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Fuzzy option value with stochastic volatility models

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Abstract

Uncertainty and vagueness play a central role in financial models and fuzzy numbers can be a profitable way to manage them. In this paper we generalize the Black and Scholes option valuation model (with constant volatility) to the framework of a volatility supposed to vary in a stochastic way. The models we take under consideration belongs to the main classes of stochastic volatility models: the endogenous and the exogenous source of risk. Fuzzy calculus for financial applications requires massive computations and when a good parametric representation for fuzzy numbers is adopted, then the arithmetic operations and fuzzy calculus can be efficiently managed.

Good in this context means that the shape of the resulting fuzzy numbers can be observed and studied in order to state fundamental properties of the model.

1 Introduction

The history of fuzzy numbers in finance starts with some very interesting papers like [1], [3] and [10] where the advantages and the critical aspects of the applications are approached.

In this paper we are mainly interested in the research of the option value when volatility is supposed to have a stochastic nature. In paper [5] the case of a plain vanilla has been analyzed, but now we take under consideration the stochastic volatility model deeply investigated in [4] and we define its fuzzy version. We believe that the introduction of fuzzy numbers helps to overcome some problems in reproducing empirical financial facts. Fuzzy numbers, in fact, are adopted in order to model the uncertainty about some key variables of financial models, and in particular of option pricing models with stochastic volatility.

We can take advantage of the parametric representation of fuzzy numbers (introduced in [11]) that is based on the use of parametrized monotonic functions to model the $\alpha - cuts$ or the membership functions. We call it the LU representation, as it models directly and works with the Lower and the Upper branches of the fuzzy numbers involved in the operations and in the fuzzy calculus. The LU-fuzzy numbers can also be viewed as a parametrized extension of the standard LR-fuzzy numbers and are related to this extension by a one-to-one correspondence. In [12] we show the advantages of the use of LU-fuzzy numbers in the principal applications of fuzzy calculus: they generalize the LR-fuzzy setting in the direction of the shape preservation but also they allow easy error-controlled approximations in fuzzy calculus.

A first attempt in the study of LU fuzzy numbers in volatility models is in [6] and now we extend the approach in a more specific way.

In section 2 we recall some fundamental properties of fuzzy calculus we shortly describe the LU model for fuzzy numbers trying to focus on its advantages.

The core of the paper is in section 3: we describe the stochastic volatility models and we investigate the most relevant properties of fuzzy stochastic differential equations in the framework of the LU parametrization. Section 4 collects some first attempt of empirical investigation; some final considerations and challenging observations conclude with section 5.

2 LU-fuzzy calculus

Fuzzy set theory has started by its invention due to Zadeh [13] in 1965. When an exact quantification of variables is not possible, fuzzy numbers represent a rigorous way to model such variables; properties of fuzzy numbers have been extensively studied since the pioneering contribution in [2] and the following [7] and [8].

The representation of fuzzy numbers is fundamental in what we are going to deal.

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Definition 1 *An LR (Left Right)-fuzzy quantity (number or interval) u has membership function of the form*

$$\mu_u(x) = \begin{cases} L(\frac{b-x}{b-a}) & \text{if } a \le x \le b \\ 1 & \text{if } b \le x \le c \\ R(\frac{x-c}{d-c}) & \text{if } c \le x \le d \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where $L, R : [0,1] \to [0,1]$ are two non-increasing shape functions such that $R(0) = L(0) = 1$ and $R(1) = L(1) = 0$. If $b = c$ we obtain a fuzzy number.

If L and R are invertible functions, then the $\alpha - cuts$ are obtained by

$$[u]_\alpha = [b - (b-a)L^{-1}(\alpha), c + (d-c)R^{-1}(\alpha)] \quad (2)$$

The usual notation for an LR-fuzzy quantity is $u = \langle a, b, c, d\rangle_{L,R}$ for an interval. We refer to functions $L(.)$ and $R(.)$ as the left and right branches (shape functions) of u, respectively.

On the other hand, the *level-cuts* of a fuzzy number are "nested" closed intervals and this property is the basis for the LU representation (L for lower, U for upper).

