the plant can be described by the finite set

$$\{a_1, a_2, \dots, a_N\} \quad (3)$$

where a_i is the sampled value at $t = iT_s$, T_s is the sampling period, N is the truncation-point of the step response model (modeling time-domain).

Based on the proportion and superposition of the linear system theory, we can get the prediction model for the system. In the convenience of calculating the increase of the control input, we can have the prediction model be the sum of $A_0 U(k-1)$ and $A \Delta U(k)$ as shown in (4): the former is the output produced by the past control input; the latter is the output produced by the future control input.

$$Y_m(k+1) = A \Delta U(k) + A_0 U(k-1) \quad (4)$$

Where, $Y_m(k+1)$ is the predicted system output when there are $\Delta u(k)$ acting on the system for the future N sampling periods.

$$Y_m(k+1) = [y_m(k+1|k), y_m(k+2|k), \dots, y_m(k+N|k)]^T \quad (5)$$

$$\Delta U(k) = [\Delta u(k), \Delta u(k+1), \dots, \Delta u(k+M-1)]^T \quad (6)$$

A is the dynamic matrix of the system, P is the prediction horizon ($N \geq P \geq M$); M is the control horizon.

$$A = \begin{bmatrix} \hat{a}_1 & 0 & \dots & 0 \\ \hat{a}_2 & \hat{a}_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{a}_P & \hat{a}_{P-1} & \dots & \hat{a}_{P-M+1} \end{bmatrix}_{P \times M} \quad (7)$$

$$A_0 = \begin{bmatrix} \hat{a}_N - \hat{a}_{N-1} & \hat{a}_{N-1} - \hat{a}_{N-2} & \hat{a}_{N-2} - \hat{a}_{N-3} & \dots & \hat{a}_3 - \hat{a}_2 & \hat{a}_2 \\ 0 & \hat{a}_N - \hat{a}_{N-1} & \hat{a}_{N-1} - \hat{a}_{N-2} & \dots & \hat{a}_4 - \hat{a}_3 & \hat{a}_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \hat{a}_N - \hat{a}_{N-1} & \dots & \hat{a}_{P+2} - \hat{a}_{P-1} & \hat{a}_{P+1} \end{bmatrix}_{P \times M} \quad (8)$$

2.2.2. Feedback Correction

As there is modeling error because of the early truncation-point or the environment disturbance, the predicted output of the system need to be corrected by the feedback of the actual output. The feedback correction is performed by using Eq. (9)

$$Y_p(k+1) = Y_m(k+1) + h e(k) \quad (9)$$

Where

$$e(k) = y(k) - y_m(k) \quad (10)$$

$$\mathbf{h} = [h_1, h_2, \dots, h_p]^T \quad (11)$$

is the feedback error correction vector.

2.2.3 Rolling Time Optimization

Optimizing the following performance measure,

$$\text{Min } J_p = [Y_p(k+1) - Y_r(k+1)]^T Q [Y_p(k+1) - Y_r(k+1)] + \Delta U(k)^T \lambda \Delta U(k) \quad (12)$$

Where $Y_r(k+1)$ is the vector of the reference for the system.

$$Y_r(k+1) = [y_r(k+1), y_r(k+2), \dots, y_r(k+p)]^T \quad (13)$$

$$Q = \text{diag}(q_1, q_2, \dots, q_p) \quad (14)$$

is the error weighting matrix.

$$\lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_M) \quad (15)$$

is the control weighting matrix.

From $\frac{\partial J_p}{\partial \Delta U(k)} = 0$, we can get the control output increase as (16)

$$\Delta U(k) = (A^T Q A + \lambda I)^{-1} A^T Q [Y_r(k+1) - A_0 U(k-1) - \mathbf{h}e(k)] \quad (16)$$

If we just actuate the current $\Delta u(k)$ as shown in (17)

$$\Delta u(k) = d_1^T [Y_r(k+1) - A_0 U(k-1) - \mathbf{h}e(k)] \quad (17)$$

$$d_1^T = C^T (A^T Q A + \lambda I)^{-1} A^T Q = [d_1, d_2, \dots, d_p] \quad (18)$$

$$C^T = [1 \ 0 \ 0 \ \dots \ 0] \quad (19)$$

At the next instant $k+1$, the similar optimization problem is performed to get $\Delta u(k+1)$.

2.3. Adaptively and Iteratively Tuning of the Control Weight

Because of the most plants in the industry process have repetitions and periods, we can use the idea of iterative learning control to tune the weight and parameter of the DMC and PID by using the input-output data of the system.

