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Exploiting a New Interpretability Index in the Multi-Objective Evolutionary Learning of Mamdani Fuzzy Rule-based Systems

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Abstract— **In this paper, we introduce a new index for evaluating the interpretability of Mamdani fuzzy rule-based systems (MFRBSs). The index takes both the rule base complexity and the data base integrity into account. We discuss the use of this index in the multi-objective evolutionary generation of MFRBSs with different trade-offs between accuracy and interpretability. The rule base and the membership function parameters of the MFRBSs are learnt concurrently by exploiting an appropriate chromosome coding and purposely-defined genetic operators. Results on a real-world regression problem are shown and discussed.**

Keywords- Mamdani Fuzzy Rule-based Systems; Multi-objective Evolutionary Algorithms; Interpretability Index; Accuracy-Interpretability trade-off; Piecewise Linear Transformation.

I. INTRODUCTION

Interpretability of Mamdani fuzzy rule-based systems (MFRBSs) is still an open issue in the fuzzy modeling community. As discussed in [1][2], there is no general agreement on a formal definition of interpretability and therefore there exists a real difficulty in formulating a measure of interpretability of an MFRBS. In general, intuitively and informally, we can state that an MFRBS is interpretable when we are able to understand how it manages to exploit the knowledge contained both in the rule base (RB) and in the data base (DB), to infer conclusions from facts.

A common approach is to distinguish between interpretability of the RB, also known as complexity, and interpretability of fuzzy partitions, also known as integrity of the DB [3]. Complexity is usually defined in terms of simple measures, such as number of rules in the RB and number of linguistic terms in the antecedent of rules [4]-[6]. On the other hand, integrity depends on some properties of the fuzzy partitions, such as coverage, distinguishability and normality, which may be difficult to measure [3].

Recently, some interpretability indices have been proposed in the specialized literature. For example, in [1] a set of heuristics for assessing the interpretability of MFRBSs are implemented in a fuzzy rule-based system, while in [2] a partition integrity index for context adaptation applications is proposed.

Preserving a high interpretability while increasing the accuracy of MFRBSs is not an easy task: indeed, these two features are in conflict between them. In the last years, multi-objective evolutionary algorithms (MOEAs) have been widely used to generate sets of MFRBSs with different trade-offs between accuracy and interpretability [7].

Several approaches have been proposed to learn the RB using a predefined DB [5][8]. Further, in [9] and [10] the RB is learnt concurrently with the membership function (MF) parameters and the partition granularities, respectively. MOEAs have been also exploited in [6][11] to perform concurrently rule selection and tuning of the DB and in [2] to adapt the DB to a specific context. Finally, in [12][13], authors propose two approaches to perform a tuning of the MFs while evolving the antecedents of the initial RB. However, none of these approaches takes RB and DB interpretability into account at the same time.

In this paper, we introduce a novel and simple interpretability index which takes both the partition integrity and the RB complexity into consideration. We exploit this index in a multi-objective evolutionary framework to learn concurrently the RB and the membership function (MF) parameters of MFRBSs. MF parameter learning is performed by using a piecewise linear transformation [14] which allows us to obtain a high modeling capability with a limited number of parameters.

Results of the application of our approach to a real world regression problem are shown and discussed. In particular, we highlight how our approach on average generates Pareto fronts with solutions characterized by good trade-offs between accuracy, RB complexity and DB integrity. We have also performed simulations exploiting the same MOEA to learn concurrently the RB and the MF parameters of MFRBSs, minimizing only the complexity without considering the partition integrity. Pareto fronts obtained with the two approaches are almost similar in terms of accuracy, but solutions generated by our approach are characterized on average by a higher partition integrity and lower complexity.

The paper is organized as follows: in Section II we briefly describe the MFRBSs. Section III introduces the technique to perform the learning of the MF parameters. In Section IV, we define our interpretability index. Section V describes the multi-objective evolutionary approach, including the chromosome coding, the fitness function and the genetic operators, used to generate the MFRBSs. Finally, Section VI shows the experimental results and Section VII draws some final conclusions.

