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A Study on Interpretability Conditions for Fuzzy Rule-Based Classifiers

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Abstract

Interpretability represents the most important driving force behind the implementation of fuzzy logic-based systems. It can be directly related to the system's knowledge base, with reference to the human user's easiness experienced while reading and understanding the embedded pieces of information. In this paper, we present a preliminary study on interpretability conditions for fuzzy rule-based classifiers on the basis of an innovative approach that relies on the concept of semantic cointesion. The approach adopted in this study consists in analysing the components of a fuzzy classifiers so that inference is carried out with the respect of logical properties. As a result, we derive some sufficient conditions and basic requirements to be verified by a fuzzy classifier in order to be tagged as interpretable in the semantic sense.

1. Introduction

The development of artificial intelligent systems can be pursued by referring to a number of different paradigms, each one characterised by some peculiar distinctive features. Among them, fuzzy logic is commonly regarded as a tool for providing the basic means to represent knowledge in a comprehensible form, thus enabling intelligent systems to express and manipulate information in agreement with the natural language spoken by humans. Therefore, interpretability represents the most important driving force behind the implementation of fuzzy logic-based systems, which are centred on the compilation of fuzzy rule bases.

Due to its subjective connotation, interpretability escapes a proper formal characterisation and the question of interpretability evaluation appears to represent an ill-posed problem. Roughly speaking, the concept of interpretability can be directly related to a knowledge base, with reference to the human user's easiness experienced while reading and understanding the embedded pieces of information. Setting aside subjective judgement, research on interpretability assessment is mainly focused on the design of automatic mechanisms of evaluation. In this context, many common attempts relies on a simplicity law (in according with the "Occam's razor" principle), so that simpler rule bases are deemed to be more comprehensible than complicated ones [2, 3, 4, 10]. On these bases, it is straightforward to derive automatic evaluation of interpretability, provided that some thresholds are established on the number of rules and conditions to indicate the boundaries between different degrees of interpretability. However, increased simplicity implies a reduction of accuracy and this kind of approach has been recently criticised, highlighting how interpretable but inaccurate models are as useless as very accurate but incomprehensible models [9].

In [1] and [11] a different approach is proposed: interpretability is ensured whenever a set of constraints is satisfied, involving the model of the knowledge base (a recent survey on this topic is reported in [8]). Even if this approach leads to the development of automatic mechanisms of evaluation, the constraints to be imposed depends from the nature of the problem at hand and human skills (and common sense) are necessary to define a constraint set. Nonetheless, it could be possible to determine a number of valid constraints applicable in most cases.

A quite different approach to interpretability assessment has been recently defined on the basis of the "logical view" concept [5]. In this case, the definition of co-intension [12] and the propositional view of a fuzzy rule base are employed to evaluate interpretability. Particularly, the evaluation process relies on determining the co-intension degree between the explicit semantics embedded into a fuzzy rule base and the implicit semantics gathered in the user's mind while reading the rules. In practice, this approach implements a minimisation of the original fuzzy rule base by means of boolean truth-preserving operators. Such a minimisation should not modify the semantics of the rules (only small distortions could be expected, due to the different nature of boolean and fuzzy rules).

Since the logical view approach is grounded on the appli-

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cation of boolean operators to the propositional structure of the rule base, a proper analysis of the involved operators is needed. In this paper, we aim at evaluating the specific components of fuzzy systems – including t-norms, t-conorms and defuzzification functions – moving from the idea that they must be compatible with the boolean logic semantics in order to pave the way for a correct interpretability assessment. Actually, the compatibility is verified by transforming the fuzzy system into a boolean system and, consequently, by choosing the suitable components in according with the requirements necessary to perform the transformation.

The focus of our research has been set on fuzzy rule-based classifiers (FRBCs), therefore in section 2 we are going to present a mathematical formalisation of FRBCs. Section 3 is devoted to illustrate the logical view approach, while section 4 presents the process for transforming a fuzzy system into a boolean one. Some conclusive remarks are reported in section 5.

