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Comprehensible model of a quasi-periodic signal

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***Abstract*—In this paper we present a new method to analyze quasi-periodic signals. This method consists of modeling these signals using a Fuzzy Finite State Machine as a particular case of a Linguistic Fuzzy Model of a dynamical system. This model defines states and transitions using a priori knowledge of the signal we want to analyze. The model is represented using fuzzy rules that make it easily comprehensible. We include a practical example analyzing quasi-periodic signals of acceleration measured during the human gait cycle where good results were achieved.**

***Keywords*-Dynamical systems; Fuzzy Finite State Machine; Human gait; Quasi-periodic signals; Signal processing.**

I. Introduction

Quasi periodicity is a property of dynamical systems that approximately retrace their paths through the state space. A quasi-periodic signal is a signal that evolves in time approximately repeating its shape and period. Quasi-periodic signals are usually modeled using statistical methods, such as Wavelet Transforms [1], Hidden Markov Models [2], [3]; or Soft Computing techniques like Neural Networks [4].

In this paper we present a Linguistic Fuzzy Model to describe quasi-periodic signals.

Linguistic Fuzzy Models were originally proposed by Zadeh [5] and further developed in subsequent papers by other authors [6], [7], [8], [9].

These models describe the system by means of a set of IF-THEN rules with vague predicates; the rule set takes the place of the usual set of equations used to characterize a dynamical system. The linguistic model is a knowledge-based system that incorporates fuzzy knowledge about a phenomenon in the real world.

As a particular case of the Linguistic Fuzzy Model we use a Fuzzy Finite State Machine where each state and transition is established using our a priori knowledge of the temporal evolution of the dynamical system we want to analyze.

II. Linguistic Models of dynamical systems

A deterministic dynamical system is described by the set of state equations:

$$\begin{cases} x[t+1] = f(x[t], u[t]) \\ y[t] = g(x[t], u[t]) \end{cases}$$

where:

- $x[t] = (x_1, x_2, ..., x_d)$ is the vector of state variables at time t, being d the dimension of the state space.
- $y[t]$ is the system's output.
- $u[t]$ is the system's input.
- $f(x[t], u[t])$ and $g(x[t], u[t])$ are mappings that describe analytically the relationships between state, input, and output variables. These functions are built using knowledge from the application domain.

We say that the model described by these equations is a Linguistic Fuzzy Model when at least one of the variables is fuzzy [10]. In the general case the variables are linguistic variables expressed by fuzzy sets [11], and $f(x[t], u[t])$ and $g(x[t], u[t])$ are replaced by logical rules operating with these linguistic variables.

A. Fuzzy Finite State Machine

In a preliminary research we have learnt that Fuzzy Finite State Machines (FFSM) are suitable tools for modeling signals which evolve following an approximately repetitive pattern [12], [13]. In this paper we refine the way of defining the FFSM and we explore its application to model the human gait signal. We will show that Finite State Machines provide an interesting paradigm to design the sets of fuzzy rules that allow us to implement the mappings $f(x[t], u[t])$ and $g(x[t], u[t])$ for modeling this type of signals.

We propose a type of FFSM as a particular case of a Linguistic Fuzzy Model of a dynamical system. We define a FFSM as a tuple:

$$\{x, Q, S, U, f, Y, g, S_0\}$$

where:

- x is a point in the original state space. Suppose that we have a two-dimensional state space defined by the state variables Temperature and Humidity (T, H) (see fig. 1).
- Q is a set of fuzzy states $\{q_1, ..., q_i, ..., q_j, ...\}$. A fuzzy state q_i is a fuzzy set of points x in the state space. For example, we can define the fuzzy state "Comfort" (q_C) by defining the degree of membership of each point of the state space (x) to the fuzzy state q_C (see fig. 1).

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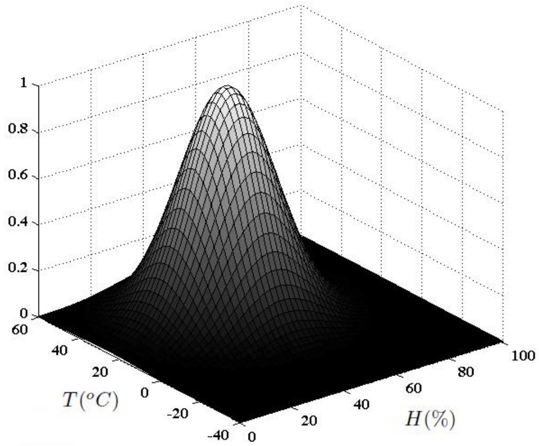

Figure 1. Degree of membership to the fuzzy state "Comfort" defined for all points x in the state space.