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II. MAMDANI FUZZY RULE-BASED SYSTEMS

Let $\mathbf{X} = \{X_1,...,X_f,...,X_F\}$ be the set of input variables and X_{F+1} be the output variable. Let U_f, with $f = 1,...,F+1$, be the universe of the f^{th} variable. Let $P_f = \{A_{f,1},...,A_{f,T_f}\}$ be a fuzzy partition of T_f fuzzy sets on variable X_f. An MFRBS is composed of M rules expressed as:

$$R_m: \text{ IF } X_1 \textbf{ is } A_{1,j_{m,1}} \text{ AND } \dots \text{ AND } X_F \textbf{ is } A_{F,j_{m,F}}$$
$$\text{THEN } X_{F+1} \textbf{ is } A_{F+1,j_{m,F+1}} \tag{1}$$

where $j_{m,f} \in [1, T_f]$ identifies the index of the fuzzy set (among the T_f fuzzy sets of partition P_f), which has been selected for X_f in rule R_m.

We adopt triangular fuzzy sets $A_{f,j}$ defined by the tuple ($a_{f,j}, b_{f,j}, c_{f,j}$), where $a_{f,j}$ and $c_{f,j}$ correspond to the left and right extremes of the support of $A_{f,j}$, and $b_{f,j}$ to the core. Further, we assume that $a_{f,1} = b_{f,1}$, $b_{f,T_f} = c_{f,T_f}$, and for $j = 2 \dots T_f - 1$, $b_{f,j} = c_{f,j-1}$ and $b_{f,j} = a_{f,j+1}$.

To take the "don't care" condition into account, a new fuzzy set $A_{f,0}$ ($f = 1,...,F$) is added to all the F input partitions P_f. This fuzzy set is characterized by a membership function equal to 1 on the overall universe [15].

The terms $A_{f,0}$ allow generating rules which contain only a subset of the input variables. It follows that $j_{m,f} \in [0, T_f]$, $f = 1,...,F$, and $j_{m,F+1} \in [1, T_{F+1}]$. Thus, an MFRBS can be completely described by a matrix $J \in \mathbb{N}^{M \times (F+1)}$ [5], where the generic element (m, f) indicates that fuzzy set $A_{f,j_{m,f}}$ has been selected for variable X_f in rule R_m. We adopt the product and the weighted average method as AND logical operator and defuzzification method, respectively.

Given a set of N input observations $\mathbf{x}_n = [x_{n,1},...,x_{n,F}]$, with $x_{n,f} \in \Re$, and the set of the corresponding outputs $x_{n,F+1} \in \Re$, $n = 1,...,N$, we apply an MOEA which generates a set of MFRBSs with different trade-offs among accuracy, complexity and integrity by learning simultaneously the RB and the MF parameters.

III. MF PARAMETERS LEARNING

We approach the problem of learning the MF parameters by using a piecewise linear transformation [14]. The transformation is described in Fig. 1 for a generic variable X_f. In the following, we assume that the interval ranges of the original and transformed variables are identical. Further, we consider each variable normalised in [0,1].

Let $t(x_f)$ be the piecewise linear transformation. We have that $A_{f,j}(x_f) = \tilde{A}_{f,j}(t(x_f)) = \tilde{A}_{f,j}(\tilde{x}_f)$, where $\tilde{A}_{f,j}$ and $A_{f,j}$ are two generic fuzzy sets from the uniform and non-uniform fuzzy partitions, respectively. In those regions where $t(x_f)$ has a high value of the derivative (high slope of the lines), the fuzzy sets $A_{f,j}$ are narrower; otherwise, the fuzzy sets $A_{f,j}$ are wider. To preserve the shape of the MFs, we force the change of slopes in $t(x_f)$ to coincide with the cores of the fuzzy sets in the partitions.