Definition 2 *An LU-fuzzy quantity (number or interval) u is completely determined by any pair $u = (u^-, u^+)$ of functions $u^-, u^+ : [0,1] \longrightarrow \mathbb{R}$, defining the endpoints of the $\alpha - cuts$, satisfying the three conditions: (i) $u^- : \alpha \longrightarrow u^-_\alpha \in \mathbb{R}$ is a bounded monotonic nondecreasing left-continuous function $\forall \alpha \in]0,1]$ and right-continuous for $\alpha = 0$; (ii) $u^+ : \alpha \longrightarrow u^+_\alpha \in \mathbb{R}$ is a bounded monotonic nonincreasing left-continuous function $\forall \alpha \in]0,1]$ and right-continuous for $\alpha = 0$; (iii) $u^-_\alpha \le u^+_\alpha$ $\forall \alpha \in [0,1]$.*

The support of u is the interval $[u^-_0, u^+_0]$ and the core is $[u^-_1, u^+_1]$. If $u^-_1 < u^+_1$ we have a fuzzy interval and if $u^-_1 = u^+_1$ we have a fuzzy number. We refer to the functions $u^-_{(.)}$ and $u^+_{(.)}$ as the lower and upper branches on u, respectively.

The obvious relation between u^-, u^+ and the membership function μ_u is

$$\mu_u(x) = \sup\{\alpha | x \in [u^-_\alpha, u^+_\alpha]\}. \quad (3)$$

In particular, if the two branches $u^-_{(.)}$ and $u^+_{(.)}$ are continuous invertible functions then $\mu_u(.)$ is formed by two continuous branches, the left being the increasing inverse of $u^-_{(.)}$ on $[u^-_0, u^-_1]$ and the right the decreasing inverse of $u^+_{(.)}$ on $[u^+_1, u^+_0]$.

The LR and the LU representations of fuzzy numbers require to use appropriate (monotonic) shape functions to model either the left and right branches of the membership function or the lower and upper branches of the $\alpha - cuts$.

The basic elements of a parametric representation of the shape functions are introduced in [12] and [11]: the parametric representations, based on monotonic Hermite-type interpolation, can be used both to define the shape functions and to calculate the arithmetic operations by error controlled approximations.

To model the monotonic branches u^-_α and u^-_α we start with an increasing shape function p such that $p(0) = 0$ and $p(1) = 1$ and a decreasing function q such that $q(0) = 1$ and $q(1) = 0$, with the four numbers $u^-_0 \le u^-_1 \le u^+_1 \le u^+_0$ defining the support $\left[u^-_0, u^+_0\right]$ and the core $\left[u^-_1, u^+_1\right]$ and we define

$$\begin{aligned} u^-_\alpha &= u^-_1 - (u^-_1 - u^-_0)p(\alpha) \text{ and} \\ u^+_\alpha &= u^+_1 - (u^+_1 - u^+_0)q(\alpha) \text{ for all } \alpha \in [0,1]. \end{aligned} \quad (4)$$

The two shape functions p and q are selected in a family of parametrized monotonic functions where the parameters are related to the first derivatives of p and q in 0 and 1; there are many ways to define p and q. The use of the mentioned parametrization allows easy arithmetic operations. In cases where u^-_α and u^+_α are required to be more flexible than a single shape function, we can always proceed piecewise over a decomposition of the interval $[0,1]$ into N sub-intervals $[\alpha_{i-1}, \alpha_i]$ for $i = 1, 2, ..., N$. For each decomposition we require (in the differentiable case) $4(N+1)$ parameters $u = (\alpha_i; u^-_i, \delta u^-_i, u^+_i, \delta u^+_i)_{i=0,1,...,N}$ satisfying the following conditions for the data and the slopes:

$$\begin{aligned} u^-_0 &\le u^-_1 \le ... \le u^-_N \le u^+_N \le u^+_{N-1} \le ... \le u^+_0 \\ \delta u^-_i &\ge 0, \delta u^+_i \le 0. \end{aligned} \quad (5)$$

and on each sub-interval $[\alpha_{i-1}, \alpha_i]$ we use the data $u^-_{i-1} \le u^-_i \le u^+_i \le u^+_{i-1}$ and the slopes $\delta u^-_{i-1}, \delta u^-_i \ge 0$ and $\delta u^+_{i-1}, \delta u^+_i \le 0$. In this way we can obtain a wide set of fuzzy numbers.

The simplest representation is obtained on the trivial decomposition of the interval $[0,1]$, with $N = 1$ (without internal points) and $\alpha_0 = 0, \alpha_1 = 1$. In this simple case, u can be represented by a vector of 8 components

$$u = (u^-_0, \delta u^-_0, u^+_0, \delta u^+_0; u^-_1, \delta u^-_1, u^+_1, \delta u^+_1) \quad (6)$$

where $u^-_0, \delta u^-_0, u^-_1, \delta u^-_1$ are used for the lower branch u^-_α, and $u^+_0, \delta u^+_0, u^+_1, \delta u^+_1$ for the upper branch u^+_α.