When the system evolves in the original state space, it could be simultaneously in several fuzzy states. The degree with which the system is in state q_i is called degree of activation of the state q_i.

- S is the state activation vector that stores, in each of its components, the degree of activation of the different states: $S = (s_1, s_2, ..., s_{N_{states}})$, being $s_i[t] = q_i(x[t])$ and therefore $s_i[t]\epsilon[0, 1]$ $\forall i$. And assuming that the system is always in a known state $\sum_{i=1}^{N_{states}} s_i[t] = 1$.
- U is the input vector $(u_1, u_2, ..., u_{N_{inputs}})$. In our case U is a set where every variable u_i is a numeric value obtained from sensors.
- f is the state activation transition function $S[t+1] = f(U[t], S[t])$. It will be explained in detail in the next section.
- Y is the output vector $(y_1, y_2, ..., y_{N_{outputs}})$. The output provided by the FFSM when leaving a state q_i can be considered as a summary of relevant characteristics of the system while remained in q_i.
- g is the output function $Y[t] = g(U[t], S[t])$. When a transition occurs, the values of the output variables are typically obtained applying e.g. the average and the standard deviation of the values of the input variables while the signal remained in the considered state.
- S_0 is the initial value of the state activation vector $S[t = 0] = S_0$.

III. The state activation transition function f

The degree of activation of the state q_i is defined not only defining constraints to interpret the value of the inputs but defining constraints to interpret the duration and the order the states occur.

The state activation transition function is implemented using a set of Takagi-Sugeno-Kang (TSK) fuzzy rules [14].

The one-order TSK fuzzy model consists of a set of fuzzy rules:

R^1: IF I_1 is B_{11} AND ... AND I_s is B_{1s} THEN
$O_1 = b_{10} + b_{11}I_1 + ... + b_{1s}I_s$
ALSO
...
ALSO
R^r: IF I_1 is B_{r1} AND ... AND I_s is B_{rs} THEN
$O_r = b_{r0} + b_{r1}I_1 + ... + b_{rs}I_s$

where B_{kl}, $1 \leq k \leq r$, $1 \leq l \leq s$ are linguistic labels and $I_1, I_2, ..., I_s$ are the values of input variables. Each of the linear functions in the rule consequents can be regarded as a linear model with crisp inputs $I_1, I_2, ..., I_s$, crisp outputs O_k and parameters b_{kl}, $1 \leq k \leq r$, $0 \leq l \leq s$. The crisp output O inferred by the fuzzy model under the TSK method is defined by the weighted average of the crisp outputs O_k of individual linear subsystems:

$$O = \frac{\sum_{k=1}^{r} \omega_k \cdot O_k}{\sum_{k=1}^{r} \omega_k}$$

where ω_k is the degree of firing of the k^{th} rule:
$\omega_k = B_{k1}(I_1) \wedge ... \wedge B_{ks}(I_s)$.

When we apply the TSK fuzzy model to the state activation transition function f, we distinguish between rules (R_{ii}) to remain in a state q_i and rules (R_{ij}) to change from the state q_i to the state q_j.

In order to define the state transition diagram we follow a simple procedure: the allowed transitions have associated fuzzy rules and simply the forbidden ones have not associated fuzzy rules.

A generic rule (R^k_{ii}) to remain in a state q_i can be explained as follows:

IF $(x[t]$ is $q_i)$ AND $(U[t]$ is $C_{ii})$ AND $(d_i[t]$ is $T_{ii})$
THEN $S_k[t+1] = (0, ..., 1, ..., 0, ...)$.