Let $b_{f,1},...,b_{f,T_f}$ and $\tilde{b}_{f,1},...,\tilde{b}_{f,T_f}$ be the cores of $A_{f,1},...,A_{f,T_f}$ and $\tilde{A}_{f,1},...,\tilde{A}_{f,T_f}$, respectively. Transformation $t(x_f)$ can be defined for $j = 2 \dots T_f$ as:

$$t(x_f) = \frac{\tilde{b}_{f,j} - \tilde{b}_{f,j-1}}{b_{f,j} - b_{f,j-1}}(x_f - b_{f,j-1}) + \tilde{b}_{f,j-1} \quad b_{f,j-1} \le x_f < b_{f,j}.$$

The cores $\tilde{b}_{f,1},...,\tilde{b}_{f,T_f}$ are fixed and therefore known. Further, $b_{f,1}$ and b_{f,T_f} coincide with the extremes of the universe U_f of X_f. Thus, $t(x_f)$ depends on $T_f - 2$ parameters, that is, $t(x_f; b_{f,2},...,b_{f,T_f-1})$. Once fixed $b_{f,2},...,b_{f,T_f-1}$, the partition $P_f = \{A_{f,1},...,A_{f,T_f}\}$ can be obtained simply by transforming the three points ($\tilde{a}_{f,j}, \tilde{b}_{f,j}, \tilde{c}_{f,j}$), which describe the generic fuzzy set $\tilde{A}_{f,j}$ into ($a_{f,j}, b_{f,j}, c_{f,j}$) applying $t^{-1}(\tilde{x}_f)$.

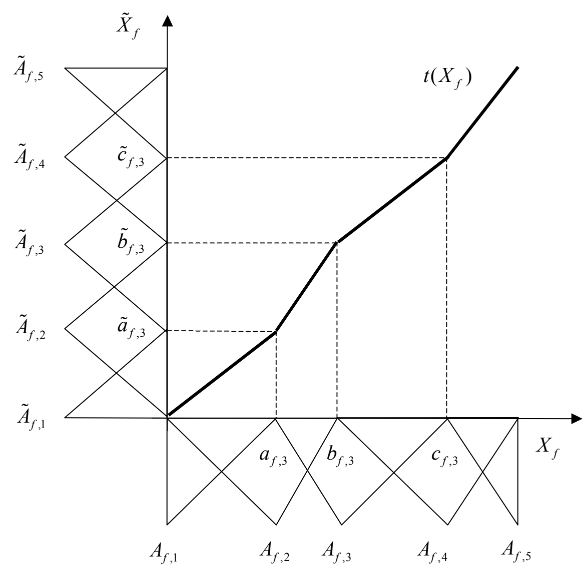

Figure 1. An example of piecewise linear transformation.

IV. THE NEW INTERPRETABILITY INDEX

The interpretability of an MFRBS relies mainly on the simplicity of the fuzzy RB and on the integrity of the fuzzy partitions [15].

To ensure the RB simplicity, both the number of fuzzy rules and the number of antecedent conditions should be maintained low. To this aim, in [5][8]-[10], authors have used a complexity measure defined as the total sum of the conditions in the antecedent of the rules during the evolutionary approach.

Further, as discussed in [3], the partition integrity depends on some properties of the fuzzy partitions such as granularity (i.e., the number of fuzzy sets), normality, coverage, distinguishability and ordering.

According to psychologists, the granularity of each linguistic variable should not be higher than 9 due to a limit of human information processing capability [1]. In the experiments, we choose to evaluate the performance of our approach setting granularity $T_f = T = 5$.

We start from uniform partitions composed of normal triangular fuzzy sets. We observe that the piecewise linear transformation preserves the normality. Finally, distinguishability and coverage are fully satisfied when partitions are uniform. On the other hand, the piecewise linear transformation tends to increase accuracy by adapting the MFs to the specific application context. Often, the MF adaptation process generates partitions which are quite far from being uniform, thus loosing in interpretability: the more the partition is different from a uniform partition, the less the partition is interpretable.

To control the partition distinguishability in the evolutionary learning of the MF parameters, we introduce, for each variable X_f, the following dissimilarity measure $d_f = \sum_{j=2}^{T-1} \left| b_{f,j} - \tilde{b}_{f,j} \right|$. Since the piecewise linear transformation only moves the cores and the extremes of the fuzzy sets without deforming their shapes, d_f can be considered a suitable measure for evaluating how much a partition generated by the MF parameter learning is different from the uniform partition.