In order to have the system has better tracking performance and robustness under the double control output, we need to set the PID parameter to the value that can have the system as an over-damped response.

Define the following performance measure as Eq. (20)

$$J_{\text{perf}}(j) = k_1 J_e(j) + k_2 J_{\text{tr}}(j) + k_3 J_{\text{ts}}(j) + k_4 J_{\delta}(j) \quad (20)$$

$$J_e(j) = \frac{1}{M} \sum_{i=1}^M (y_r(i) - y(i))^2 \quad (21)$$

$$J_{\delta}(j) = \frac{y_{\text{max}} - y_{\infty}}{y_{\infty}} \times 100\% \quad (22)$$

$J_e(j)$ is the mean error for the system during the j th period. $J_{\text{tr}}(j)$ is the rising time for the system (the time when the system output reaches the steady state for the first time). $J_{\text{ts}}(j)$ is the settling time of the system (the time when the error of the output steady state and the reference is between 2%-5% , and keeps this state for required period of time). $J_{\delta}(j)$ is the overshoot percentage of the system. k_1, k_2, k_3, k_4 is set to keep the four performance to a comparable value.

Taking the tuning of DMC weight for an example, we need keep the PID weight and parameter as constant at first. Then set the initial value of the DMC weight w_2^0 as 0.5. Then have both the PID and DMC acting on the plant to get the system response. Calculate the performance measure $J_{\text{perf}}(0)$ according to Eq. (18). And then increase the DMC weight using step size α ($0 < \alpha \leq 0.1$), $w_2(j) = w_2^0 + \alpha$ to get the closed loop system response $J_{\text{perf}}(j)$, $j = 1, 2, \dots$.

If $J_{\text{perf}}(j) - J_{\text{perf}}(j-1) < 0$, which means that increasing the weight of DMC can increase the tracking performance, then $w_2(j+1) = w_2(j) + \alpha$.

Otherwise if $J_{\text{perf}}(j) - J_{\text{perf}}(j-1) > 0$, which means that increasing the weight of DMC will decrease the tracking performance, then $w_2(j) = w_2^0 - \alpha/2$ or $w_2(j) = w_2^0 + \alpha/2$. The weight of DMC is adaptively and iteratively tuned so that the performance reaches the optimum.

The PID weight tuning is equivalent to the parameter tuning. It can be tuned the same way as the DMC weight when the DMC is set to be constant.

2.4. Synthesis of the Control Algorithm

As discussed above, the overall control algorithm can be summarized as following:

Step 1. Off-line identification of the step response of the plant, appropriately choose the sampling period T_s and the truncation-point N to reduce the dimension of the dynamic matrix so that reduce the amount of calculation.

Step 2. Set the PID controller parameters to make the system response as over-damped and satisfying the steady state.

Step 3. Keep the PID parameters constant and

choose proper DMC parameters to get the DMC output from Eq.(4)-(17). Set the initial DMC weight as 0.5.

Step 4. Adaptively and iteratively tune the control weight of DMC as describe in Section C.

Step 5. Keep the DMC weight constant and iteratively tune the PID weight or PID parameters. Or go to Step 4 till the satisfying performance.

3. Simulation Example and Analysis

The plant under study in this paper involves parameter uncertainties, unstructured nonlinear uncertainties, stochastic disturbance. In this section, a numerical example is presented to demonstrate the effectiveness of the proposed controller on the system with uncertainties.

Consider the following system as shown in (23). The sampling period is 0.5s.

$$y(k+1) = 0.4243u(k) + 0.3893u(k-1) + 0.9652y(k) - 0.7788y(k-1) + 0.25u(k)y(k) + \xi(k) \quad (23)$$

Where there is an unknown nonlinear part $0.25u(k)y(k)$ exists in the control output. $\xi(k)$ is random noise signal with 0 mean and amplitude between $[-0.05, 0.05]$. The step response of the plant is shown as in Fig 2.