This rule consists of a TSK fuzzy rule of zero-order (where the linear function is reduced to be constant). In this particular case we have the following vector constant output: $s_{k_i}[t+1] = 1$ and $s_{k_j}[t+1] = 0$, $\forall j \neq i$. We have three antecedents defined by:

- The function $q_i(x[t]) = s_i[t]$ that provides the degree of activation of the state q_i.
- The function $C_{ii}(U[t])$ that provides the degree of satisfaction of the system inputs of the conditions of amplitude to remain in the state q_i.
- The function $T_{ii}(d_i[t])$, where $d_i[t]$ is the duration of the state q_i (which is defined as the time that $s_i[t] > 0$),

provides the maximum time that the signal is expected to remain in the state q_i. It is a trapezoidal membership function which initially takes a value of 1 and loses value until it reaches 0 after a certain amount of time which is the maximum duration calculated for that state (see in fig. 4 the continuous line membership function).

A generic rule (R_{ij}^k) to change from the state q_i to the state q_j can be explained as follows:

IF ($x[t]$ is q_i) AND ($U[t]$ is C_{ij}) AND ($d_i[t]$ is T_{ij})
THEN $S_k[t+1] = (0, ..., 0, ..., 1, ...)$.

If we consider this rule in the context of the TSK zero-order fuzzy rules we can see that we have the vector constant output: $s_{k_j}[t+1] = 1$ and $s_{k_i}[t+1] = 0, \ \forall i \neq j$, and the following antecedents defined by:

- The function $q_i(x[t]) = s_i[t]$.
- The function $C_{ij}(U[t])$ that provides the degree of satisfaction of the system inputs of the conditions of amplitude to change from the state q_i to the state q_j.
- The function $T_{ij}(d_i[t])$ that provides the minimum time that the signal is expected to remain in the state q_i before changing to the state q_j. It is a trapezoidal membership function which initially takes a value of 0 and raises value until it reaches 1 after a certain amount of time which is the minimum duration calculated for that state (see in fig. 4 the dotted line membership function).

Once we have all the rules, we can calculate the final output weighted average of the crisp outputs S_k of individual linear subsystems:

$$S[t+1] = \frac{\sum_{k=1}^{\#Rules} \omega_k \cdot S_k[t+1]}{\sum_{k=1}^{\#Rules} \omega_k}$$

where ω_k is the degree of firing of the k^{th} rule.

Note that this step produces automatically the normalization of the state activation vector.

IV. A Practical Example

As an application of the idea described above, we will create a model of the quasi-periodic signals obtained when measuring the accelerations during the human gait cycle.

The human gait is defined as the interval between two similar events (usually heel contact) of the same foot. It is characterized by a stance phase (60 % of the total gait cycle), where at least one foot is in contact with the ground, and a swing phase (40 % of the total gait cycle) during which one limb swings through to the next heel contact [15].

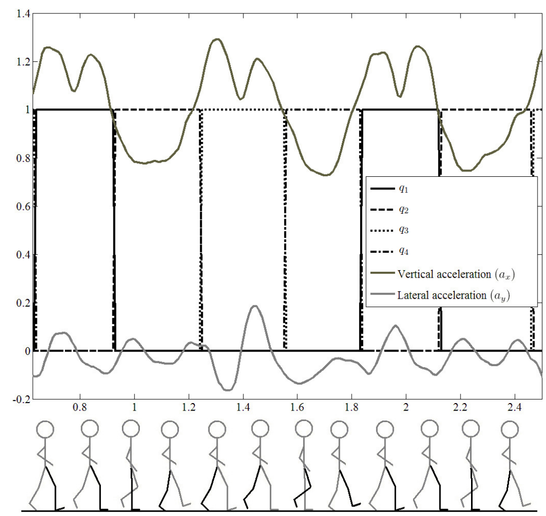

Figure 2. Vertical and lateral acceleration during the four states of the human gait cycle.

A. Defining the states

Fig. 2 shows the vertical and lateral acceleration and the four states of the FFSM describing the evolution of the human gait. The description of the states is:

- q_1: Reference foot stance phase and opposite foot stance phase (double limb support). This state covers a 10-15 % of the total period.
- q_2: Reference foot stance phase and opposite foot swing phase (reference limb single support). This state covers a 35-40 % of the total period.
- q_3: Reference foot stance phase and opposite foot stance phase (double limb support but different of q_1 because of the feet position). This state covers a 10-15 % of the total period.
- q_4: Reference foot swing phase and opposite foot stance phase (opposite limb single support). This state covers a 35-40 % of the total period.

B. Defining the linguistic labels

We use a three-dimensional accelerometer in a belt, centered at the back, that provides us the three orthogonal accelerations. During a first analysis of data it was realized that the vertical acceleration (a_x) and the lateral acceleration (a_y) were indicative for the states we wanted to distinguish.