In order to take both the DB integrity and the RB complexity into account, we define the following index:

$$I = \sum_{m}^{M} \sum_{f}^{F+1} (1 + d_f) \cdot u(j_{m,f}) \tag{2}$$

where $u(j_{m,f}) = \begin{Bmatrix} 1 \ if \ j_{m,f} > 0 \\ 0 \ if \ j_{m,f} = 0 \end{Bmatrix}$.

The value of this index increases with the increasing of the number of rules and the number of antecedent conditions in the rules, and with the increasing of the values of dissimilarity d_f between the actual and the uniform partitions for each linguistic variable X_f. Thus, the higher the value of the index, the lower the MFRBS interpretability. If each linguistic variable is uniformly partitioned, then the values of all the dissimilarity measures d_f are equal to zero and therefore the value of the index coincides with the sum of the number of rules and of the complexity index defined in [5].

We note that, since the RB cannot be composed by rules with no condition in the antecedents, index I can never be equal to zero. From simple mathematical considerations, we derive that $2M_{\min} \le I \le M_{\max}(F+1) \cdot [1 + \frac{1}{2}(T-2)]$, where $M_{\min}$ and $M_{\max}$ are the possible minimum and maximum numbers of rules. Based on index I, we introduce the following index μ (*interpretability index*) to globally evaluate the interpretability of a knowledge base of an MFRBS:

$$\mu = 1 - \frac{I - 2M_{\min}}{M_{\max}(F+1) \cdot [1 + \frac{1}{2}(T-2)] - 2M_{\min}}.$$

Index μ varies from 0 (minimum level of interpretability) to 1 (maximum level of interpretability). The maximum value corresponds to an RB composed by the minimum number of rules with only one condition in the antecedent and to a DB with uniform partitions for each linguistic variable.

To increase interpretability, that is, to enhance partition integrity and to reduce complexity, and to increase accuracy are often conflicting objectives. Thus, we approach the generation of MFRBSs by using a two-objective evolutionary algorithm, where the two objectives are the MSE computed as in [10] and the interpretability index μ defined in (2), respectively.

V. THE MULTI-OBJECTIVE EVOLUTIONARY APPROACH

We adopt the (2+2)M-PAES proposed in [5]. Each solution is codified by a chromosome C composed of two parts (C_1, C_2), which define the RB and the piecewise linear transformations of all the variables, respectively. C_1 codifies matrix J described in [5] and is composed of $M \cdot (F+1)$ natural numbers where M is the number of rules currently present in the RB. C_2 is a vector containing $F+1$ vectors of $T-2$ real numbers: the f^{th} vector contains the $\left[b_{f,2}, ..., b_{f,T_f - 1} \right]$ points which define the piecewise linear transformation for the linguistic variable X_f.

In order to generate the offspring populations, we exploit both crossover and mutation. We apply the one-point crossover defined in [5] to C_1 and the BLX-α crossover, with α = 0.5, to C_2. Possibly, we reorder the cores so as to preserve the label ordering. To constrain the search space, we fix $M_{\min}$ and $M_{\max}$ to 5 and 50, respectively. The crossover is applied with probability 0.5.

As regards mutation, we apply for C_1 the two mutation operators described in [5]. If the crossover is not applied, the mutation is always applied to C_1; otherwise the mutation is applied with probability 0.2. The two mutation operators are applied with probabilities 0.55 and 0.45, respectively. The mutation applied to C_2 first chooses randomly a variable $f \in [1, F+1]$, then extracts a random value $j \in [2, T_f - 1]$ and changes the value of $b_{f,j}$ to a random value in the interval $\left[b_{f,j-1}, b_{f,j+1}\right]$. The probability of applying the mutation to C_2 is 0.2.

VI. Experimental Results

We tested our approach on the real world regression problem described in [16] that consists of estimating the maintenance costs of medium voltage lines in a town. The data set contains 1059 patterns (4 input and 1 output variables). In order to assess the reliability of our approach, we performed a five-fold cross-validation, using each fold six times with different seeds for the random function generator (thirty trials in total). We fixed the archive size and the maximum number of iterations to 64 and 300,000, respectively.