For $N \ge 1$, an array of $4(N+1)$ parameters is available for the lower branch u^-_α (monotonic increasing) and the upper branch u^+_α (monotonic decreasing).

The set of LU-fuzzy numbers (for a fixed monotonic-shaped model) is denoted by

$$F_N = \left\{(u^-_i, \delta^-_i, u^+_i, \delta^+_i)_{i=0,1,...,N} \mid u^-_i \nearrow, u^+_i \searrow, \delta^-_i \ge 0, \delta^+_i \le 0\right\}.$$

The arithmetic operations, the Zadeh's fuzzy extensions, the fuzzy integral and derivative and other elements of fuzzy calculus can be defined in F_N.

As an example, the fuzzy multiplication is obtained by a relatively simple algorithm: denote $uv = w = \left(w_i^-, f_i^-, w_i^+, f_i^+\right)_{i=0,1,\dots,N}$, and

$$(uv)_i^- = \min\{u_i^- v_i^-, u_i^- v_i^+, u_i^+ v_i^-, u_i^+ v_i^+\} \tag{7}$$

$$(uv)_i^+ = \max\{u_i^- v_i^-, u_i^- v_i^+, u_i^+ v_i^-, u_i^+ v_i^+\}; \tag{8}$$

let (p_i^-, q_i^-) be the pair associated to the combination of + and - of $u_i^\pm v_i^\pm$ giving the minimum for $(uv)_i^-$ in (7), and similarly let (p_i^+, q_i^+) be the pair associated to the combination of + and - of $u_i^\pm v_i^\pm$ giving the maximum for $(uv)_i^+$ in (8), then (for the values and the slopes of $w = uv$)

$$\begin{cases} w_i^- = u_i^{p_i^-} v_i^{q_i^-} \ , \ w_i^+ = u_i^{p_i^+} v_i^{q_i^+} \\ f_i^- = \delta_i^{p_i^-} v_i^{q_i^-} + u_i^{p_i^-} e_i^{q_i^-} \ , \ f_i^+ = \delta_i^{p_i^+} v_i^{q_i^+} + u_i^{p_i^+} e_i^{q_i^+}. \end{cases}$$

Analogous simple rules are valid for the other operations.

In [12] we show that LU-fuzzy numbers generalize the LR-fuzzy setting in the direction of the shape preservation but also they allow easy error-controlled approximations in fuzzy calculus. We provide an extensive list of applications of LU-fuzzy numbers and we show the computational advantages associated to their adoption. It is argued that they are very flexible and rich in the shapes that are representable and it is extremely easy to implement algorithms of fuzzy calculus that reproduce shape-preserved results.

3 Fuzzy stochastic differential equations

We believe that the study of fuzzy stochastic differential equation of the most general form have a lot to say in terms of modelling of financial objects, and that the LU-parametric representation can simplify the numerical approximation of the solution.

In [6] we show some examples of fuzzy random variables that have properties not far from those of the Brownian motion and that suggest a link between stochastic differential equations and fuzzy numbers.

We refer to [9] for additional references about stochastic integrals.

Given a Brownian motion B_t and two fuzzy functions defined as follows:

$$G(t,u) : \mathbb{R}^+ \times \mathbb{F}_\mathbb{X} \longrightarrow \mathbb{F}_\mathbb{X}$$

$$F(t,u) : \mathbb{R}^+ \times \mathbb{F}_\mathbb{X} \longrightarrow \mathbb{F}_\mathbb{X}$$

the following FSDE can be defined

$$\begin{cases} dX_t &= F(t, X_t)\,dt + G(t, X_t)\,dB_t \\ X_0 &= x_0 \in \mathbb{F}_\mathbb{X}(\omega) \end{cases}$$

and it can be written in the equivalent integral form:

$$X_t = X_0 + \int_0^t F(s, X_s)\,ds + \int_0^t G(s, X_s)\,dB_s \tag{9}$$

where the first one is an ordinary fuzzy integral and the second one is a stochastic fuzzy integral.