2. Fuzzy rule-based classifiers

Let us consider a classifier as a system computing a function of the following type:

$$f : \mathbf{X} \longrightarrow \Lambda, \tag{1}$$

where $\mathbf{X} \subseteq \mathbf{R}^n$ is an n-dimensional input space, and $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_c\}$ is a set of class labels.

A fuzzy rule-based classifier (FRBC) is a system that carries out classification (1) through inference on a knowledge base. The knowledge base includes the definition of a linguistic variable for each input. Thus, for each $j = 1, 2, \ldots, n$, linguistic variables are defined as:

$$V_j = (v_j, X_j, Q_j, S_j, I_j), \tag{2}$$

being:

- v_j the name of the variable;
- X_j the domain of the variable (it is assumed that $\mathbf{X} = X_1 \times X_2 \times \cdots \times X_n$);
- $Q_j = \{q_{j1}, q_{j2}, \ldots, q_{jm_j}, \text{ANY}\}$ is a set of labels denoting linguistic values for the variable (e.g. SMALL, MEDIUM, LARGE);
- $S_j = \{s_{j1}, s_{j2}, \ldots, s_{jm_j+1}\}$ is a set of fuzzy sets on X_j, $s_{jk} : X_j \to [0, 1]$;
- I_j associates each linguistic value q_{jk} to a fuzzy set s_{jk}. We will assume that $I_j(q_{jk}) = s_{jk}$.

We assume that each linguistic variable contains the linguistic value "ANY" associated to a special fuzzy set $s \in S_j$ such that $s(x) = 1, \forall x \in X_j$.

The knowledge base of a FRBC is defined by a set of R rules. Each rule can be represented by the schema:

$$\begin{array}{l}\text{IF } v_1 \text{ IS } [\text{NOT}]\; q_1^{(r)} \text{ AND } \cdots \text{ AND } v_n \text{ IS } [\text{NOT}]\; q_n^{(r)} \\ \text{THEN } \lambda^{(r)},\end{array} \tag{3}$$

being $q_j^{(r)} \in Q_j$ and $\lambda^{(r)} \in \Lambda$. Symbol NOT is optional for each linguistic value. If for some j, $q_j^{(r)} = \text{ANY}$, then the corresponding atom "v_j IS ANY" can be removed from the representation of the rule.[1]

Inference is carried out as follows. When an input $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is available, the strength of each rule is calculated as:

$$\mu_r(\mathbf{x}) = s_1^{(r)}(x_1) \otimes s_2^{(r)}(x_2) \otimes \cdots \otimes s_n^{(r)}(x_n), \tag{4}$$

being $s_j^{(r)} = \nu_j^{(r)}(I_j(q_j^{(r)}))$, with $j = 1, 2, \ldots, n$, $r = 1, 2, \ldots, R$. Function $\nu_j^{(r)}(t)$ is $1 - t$ if NOT occurs before $q_j^{(r)}$, otherwise it is defined as t. The operator $\otimes : [0, 1]^2 \to [0, 1]$ is usually a t-norm, such as minimum or product.

A defuzzification process is necessary to evaluate the membership of input $\mathbf{x}$ to class λ_i. A number of defuzzification methods have been proposed in literature, among them the Centre of Gravity (COG) method can be approximated by the Weighted Average (WA) method, which is performed by means of the following formula:

$$\mu_{\lambda_i}(\mathbf{x}) = \frac{\sum_{r=1}^{R} \mu_r(\mathbf{x}) \chi(\lambda_i, \lambda^{(r)})}{\sum_{r=1}^{R} \mu_r(\mathbf{x})}, \tag{5}$$

being $\chi(a, b) = 1$ iff $a = b$ and 0 otherwise. Another option for the defuzzification process is given by the maximum criteria, which can be expressed by the formula:

$$\mu_{\lambda_i}(\mathbf{x}) = \max\ \mu_r(\mathbf{x}) \chi(\lambda_i, \lambda^{(r)}). \tag{6}$$