Fig. 2 shows an example of the Pareto fronts achieved by the algorithm on the training and test sets, respectively. As expected, we can observe that, when the accuracy increases, our interpretability index decreases (indeed, the complexity of the rule base and the dissimilarities d_f increase).

To assess the advantages of exploiting our interpretability index, we compared the results achieved by our approach with the results obtained by applying the (2+2)M-PAES to minimize only the RB complexity, together with the MSE, without considering the partition integrity. We denote these two approaches as PAES-SFI and PAES-SF, respectively.

To perform the comparison statistically and not on a single trial, we exploit the idea of average Pareto fronts. These fronts are obtained as follows. First, the solutions in the Pareto front approximations produced on the training set in each of the thirty trials are ordered for increasing MSE values. Then, the corresponding solutions are averaged on the thirty Pareto front approximations.

We plot for both PAES-SFI and PAES-SF the twenty solutions with the lowest MSEs (the choice of considering only the twenty solutions with the lowest MSEs was motivated by the observation that the other solutions are in general characterized by quite high MSEs which make these solutions impractical).

Figure 3 shows the average Pareto fronts achieved by the two algorithms, on the training and test sets, in the complexity-MSE plane. The complexity is measured as in [5]. We can observe that the average Pareto fronts generated by PAES-SFI dominate the average Pareto fronts generated by PAES-SF, both on training and test sets.

In Table I we show the average MSEs corresponding to three representative points of the average Pareto fronts: the first (the most accurate), the median and the last (the least complex) point. We refer to these average values as First, Median and Last, respectively.

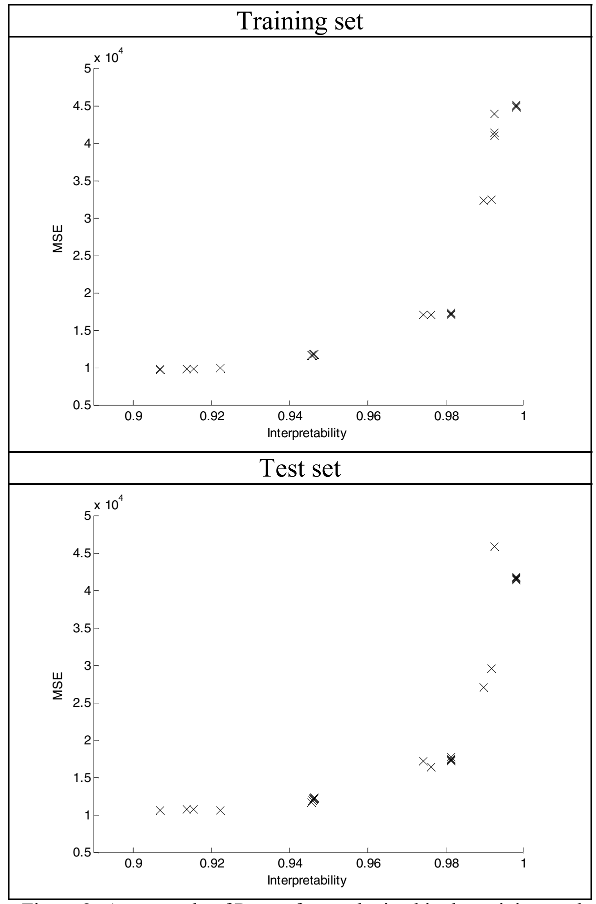

Figure 2. An example of Pareto fronts obtained in the training and test sets.