In order to make the system valid for any gait we have normalized the signals. First we subtract the respective average values. Then we rescale the signals in the range given by their standard deviations. We use three trapezoidal linguistic labels for each acceleration (Negative, Zero, and Positive) so we have six linguistic labels $\{N_x, Z_x, P_x\}\{N_y, Z_y, P_y\}$, each one covering a third of the total amplitude. Fig. 3 shows an example of the vertical acceleration.

We have applied self-correlation analysis of the vertical acceleration to obtain an approximation to the signal period

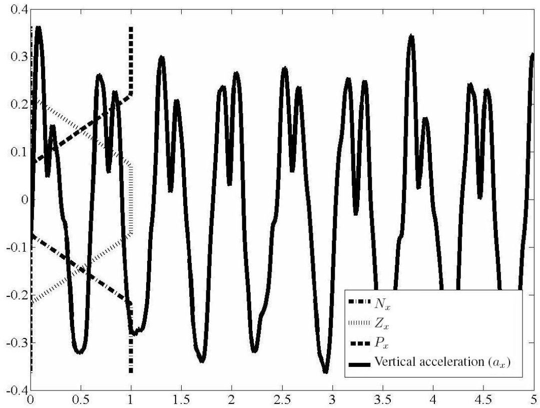

Figure 3. Trapezoidal linguistic labels for the vertical acceleration.

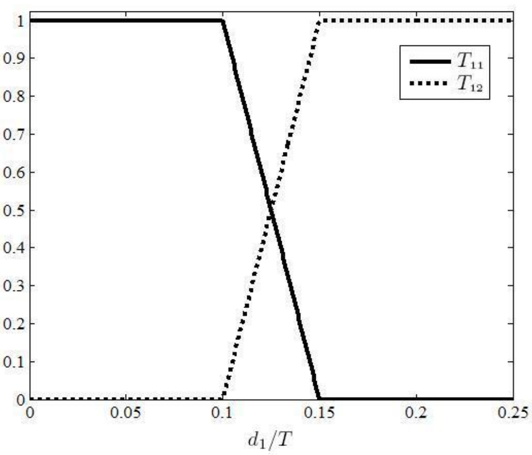

Figure 4. Temporal constraints for the state q_1 related with the signal period T.

(T). We assign for each state the percentage of the total period that corresponds with the description of the typical human gait cycle. The temporal constraints are based on the linguistic labels T_{ii} and T_{ij}. In fig. 4 the temporal constraints for the state q_1 related with the total signal period are shown.

C. Defining the rules

Using the information shown in fig. 2 and our knowledge about he human gait, we can deduce the conditions to remain in a state or to change between states. Once we have these conditions, we can define the 8 rules (4 to remain in each state and 4 to change between states) with their inputs, linguistic labels and outputs:

R^1_{11} : IF ($x[t]$ is q_1) AND ($a_x[t]$ is P_x) AND ($a_y[t]$ is P_y) AND ($d_1[t]$ is T_{11}) THEN $S_1[t+1] = (1,0,0,0)$.

R^2_{12} : IF ($x[t]$ is q_1) AND ($a_x[t]$ is N_x) AND ($a_y[t]$ is Z_y) AND ($d_1[t]$ is T_{12}) THEN $S_2[t+1] = (0,1,0,0)$.

R^3_{22} : IF ($x[t]$ is q_2) AND ($a_x[t]$ is N_x) AND ($a_y[t]$ is Z_y) AND ($d_2[t]$ is T_{22}) THEN $S_3[t+1] = (0,1,0,0)$.

R^4_{23} : IF ($x[t]$ is q_2) AND ($a_x[t]$ is P_x) AND ($a_y[t]$ is N_y) AND ($d_2[t]$ is T_{23}) THEN $S_4[t+1] = (0,0,1,0)$.

R^5_{33} : IF ($x[t]$ is q_3) AND ($a_x[t]$ is P_x) AND ($a_y[t]$ is N_y) AND ($d_3[t]$ is T_{33}) THEN $S_5[t+1] = (0,0,1,0)$.