In terms of the $\alpha - cuts$ the FSDE in (9) can be written as follows:

$$[X_t]_\alpha = [X_0]_\alpha + \left[\int_0^t [F(s, X_s)]_\alpha^- \,ds, \int_0^t [F(s, X_s)]_\alpha^+ \,ds\right] +$$

$$+ \left[\int_0^t [G(s, X_s)]_\alpha^- \,dB_s, \int_0^t [G(s, X_s)]_\alpha^+ \,dB_s\right]$$

or in the equivalent form ($\forall \omega \in \Omega$ and $\forall \alpha \in [0,1]$)

$$\begin{cases} x_{t,\alpha}^-(\omega) = x_{0,\alpha}^-(\omega) + \displaystyle\int_0^t F(s, x_s(\omega))_\alpha^- \,ds + \int_0^t G(s, x_s(\omega))_\alpha^- \,dB_s \\ x_{t,\alpha}^+(\omega) = x_{0,\alpha}^+(\omega) + \displaystyle\int_0^t F(s, x_s(\omega))_\alpha^+ \,ds + \int_0^t G(s, x_s(\omega))_\alpha^+ \,dB_s \end{cases} \tag{10}$$

Consider now an LU-model for the fuzzy numbers in (10):

$$u(t) = \left(\alpha_i; u_i^-(t), \delta_i^-(t), u_i^+(t), \delta_i^+(t)\right)_{i=0,1,\dots,N}$$

based on the $\alpha-$decomposition

$$0 = \alpha_0 < \alpha_1 < \dots < \alpha_N = 1$$

then it is possible to write, for $i = 0, 1, \dots, N$

$$\begin{cases} x_{t,i}^-(\omega) = x_{0,i}^-(\omega) + \displaystyle\int_0^t F(s, [x_s]_i)_i^- \,ds + \int_0^t G(s, [x_s]_i)_i^- \,dB_s \\ x_{t,i}^+(\omega) = x_{0,i}^+(\omega) + \displaystyle\int_0^t F(s, [x_s]_i)_i^+ \,ds + \int_0^t G(s, [x_s]_i)_i^+ \,dB_s \end{cases} \tag{11}$$

and if

$$[x_s]_i = \left[x_{s,i}^-, x_{s,i}^+\right]$$

is the $\alpha_i - cut$ interval of the fuzzy number x_s then

$$\delta F(s, [x_s]_i)_i^-, \delta F(s, [x_s]_i)_i^+$$

and

$$\delta G\left(s,[x_s]_i\right)_i^-, \delta G\left(s,[x_s]_i\right)_i^+$$

are the slopes corresponding to $F\left(s,[x_s]_i\right)_i^-, F\left(s,[x_s]_i\right)_i^+$ and $G\left(s,[x_s]_i\right)_i^-, G\left(s,[x_s]_i\right)_i^+$ respectively, we also have

$$\begin{cases} \delta x_{t,i}^-(\omega) = \delta x_{0,i}^-(\omega) + \\ +\int\limits_0^t \delta F\left(s,[x_s]_i\right)_i^- ds + \int\limits_0^t \delta G\left(s,[x_s]_i\right)_i^- dB_s \\ \delta x_{t,i}^+(\omega) = \delta x_{0,i}^+(\omega) + \\ +\int\limits_0^t \delta F\left(s,[x_s]_i\right)_i^+ ds + \int\limits_0^t \delta G\left(s,[x_s]_i\right)_i^+ dB_s \end{cases} \tag{12}$$

The differential form for (11) and (12) can be written as follows:

$$\begin{cases} dx_{t,i}^- = & F\left(t,[x_t]_i\right)_i^- dt & +G\left(t,[x_t]_i\right)_i^- dB_t \\ dx_{t,i}^+ = & F\left(t,[x_t]_i\right)_i^+ dt & +G\left(t,[x_t]_i\right)_i^+ dB_t \\ d\left(\delta x_{t,i}^-\right) = & \delta F\left(t,[x_t]_i\right)_i^- dt & +\delta G\left(t,[x_t]_i\right)_i^- dB_t \\ d\left(\delta x_{t,i}^+\right) = & \delta F\left(t,[x_t]_i\right)_i^+ dt & +\delta G\left(t,[x_t]_i\right)_i^+ dB_t \end{cases}$$

$$i = 0, 1, ..., N$$

They are $4(N+1)$ (crisp) stochastic differential equations that are independent for different values of i (different $\alpha_i - cuts$) and can be solved by using ordinary numerical SDE-solvers.

Some conditions have to be satisfied in order to obtain a fuzzy solution of the system.