Table I. Average MSEs on training and test sets

	MSE_{TR}	$t\text{-}t_{TR}$	MSE_{TS}	$t\text{-}t_{TS}$
First				
PAES-SF	13345±4221	=	16180±6614	=
PAES-SFI	**13282±2593**	*	**15484±4248**	*
Median				
PAES-SF	14072±4716	=	16991±7015	=
PAES-SFI	**13813±2702**	*	**16081±4434**	*
Last				
PAES-SF	18266±13853	=	20972±16939	=
PAES-SFI	**16290±4423**	*	**18178±5187**	*

We verified that the MSE distributions generated with the thirty trials, both on training and test sets, can be considered as normal distributions. On the basis of this assumption, in order to assess whether the differences between the solutions are statistically significant, we applied the t-student test with 95% confidence (column $t\text{-}t_{TR}$ and $t\text{-}t_{TS}$ for the training and test sets, respectively).

The interpretation of the t-t columns is the following:

* * represents the best result;
* + means that the best result has better performance than that of the corresponding row;
* = means that the best result has performance comparable to that of the corresponding row.

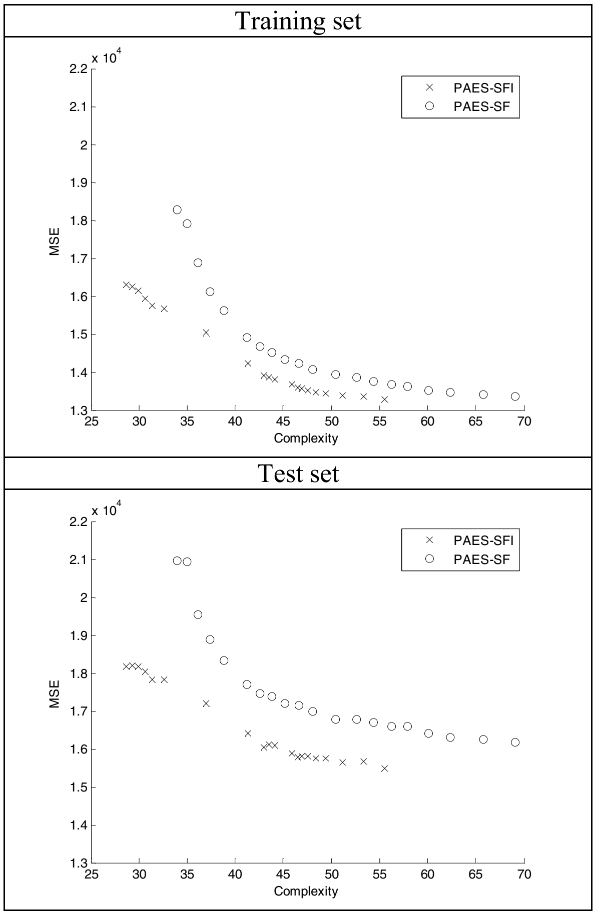

Figure 3. Average Pareto fronts in the Complexity-MSE plane on training and test sets.

Analyzing the results of the t-student test performed on the three representative points of the average Pareto fronts, we can affirm that the MFRBSs generated by both approaches are statistically equivalent in terms of MSE, even though the average Pareto fronts provided by PAES-SFI lie in a region characterized by a lower complexity.

In order to discuss the interpretability of the MFRBSs generated by the two approaches, we report in Table II the average values of complexity, number of rules and average dissimilarity D, for the first, median and last solutions. The average dissimilarity is defined as $D = \frac{1}{F+1}\sum_{f=1}^{F+1} d_f$. This dissimilarity expresses how much on average the partitions generated by the MF parameter learning are different from the uniform partitions. The higher the value of D, the lower the partition integrity.

We can observe that the MFRBSs generated by PAES-SFI are characterized on average by lower values of D, thus confirming the validity of exploiting the proposed interpretability index rather than using only the complexity as an objective to be optimised together with the MSE.

Figures 4 and 5 show two examples of fuzzy partitions for the most accurate MFRBSs generated on a representative fold by PAES-SFI and PAES-SF. We represent the uniform and the transformed partitions with dashed and continuous lines, respectively.