R^6_{34} : IF ($x[t]$ is q_3) AND ($a_x[t]$ is N_x) AND ($a_y[t]$ is Z_y) AND ($d_3[t]$ is T_{34}) THEN $S_6[t+1] = (0,0,0,1)$.

R^7_{44} : IF ($x[t]$ is q_4) AND ($a_x[t]$ is N_x) AND ($a_y[t]$ is Z_y) AND ($d_4[t]$ is T_{44}) THEN $S_7[t+1] = (0,0,0,1)$.

R^8_{41} : IF ($x[t]$ is q_4) AND ($a_x[t]$ is P_x) AND ($a_y[t]$ is Z_y) AND ($d_4[t]$ is T_{41}) THEN $S_8[t+1] = (1,0,0,0)$.

The total output of the rules is the weighted average of the crisp outputs $S_k[t+1]$ of individual linear subsystems:

$$S[t+1] = \begin{cases} \dfrac{\sum_{k=1}^{8} \omega_k \cdot S_k[t+1]}{\sum_{k=1}^{8} \omega_k} & \text{if } \sum_{k=1}^{8} \omega_k \neq 0 \\ S[t] & \text{if } \sum_{k=1}^{8} \omega_k = 0 \end{cases}$$

where the degree of firing for each rule (ω_k) is calculated using the minimum for the AND operator.

Fig. 2 shows how these rules model sharply the evolution of the signal through the four states.

D. The output of the FFSM

Our aim is to build a summary of the human gait where the relevant aspects should be remarked. Once the four phases in the signal have been identified, we calculate the average and the standard deviation of the values taken by input variables while the signal remained in the considered state. In this example, the output function g is implemented as a set of equations that calculate these values:

- $\overline{t_i}$: Temporal "center of gravity" of the input u in the state q_i.
  $$\overline{t_i} = \frac{\sum_{t=0}^{T} t \cdot u[t] \cdot s_i[t]}{\sum_{t=0}^{T} u[t] \cdot s_i[t]}$$
- $\overline{u_i}$: Average of the input u during state q_i.
  $$\overline{u_i} = \frac{\sum_{t=0}^{T} u[t] \cdot s_i[t]}{\sum_{t=0}^{T} s_i[t]}$$
- σ_{t_i}: Standard deviation of the temporal distributions of the input u in the state q_i.

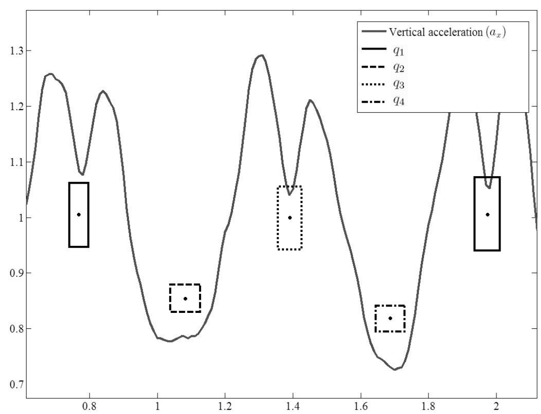

Figure 5. The output of the FFSM.

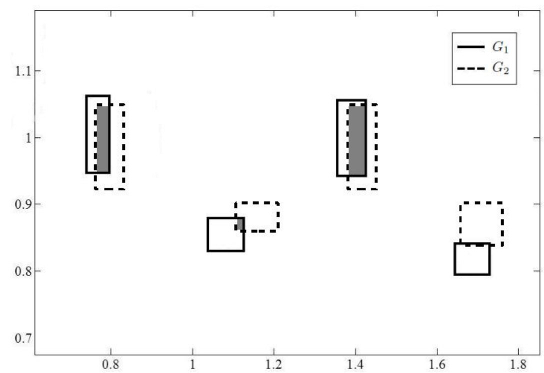

Figure 6. Calculating the degree of matching between two gaits (G_1 and G_2).

$$\sigma_{t_i}^2 = \frac{\sum_{t=0}^{T}(t-\overline{t_i})^2 \cdot u[t] \cdot s_i[t]}{\sum_{t=0}^{T} u[t] \cdot s_i[t]}$$

- σ_{u_i}: Standard deviation of the input u during the state q_i.

$$\sigma_{u_i}^2 = \frac{\sum_{t=0}^{T}(u[t]-\overline{u_i})^2 \cdot s_i[t]}{\sum_{t=0}^{T} s_i[t]}$$

where:

- $u[t]$ is the input variable at the instant t.
- $s_i[t]$ is the degree of activation of the state q_i at the instant t.
- T is the duration of a complete cycle.