After defuzzification, since just one class label has to be assigned to the input $\mathbf{x}$, the FRBC assigns the class label with highest membership (ties are solved randomly):

$$f_{FRBC}(\mathbf{x}) = \lambda \Leftrightarrow \mu_\lambda(\mathbf{x}) = \max_{i=1,2,\ldots,c} \mu_{\lambda_i}(\mathbf{x}). \tag{7}$$

3. Logical View Approach

The logical view approach, relying on the formal structure of FRBCs, has been recently introduced in some previous papers of ours [6, 7]. The rationale behind this approach comes from the observation that the rule base is the linguistic interface of the FRBC to the user. For an interpretable knowledge base, the user should be able to understand the classification rules by simply observing their linguistic representation. All the semantic information (fuzzy

[1] The sequence NOT ANY is not allowed.

sets attached to linguistic values, t-norm used for conjunction, etc.) should be hidden to the user because - this is the key point of interpretability - the semantics of FRBC knowledge should be co-intensive with the user's knowledge, recalled by the linguistic terms.

To assess interpretability, the cognitive structures shared by human users and FRBCs are sought and explored. In particular, a strict affinity can be recognised between the rule base of a FRBC and the logical propositions. Actually, rules are constructed in order to resemble propositions, so that they can be understood by users. As a consequence, the propositional view of rules stands as the cognitive structure shared by users and FRBCs, and the logical view can be exploited as the common ground to analyse both the explicit semantics embedded into the rule base and the implicit semantics conveyed to the reader. By doing so, we are able to translate the interpretability evaluation problem into a formal process: being like propositions, rules can be modified by truth-preserving operators and the consequent distortion of their fuzzy semantics can be prefigured to a reduced amount, due to the shared propositional view between the FRBC and the user.

The core of the logical view approach is the following: given a rule base of a FRBC, it is represented as a collection of logical propositions which, in turn, can be modified by applying truth-preserving operators. In this way, a new set of propositions is obtained constituting a rule base different from the original one. Then the rule bases are compared on the basis of their classification performance: if they do not differ too much, we recognise that the logical view of the original FRBC (which is shared with the human user) is in agreement with the explicit semantics exhibited by the fuzzy rules. In other words, co-intension with the user's knowledge is verified and the FRBC can be deemed interpretable. On the other hand, if the two rule bases are characterised by notably different accuracy values, then the logical view of the FRBC is not compatible with the explicit semantics of fuzzy rules, therefore the knowledge base is not co-intensive with user's knowledge and it can be deemed as not interpretable. This means that any attempt at reading the linguistic labels would be misleading and the classification capability of the original FRBC only relies on the mathematical configuration of its parameters (without any engagement of comprehensible information).

The choice of the truth-preserving operators to be applied on the logical propositions deserves a special mention. Actually, several modifications can be conceived to convert the original fuzzy rule base, among them the one minimising the number of the involved linguistic terms appears to be mostly suitable. In fact, this choice produces two kinds of benefits. First, by eliminating as many terms as possible, it is possible to verify the preservation of the logical view in the specific condition where only the minimum required information is available. Second, if assessment leads to positive results, the simplified rule base can be retained in place of the original one (it should be preferred by reason of its increased compactness).

4. Interpretability evaluation

One of the major issues in fuzzy modelling concerns the appropriate choice for the fuzzy system parameters, including the definition of membership functions, t-norms and t-conorms, defuzzification methods, etc. Different choices in the combination of the involved parameters obviously determines the realisation of different systems, each one characterised by a specific working engine. As previously asserted, the logical view method for interpretability assessment is based on boolean logic and the minimisation process performed over the original rule base is expected to introduce negligible modifications in the semantics of the rules. However, the adoption of boolean operators in the minimisation process could be questionable since the fuzzy inference performed over the knowledge base is different in its nature.