Euler scheme is the simplest strong Taylor approximation. Given an Ito process:

$$X = \{X_t, t_0 \le t \le T\} \tag{13}$$

that satisfies the scalar SDE in the general form:

$$dX_t = b\left(t, X_t\right) dt + \sigma\left(t, X_t\right) dB_t \tag{14}$$

on the time interval $t_0 \le t \le T$, with initial value $X_{t_0} = X_0$ and given a discretization:

$$t_0 = \tau_0 < \tau_1 < \ldots < \tau_n < \ldots < \tau_N = T \tag{15}$$

of the time interval $[t_0, T]$, an Euler approximation is a stochastic process in continuous satisfying the iterative scheme:

$$\begin{aligned} Y_{n+1} &= Y_n + b\left(\tau_n, Y_n\right)\left(\tau_{n+1} - \tau_n\right) + \\ &\quad +\sigma\left(\tau_n, Y_n\right)\left(B_{\tau_{n+1}} - B_{\tau_n}\right) \\ y_0 &= x_0 \end{aligned} \tag{16}$$

for $n = 0, 1, 2, \ldots, N-1$

In general the time discretization points are considered equidistant:

$$\begin{aligned} \tau_n &= t_0 + n\Delta, \\ \Delta &= \frac{(T - t_0)}{N} \end{aligned}$$

A linear interpolation is often adopted to connect the approximation points computed via Euler scheme.

The random increments of the Brownian motion are obtained as usual by generating a sequence of pseudo-random numbers.

Euler scheme has strong order of convergence $\gamma = 0.5$ and a weak order of convergence $\beta = 1$. Euler scheme produces good numerical results when the drift and diffusion coefficients have good properties, but the use of higher order schemes is often recommended.

Milstein scheme can be written by adding the following term of the Ito-Taylor expansion:

$$\frac{1}{2}\sigma\sigma'\left\{\left(\triangle B_n\right)^2 - \triangle_n\right\} \tag{17}$$

so that the iterative scheme takes the form:

$$Y_{n+1} = Y_n + b\Delta_n + b\Delta B_n + +\frac{1}{2}\sigma\sigma'\left\{\left(\triangle B_n\right)^2 - \triangle_n\right\} \tag{18}$$

Under the hypothesis of continuity of the first derivative of drift coefficient b and of continuity of the first and second order derivative of diffusion coefficient, it is possible to show that Milstein scheme has a strong order of convergence equals to 1.0. So by adding only one term of the Ito-Taylor expansion is possible to augment the order from 0.5 to 1.0.

4 Financial Examples

Fuzzy modelling adds a source of uncertainty to the classical stochastic modelling of volatility and option value.

Option pricing models in continuous time are generally introduced in a simple setting in which only one risky asset (a stock) is available. They describe the stock price dynamics by means of a stochastic differential equation (SDE) of the following type:

$$d\ln S_t = \alpha(t, S_t)dt + \sigma(t, S_t)dB_t \tag{19}$$

where S_t is the stock price at time t, α is the drift function and σ is the so called volatility function of the price process. Each model is characterized by the functional form of α and σ; for example, in Black and Scholes model, α and σ are constant parameters. Note that in most cases the stochastic differential equation in (19) is autonomous (i.e. α and σ do not depend explicitly on time).

The models which take into account random volatility can be divided into two broad classes: the level dependent, where volatility is considered as a function of the stock price level S_t (i.e. $\sigma(t,S_t)=\sigma(S_t)$), and the exogenous stochastic volatility (SV), where another SDE is introduced, driven by a second source of risk, to describe the randomness of the volatility process.

In the assumption of a volatility that evolves as a stochastic process, the standard framework assumes a bivariate diffusion process in which the processes of the underlying asset S and of the volatility σ have to be jointly specified. In [4] we study the capability of Hobson and Rogers model (more details in [**?**]) to capture option price. Hobson and Rogers model can be viewed as a good compromise between the level-dependent and the SV approaches; it suggests that the random volatility σ may depend on the entire path of the stock price S_t:

$$\sigma_t=\sigma(S_u,u\le t).$$

This assumption allows for more flexibility in the volatility process, without the need of a new source of randomness.
More precisely, Hobson and Rogers specify the volatility as a function of a vector of state variables $\left(X_t^{(1)},X_t^{(2)},...,X_t^{(d)}\right)$, which are called offsets, defined as follows:

$$X_t^{(m)}=\int_0^\infty \nu e^{-\nu u}\left(\ln\left(\frac{e^{-rt}S_t}{e^{-r(t-u)}S_{t-u}}\right)\right)^m du \qquad (20)$$