With these values we built the output vector Y which is calculated for each state in each cycle of the human gait:

$Y = (\overline{t_1}, \overline{u_1}, \sigma_{t_1}, \sigma_{u_1}, \overline{t_2}, \overline{u_2}, \sigma_{t_2}, \sigma_{u_2}, \overline{t_3}, \overline{u_3}, \sigma_{t_3}, \sigma_{u_3}, \overline{t_4}, \overline{u_4}, \sigma_{t_4}, \sigma_{u_4})$

Fig. 5 shows the graphical representation of the output vector Y calculated being the input $u[t]$ the vertical acceleration (a_x) as it was obtained from the sensor.

E. Results

We have converted the signals evolving in time (the vertical and the lateral accelerations) to a set of rectangles obtained from the output vector. To calculate the output vector we have used the vertical acceleration (a_x) as the input $u[t]$. These rectangles characterize the gait of each person. We can say that the FFSM gives us a summary of a stream of data captured using sensors.

For example, comparing the areas of the rectangles among different states we can obtain parameters like the symmetry (the degree of the movement of a limb is similar to the other one) or the homogeneity (the degree with which the gait profile repeats in time).

Other result that we can obtain from the output vector is the degree of matching between different gaits. First we calculate the four rectangles representative of the gait of a specific person. Then we can calculate the area intersected by the rectangles defined by the points of the output vector of two different gaits (G_l and G_m), and comparing them with the area of the rectangles that we want to test. Let $I_i(l,m)$ be the intersection between the areas of rectangles corresponding to different gaits (G_l and G_m) but the same state q_i, and let $A_i(m)$ be the area of the rectangle corresponding to the gait we want to test (G_m). The degree of matching ($DM(l,m)$) of the gait (G_m) with the gait (G_l) is defined by:

$$DM(l,m) = \frac{\sum_{i=1}^{4} I_i(l,m)}{\sum_{i=1}^{4} A_i(m)}$$

In fig. 6 an example of two sets of rectangles used to calculate the degree of matching between two gaits (G_1 and G_2) can be seen.

Finally, table I shows the average degree of matching among gaits of 10 different people. We used 5 samples to learn the mean of the output vector of the gait of each person, and 5 additional samples were used to calculate the average degree of matching of the gait of each person with the others. It can be seen that the degree of matching between gaits of the same person (in bold) is almost always greater than the degree of matching between gaits of different people.

V. Conclusions and Future Works

This paper contributes to the field of signal analysis providing a new way of modeling quasi-periodic signals using a FFSM.

The model has been presented formally as a particular case of a Linguistic Fuzzy Model of a dynamical system.

Not only is this model comprehensible, but it also provides good results that demonstrate the usability of the proposal.

PERSON	1	2	3	4	5	6	7	8	9	10
1	**0.95**	0.67	0.29	0.29	0.53	0.67	0.49	0.55	0.53	0.57
2	0.81	**0.95**	0.47	0.32	0.66	0.71	0.62	0.50	0.60	0.64
3	0.42	0.49	**0.83**	0.57	0.55	0.41	0.74	0.33	0.42	0.49
4	0.32	0.25	0.35	**0.83**	0.33	0.26	0.37	0.26	0.33	0.33
5	0.74	0.76	0.54	0.40	**0.89**	0.66	0.80	0.59	0.79	0.84
6	0.87	0.78	0.41	0.34	0.69	**0.93**	0.64	0.78	0.63	0.66
7	0.56	0.56	0.71	0.48	0.70	0.49	**0.88**	0.47	0.61	0.65
8	0.79	0.64	0.37	0.37	0.66	0.75	0.59	**0.91**	0.67	0.67
9	0.83	0.63	0.36	0.38	0.73	0.65	0.64	0.74	**0.90**	0.86
10	0.62	0.50	0.32	0.34	0.53	0.49	0.53	0.49	0.61	**0.91**

Table I
DEGREE OF MATCHING AMONG GAITS OF 10 DIFFERENT PEOPLE

A comparison with other methods, where the comprehensibility of the system is not important, e.g. Neural Networks or Hidden Markov Models, will be considered in future works.

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