Moving from the above considerations, we resolve to follow up the steps for transforming a FRBC into a boolean classification system in order to identify the operators which should be adopted to keep equivalence between the two systems (the equivalence is intended as obtaining the same output from both systems starting from the same input values). Once a set of operators enabling such a transformation is detected, we can assume that the underlying semantics of both systems is identical and, therefore, the minimisation process performed over the original FRBC does not carry out any kind of modification in the involved semantics.

The basic difference between boolean and fuzzy logic is in the range of the admitted values. While boolean logic relies on a couple of values $\{0,1\}$, fuzzy logic refers to an extended range [0,1], where all the in-between values are admissible. To move from a fuzzy perspective to the boolean dichotomy we need to transform fuzzy sets into crisp counterparts. This can be achieved through α-cuts, but the choice of the parameter α is crucial to transform a FRBC into an equivalent boolean classifier.

In the following sections we are going to track the transformation process of a FRBC into a boolean classifier. To this aim, various steps will be considered, from the investigation of the single clauses composing the fuzzy rules to the analysis of the defuzzification method.

4.1. Step 1: Clauses

The general format of a fuzzy rule (as expressed in (3)) comprises an antecedent and a consequent part. Focusing attention over the antecedent (delimited by the "IF"-

"THEN" keywords), we notice how it is composed by a number of clauses connected by the "AND" conjunction. In order to express each single clause (v_j IS q_j) of a fuzzy rule in a propositional form, we must refer to the α-cuts evaluated over the involved fuzzy sets. Particularly, the following formula has to be applied for all $x \in X_j$:

$$(v_j \text{ IS } q_{jk}) = \begin{cases} 1 & s_{jk}(x) \geq \alpha, \\ 0 & s_{jk}(x) < \alpha. \end{cases} \quad (8)$$

By means of (8) it is possible to express boolean conditions over the range of input values for which the fuzzy sets s_{jk} corresponding to each clause is activated with a membership degree greater than α. For example, if according to equation (8) a clause (v_j IS q_j) is equal to one for all input values ranging between 1 and 5, then the corresponding boolean form can be expressed as $(x \geq 1 \wedge x \leq 5)$. Obviously, combinations of ranges can be considered for input values in order to derive a boolean condition: in such cases the logical operator OR is to be employed. For example, if the input values range inside $[1, 2]$ and $[3, 4]$, then the boolean form can be expressed as $((x \geq 1 \wedge x \leq 2) \vee (x \geq 3 \wedge x \leq 4))$.

It is important to underline that the choice of the parameter α should be made so as to guarantee that for each linguistic variable v_j and for each input $x \in X_j$ only one fuzzy set s_{jk} (with the exclusion of the fuzzy set labelled ANY) is such that $s_{jk}(x) \geq \alpha$. This property is called "α-distinguishability" and it is commonly preserved in interpretability-oriented fuzzy models.

4.2. Step 2: t-norm and t-conorm

Once a propositional form has been derived for each single clause, we should turn to consider the entire antecedent part of the fuzzy rule. Clauses are combined by the "AND" connective which finds a direct counterpart in the logical conjuction ($\wedge$) of boolean logic. In the context of fuzzy logic, a number of mathematical functions can be employed to translate the "AND" connective, all of them falling under the general family of the t-norm operators. For our purposes, we should seek for the most appropriate t-norm operator ensuring the equivalence between a fuzzy and a boolean system. To this aim, we can consider the truth table of the "AND" operator (as it can be derived by employing the logical $\wedge$ conjuction): a conjunctive proposition is true if and only if all its composing clauses are true. With reference to the antecedent part of a fuzzy rules, the previous condition holds whenever each composing clause is assigned value 1 by means of equation (8). Since we determined to adopt α-cuts for deriving the crisp counterparts of fuzzy sets, a specific fuzzy value should be considered true when its membership degree is greater than α: that is the condition to verify while choosing a suitable t-norm for the "AND" connective. Now, we observe that if all the membership degrees of conjunctive clauses in a fuzzy rule are greater than α the same holds for the minimum among them. Therefore, the minimum function shares the same semantics of the logical conjunction ($\wedge$), thus being the best candidate for our choice of the t-norm operator (it is straightforward to observe that a different t-norm choice, such as the product function, would not be suitable since, from being $a, b \geq \alpha$, condition $a * b \geq \alpha$ is not guaranteed).