TABLE II: AVERAGE INTERPRETABILITY VALUES

	Complexity	# Rules	D
First			
PAES-SF	69.2±22.4	29.2±8.2	0.58±0.10
PAES-SFI	55.5±19.3	24.3±6.9	0.24±0.07
Median			
PAES-SF	48.1±16.6	22.8±6.6	0.58±0.10
PAES-SFI	44.1±16.6	20.8±6.1	0.24±0.07
Last			
PAES-SF	34.0±15.2	18.0±6.4	0.59±0.10
PAES-SFI	28.7±11.4	15.3±4.8	0.25±0.07

We observe that, for each linguistic variable, all the partitions generated by PAES-SF are very far from being uniform and result to be very hard to interpret. On the other hand, the fuzzy partitions generated by PAES-SFI preserve a high interpretability level. In particular the first, the third and the output partitions are very close to a uniform partition.

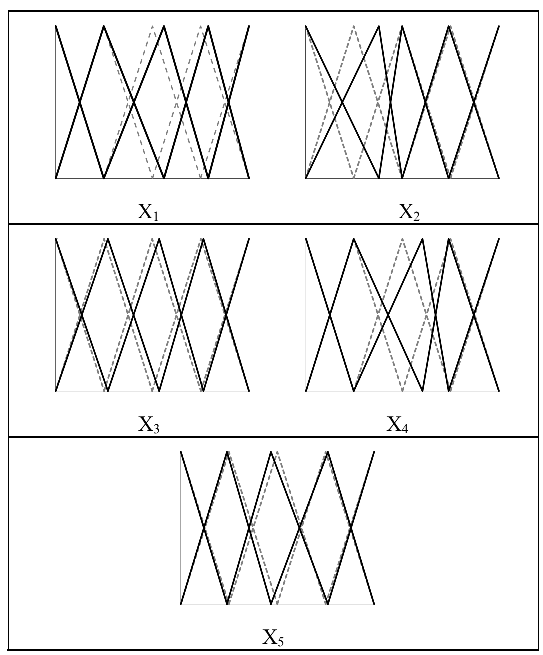

Figure 4. An example of fuzzy partitions generated by PAES-SFI on a representative fold.

VII. CONCLUSIONS

In this paper we have proposed a new MFRBS interpretability index, which takes both the rule base complexity and the partition integrity into account. This index and accuracy have been used as objectives in a multi-objective evolutionary learning of rules and MF parameters of MFRBSs. To this aim, we have adopted a modified version of the well-known (2+2)PAES and a chromosome consisting of two parts which codify, respectively, the RB, and, for each variable, the parameters of a piecewise linear transformation of the membership functions. This approach has proved to be very efficient and effective, allowing both a good exploitation of the solutions and an accurate exploration of the search space.

The algorithm has been tested on a real world regression problem and compared with a similar (2+2)PAES-based approach which uses the RB complexity and the accuracy as objectives.

On average, the solutions generated by the two approaches have proved to be statistically equivalent in terms of accuracy. On the other hand, the set of MFRBSs obtained by exploiting the proposed interpretability index have shown a higher partition integrity level.

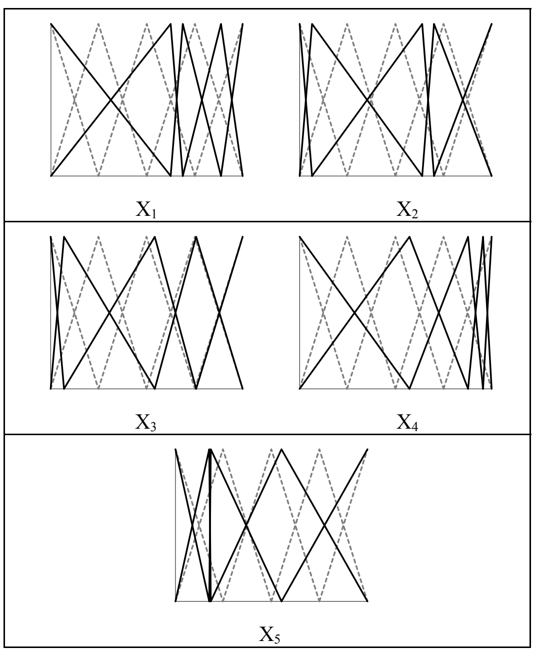

Figure 5. An Example of fuzzy partitions generated by PAES-SF on a representative fold.

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