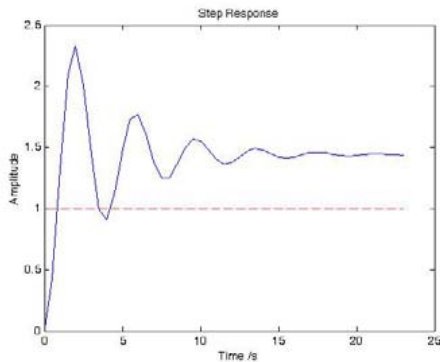


Fig 2. Step response of the plant

The DMC controller is designed using the following parameters. $N=40, P=15, M=10, Q=4 \times I_{P \times P}, \lambda=1 \times I_{M \times M}$

The weight of each controller is adaptively and iteratively tuned to 0.8 for DMC and 1 for PID. The simulation results are shown respectively in Fig. 3 the performance of the DMC-PID paralleled controller and Fig. 4 the performance of DMC. What's more, the comparison of different weight of DMC and constant weight of PID is shown in table 1. The comparison of different weight of PID and constant weight of DMC is shown in table 2. From the results we can obviously see the advantage of the proposed method over the

other method in the tracking ability and overcoming the model uncertainty.

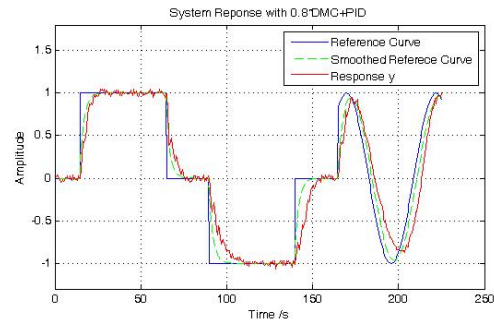


Fig 3. The system response of DMC-PID paralleled control

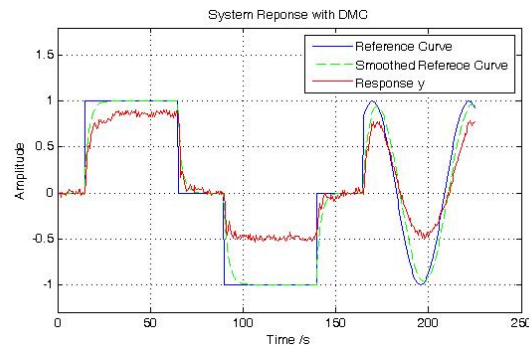


Fig 4. The system response of traditional DMC

4. Conclusion

In this paper, we presented the design of an adaptive paralleled DMC-PID controller for a class of system with uncertainties. The method proposed in this paper has the following advantages as it is shown from the design of the control system and the simulation results.

- 1) Combined with the model-based control and model-free control, this controller design method greatly improves the tracking performance.
- 2) This controller design method simplifies the DMC controller design procedure.
- 3) This controller uses the model believable degree as the weight of DMC. The paralleled PID controller can complement the un-modeled dynamics of the plant such as the high order or the nonlinearity. This controller design gives better robustness.
- 4) Because of the control output part of model-based DMC, the work of PID parameter tuning is also reduced which can save the time and the complexity of the engineer.

5) This method has simple structure and great extensibility, which is very suitable for the control application.

However the method also has some disadvantages as described in the following which need to be improved.

1) The precision of the modeling controlled plant need to be improved so that the DMC control output weight can be further improved.

2) So far the plant considered in this paper is limited to system with unknown weak nonlinearity or the high order system which can be regarded as a lower order system. To those system with big modeling error and oscillate under the DMC controller, this controller design method proposed in this paper may be invalid in that case. To specify the range of the model uncertainties will be an important direction for research.

3) If the PID parameter can be adaptively tuned or nonlinear PID can be used in this controller design, the control performance can be further improved.

4) The application research needs to be enhanced. The controller can be modularized for easy use.

Our future research will mainly focus the definition of the uncertainty range and the application of this method.

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TABLE I
PERFORMANCE COMPARISON
BY PURELY TUNING THE DMC WEIGHT

DMC Weight	Mean Error	Overshoot Percentage	Rising Time s	Settling Time s
0	0.1186	0	25	32
0.1	0.0816	4.18	12.5	15.5
0.3	0.0671	5.29	12	29
0.5	0.0589	6.92	11.5	21
0.7	0.0472	5.46	10.5	44
0.9	0.0372	6.51	8.5	31.5

TABLE II
PERFORMANCE COMPARISON
BY PURELY TUNING THE PID WEIGHT

PID Weight	Mean Error	Overshoot Percentage	Rising Time s	Settling Time s
0.1	0.0852	0	37.5	>50
0.3	0.0564	3.18	16.5	19
0.5	0.0478	5.86	12.5	40
0.7	0.0447	6.88	10.5	23
1	0.0432	6.00	9.5	21.5