Analogously, the "OR" connective, correlating all the rules in the fuzzy rule base in disjunctive form, should be associated to a specific mathematical function to be chosen among the family of t-conorm operators. Actually, the "OR" connective has a counterpart in the logical disjunction ($\vee$) of the boolean logic, whose truth table assigns true value to any disjunctive proposition where at least one clause is true. If at least one membership degree of the fuzzy rules composing the rule base in a disjunctive form is greater than α, then the same holds for the maximum among them. Therefore, the maximum function shares the same semantics of the logical disjunction ($\vee$), thus being the best candidate for our choice of the t-conorm operator.

Summing up, we identified in the minimum and maximum functions the suitable modelling choices for implementing respectively the t-norm and t-conorm operators in a FRBC. As concerning translation of connectives, this stands as a sufficient condition for ensuring equivalence between fuzzy and boolean systems.

4.3. Step 3: Rules

The antecedent of a fuzzy rule is correlated with the consequent by means of an implication identified by the label "THEN" appearing in (3). However, this kind of implication can not be directly related with classical logical implication. In fact, the latter is characterised by a true value whenever the antecedent is false, while the rule implication should not adhere to such a condition. Therefore, we can not refer to the truth table of the logical implication to analyse the structure of the fuzzy rule: we should turn to consider how classification is performed by a boolean system, instead.

Boolean classification assumes the antecedent part of rule as a condition which, when it is verified, must be related with a specific class. In this sense, the implication is more properly translated into an assignment process: when an input vector enables true conditions, then it can be assigned to the corresponding class indicated by the consequent part of the rule. A problem arises with fuzzy rule bases where a number of rules relate the same consequent to different antecedents (which is mostly the case): in such situations the boolean classifier would not be able to assign a single condition to each class. This kind of problem can be discovered in the following rule base example:

R1: IF TEMPERATURE IS LOW AND HUMIDITY IS LOW THEN NORMAL
R2: IF TEMPERATURE IS HIGH AND HUMIDITY IS LOW THEN NORMAL
R3: IF TEMPERATURE IS HIGH AND HUMIDITY IS HIGH THEN WARNING
R4: IF TEMPERATURE IS LOW AND HUMIDITY IS HIGH THEN WARNING
R5: IF TEMPERATURE IS MEDIUM AND HUMIDITY IS HIGH THEN WARNING

Semantics of rules is not distorted while rewriting the rule base as follows:

R1 : IF (TEMPERATURE IS LOW OR TEMPERATURE IS HIGH) AND HUMIDITY IS LOW THEN NORMAL
R2 : IF (TEMPERATURE IS HIGH OR TEMPERATURE IS LOW OR TEMPERATURE IS MEDIUM) AND HUMIDITY IS HIGH THEN WARNING

In this way, rules with the same consequent part are aggregated using the "OR" connective, whose corresponding t-conorm has been assessed in the previous section.

4.4. Step 4: Defuzzification

Defuzzification is a crucial step in fuzzy modelling, since it determines the single output value to be assigned to each input instance starting from the analysis of the aggregated activation strengths of the rules. This represents a main difference with respect to boolean systems, where each rule must express a crisp output value for every input instance. In order to derive a boolean classifier that is equivalent to the FRBC, we need to constraint the fuzzy classifier so that two rules do not overlap too much. Formally this is achieved by imposing:

$$\forall_r^{r'}, r \neq r' : M_r^\alpha \cap M_{r'}^\alpha = \emptyset, \qquad (9)$$

where $M_r^\alpha = \{\mathbf{x} \in \mathbf{X} : \mu_r(\mathbf{x}) \geq \alpha\}$. The usefulness of such a constraint is straightforward. In fact, by means of (9) the input space is partitioned in such a way that only a single classification rule is activated with a strength degree greater than the α value when each input is presented, thus avoiding situations where multiple classes would be assigned to the same input instance (which is implausible for boolean classification).