where ν is a positive constant parameter referred to as the feedback parameter.
The state variables in (20) represent the exponentially weighted moments of the historical logreturns according to different time scales. In practice the logreturn for the time interval $(t-u,t)$ becomes less significant in the definition of the offsets as long as the time-lag u increases; the feedback parameter ν represents the rate at which past information on logreturns is actually discounted.
The integral in (20) can be simplified as follows when the order of offsets, as the authors suggest, is considered equal to one:

$$\begin{aligned}X_t &= \int_0^\infty \nu e^{-\nu u}\ln\left(\frac{e^{-rt}S_t}{e^{-r(t-u)}S_{t-u}}\right)du=\\ &= \int_0^\infty \nu e^{-\nu u}\left[-rt+\ln S_t+r\left(t-u\right)-\ln S_{t-u}\right]du=\end{aligned}$$

$$=\ln S_t\int_0^\infty \nu e^{-\nu u}du-r\int_0^\infty \nu u e^{-\nu u}du-\int_0^\infty \nu e^{-\nu u}\ln S_{t-u}du \qquad (21)$$

Due to the fact that the first ad second integral in (21) have values respectively 1 and $\frac{1}{\nu}$and that the third integral in (21) can be approximated by a Gauss-Laguerre quadrature formula , after the transformation $x=\nu u$ it follows:

$$\int_0^\infty \nu e^{-\nu u}\ln S_{t-u}du=\int_0^\infty e^{-x}\ln S_{t-\frac{x}{\nu}}dx\cong\sum_{i=1}^n w_i\ln S_{t-\frac{x_i}{\nu}}$$

where (w_i,x_i) can be given in a table for $n=5,10,15,20,...$
The offset in (20) assumes then the following approximating form:

$$X_t\cong\ln S_t-\frac{r}{\nu}-\sum_{i=1}^n w_i\ln S_{t-\frac{x_i}{\nu}} \qquad (22)$$

The risk neutral dynamics in Hobson and Rogers model is described as follows:

$$\begin{aligned}d\ln S_t &= rdt+\sigma(X_t)dB_t\\ \sigma\left(X_t\right) &= \min\left\{\eta\sqrt{1+\epsilon X_t^2},M\right\}\end{aligned}$$

where M is a constant which is introduced in order to avoid the explosion of the diffusion X_t.

Hobson and Rogers explain that this simple setting accounts for the possibilities of smiles and skews in the implied volatility structure; in particular, the size of the smile in the term structure of volatility is directly related to the parameter ϵ and inversely to the parameter ν (larger values of ν are associated with a shorter half-life lookback period). We notice that an exact analytical pricing formula is not available for options for this model specification.

The fuzzy extension of the two logaritmic terms in (22) is simple to obtain due to the monotonicity of the function $x\to\ln x$.

In this context we are not interested in the calibration of the model, we take parameters as constant values and we simulate the fuzzy version of the model.

The key parameters of the model are: the feedback parameter ν and the two volatility parameters η and ϵ. Their value is not known and we assume them to be triangular (linear shaped) symmetric fuzzy numbers, centered at the crisp values and with the support being the interval $[crisp-0.1crisp,crisp+0.1crisp]$, corresponding to a symmetric uncertainty of 10% of the values of the parameters.

The properties of the LU parametric representation make possible the implementation of the stochastic volatility model because arithmetic operations and required fuzzy calculus can be efficiently solved. The crisp values for the parameters ν, η and ϵ vary depending on the data set considered for the research of option prices. Including fuzziness avoids the calibration of the parameters that has shown to have some critical aspects in robustness.

We test the pricing performance of the fuzzy Hobson and Rogers model on the S&P500 options, using the same data as in [4]. The crisp value for the feedback parameter ν is chosen equal to 12.6, obtained with the econometric procedure described in [4]. The crisp values for the volatility parameters are again estimated by simulating the volatility behavior of real data.

The call option price is a nonlinear fuzzy number; in fact, due to the calculus involved, the linear input produce an output with a not trivial shape to be analyzed.

5 Conclusions

We study the peculiarities of LU parametric representation in the fuzzy version of the Hobson and Rogers stochastic volatility model. In details we show the advantages of LU-fuzzy numbers when finding the call option price where some input variables are taken as fuzzy numbers.

This paper starts a research area that we believe will become fertile in the scenario of financial models; we are working on the numerical solution of fuzzy stochastic differential equations and we observe that the desirable properties of the LU representation improve the performance of the scheme. It is well known how large is the field of financial applications of SDE and how relevant can be the introduction of uncertainty variables in many models.

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