The constraint expressed by (9) represents a necessary condition for keeping equivalence between a fuzzy and a boolean classifier, but a further analysis must be performed for choosing a defuzzification method among those available in literature. Particularly, we are going to empirically show how the adoption of different defuzzification functions strongly influences the final output of a classification system and in some cases compromises the equivalence we are pursuing.

Let us consider the 5 illustrative rules introduced in the previous section and the following membership degrees pertaining to a specific input instance:

- TEMPERATURE IS LOW: 0.3
- TEMPERATURE IS MEDIUM: 0.5
- TEMPERATURE IS HIGH: 0.7
- HUMIDITY IS LOW: 0.6
- HUMIDITY IS HIGH: 0.4

The corresponding activation strengths of the rules are as follows:

R1 : NORMAL = 0.3 AND 0.6 = 0.3
R2 : NORMAL = 0.7 AND 0.6 = 0.6
R5 : WARNING = 0.7 AND 0.4 = 0.4
R3 : WARNING = 0.3 AND 0.4 = 0.3
R4 : WARNING = 0.5 AND 0.4 = 0.4

By assuming $\alpha = 0.5$, it can be observed that the constraint expressed by (9) is verified: only the second rule is activated with a degree greater than the α value and class "NORMAL" would be assigned to the input instance by a boolean classifier. However, if the WA defuzzification method expressed in (5) is applied to perform fuzzy inference, the following results can be obtained by the FRBC:

$$\begin{aligned} normal &= \frac{\mu_{r1} + \mu_{r2}}{\mu_{r1} + \mu_{r2} + \mu_{r3} + \mu_{r4} + \mu_{r5}} \\ &= \frac{0.3 + 0.6}{0.3 + 0.6 + 0.3 + 0.4 + 0.4} \\ &= \frac{0.9}{2} \\ &= 0.45 \end{aligned}$$

$$\begin{aligned} warning &= \frac{\mu_{r3} + \mu_{r4} + \mu_{r5}}{\mu_{r1} + \mu_{r2} + \mu_{r3} + \mu_{r4} + \mu_{r5}} \\ &= \frac{0.3 + 0.4 + 0.4}{0.3 + 0.6 + 0.3 + 0.4 + 0.4} \\ &= \frac{1.1}{2} \\ &= 0.55 \end{aligned}$$

The "WARNING" class would be assigned to the input instance, which is not in agreement with the boolean classification results. This means that the modelling choice to be applied for implementing the defuzzification process must be related to the maximum criteria expressed by (6). By doing so, the equivalence between the FRBC and the boolean classifier can be preserved, since the maximum criteria allows the choice of the rule characterised by the maximum activation strength, which is the same rule determined by the boolean classifier.

5. Conclusions

Interpretability of fuzzy models is a topical issue, as it motivates the preference of these models over black-box models. However, defining and assessing interpretability is difficult, because of its blurry nature that eludes any formalization. In this paper we embrace the cointension-based approach for dealing with interpretability, and we use the logical-view as a formal basis to analyse fuzzy rule-based classifiers. In this context, we emprically derive some sufficient properties and basic requirements to be verified by a FRBC in order to adhere to the logical view. We show that the choice of minimum as t-norm, maximum as t-conorm and maximum for defuzzification is sufficient to adehere to the logical view, provided that the fuzzy sets in each linguistic variables, as well as all rules of the classifers, are distinguishable. This results can be used as an aid for designing interpretable FRBC, although they should be considered as a starting point to the study of the semantic properties of fuzzy models to actually achieve interpretability.

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