

# Iterative EM Reconstruction of Cardiac Small Animal PET Images Using System Point Spread Function Modelling and MAP with Anatomical Priors

AE Spinelli<sup>1,2</sup>, G Fiacchi<sup>2,4</sup>, D D'Ambrosio<sup>1,2</sup>, P Cilibrizzi<sup>2</sup>, C Lamberti<sup>4</sup>,  
G Baldazzi<sup>3</sup>, S Boschi<sup>5</sup>, R Franchi<sup>5</sup>, M Marengo<sup>2</sup>

<sup>1</sup>Scuola di Specializzazione in Fisica Sanitaria, Università di Bologna, Italy

<sup>2</sup>Servizio di Fisica Sanitaria, Policlinico S Orsola - Malpighi, Bologna, Italy

<sup>3</sup>Dipartimento di Fisica, Università di Bologna, Italy

<sup>4</sup>DEIS, Università di Bologna, Italy

<sup>5</sup>Servizio di Medicina Nucleare, Policlinico S Orsola - Malpighi, Bologna, Italy

## Abstract

*The goal of this work was to improve the image quality of small animal PET images by introducing in the reconstruction process the true system point spread function (PSF) and an anatomical image prior. Simulations were performed using a mouse heart phantom (myocardium and ventricles) and a comparison between standard EM reconstruction and EM with PSF modelling and anatomical prior was performed. The system PSF was assumed to be a Gaussian function and its Full Width Half Maximum (FWHM) was modelled to be spatially variant in order to simulate the different spatial resolution inside the scanner field of view. A visual comparison of the images reconstructed with the standard EM and with the proposed image reconstruction method showed that the reconstructed images look much sharper and are very close to the true ones when using EM with PSF modelling and anatomical prior.*

## 1. Introduction

Reconstruction of small animal Positron Emission Tomography (PET) images is commonly performed using iterative algorithms such as the Expectation Maximization (EM) algorithm. Such algorithms are interesting because it is possible to include in the reconstruction process different effects such as attenuation, scatter, system resolution, anatomical information etc.

In this work attention was focused on the modeling of the system Point Spread Function (PSF) combined with a Maximum A Posteriori (MAP) reconstruction scheme. The prior was obtained using a computed tomography (CT) image coregistered with the PET image. The basic

equation of the standard EM algorithm is [1]:

$$n_j^{k+1} = \frac{n_j^k}{\sum_{i=1}^I a_{ij}} \left[ \sum_{i=1}^I a_{ij} \cdot \frac{m_i}{q_i^k} \right]$$

with  $q_i^k = \sum_{j=1}^J a_{ij} n_j^k$ . Where  $q_i^k$  represents the estimated projection of the image along the  $i$ -th Line Of Response (LOR),  $n_j^k$  represents the reconstructed image at the  $k$ -th step, while  $n_j^{k+1}$  is the reconstructed image at the next iteration;  $a_{ij}$  is an element of the system matrix  $A$ , and represents the probability that an emission from voxel  $j$  will be detected along LOR  $i$ ,  $m_i$  is the measured projection of the image along the LOR  $i$ .  $I$  is the number of LORs while  $J$  is the number of pixels of the image. The system matrix can be factorized as  $A = PX$  where  $X$  is the Radon transform and the matrix  $P$  describes the system blurring in the projection space [2]. A spatially variant PSF was included in forward step by blurring pixel by pixel the sinogram [3]. The estimated projections are thus given by the following equation:  $q_i^k = \sum_l q_l PSF_{i,l}$ . With this approach a better modeling of the system resolution was achieved. Since noise typically increases when resolution is improved an anatomical prior is introduced in order to reduce the amount of noise in the reconstructed image. The prior was chosen equal to the logarithm of the ratio between the reconstructed image at the previous step of the iterations and the corresponding anatomical image [4]. The EM algorithm including the prior can be rewritten as follows:

$$n_j^{k+1} = \frac{n_j^k}{\sum_{i=1}^I a_{ij}} \left[ \sum_{i=1}^I a_{ij} \cdot \frac{m_i}{q_i^k} - \beta \cdot \ln \frac{n_j^k}{t_j} \right]$$

where the constant  $\beta$  takes into account the weight of the cross-entropy term  $\ln \frac{n_j^k}{t_j}$  prior and  $t_j$  represents the anatomical CT image.

## 2. Methods

### 2.1. PET scanner PSF measurement

The PSF of the small animal PET scanner (GE eXplore Vista) installed at our institution was measured using a small (0.2 mm) 18-F point source. The source was placed at the center of the field of view (FOV) and several measurements were taken by moving the source along the vertical axis with an increment of 2 mm. The resulting PSF (measured from the sinogram) was then fitted using a Gaussian function in order to obtain the Full Width Half Maximum (FWHM) at different positions.

### 2.2. Image simulations and acquisitions

In order to test the proposed reconstruction algorithm a set of numerical simulations were performed and true images of the mini Derenzo phantom were acquired. Comparison were performed between standard EM algorithm and EM with PSF modeling and MAP using the same number of iterations (60). The first set of simulations (data not showed) was obtained using a digital phantom containing three arrays of squares with different dimensions (2, 3 and 4 pixels). The sinogram has been then blurred using a spatially variant PSF with a FWHM ranging from 1.5 mm at the center of the FOV to 2mm at the edges. The second set of simulations was obtained using the heart of the MOBY mouse digital phantom (see figure 1). In order to investigate the performance of the proposed EM method when reconstructing dynamic images a typical cardiac FDG dynamic scan was simulated. As before the sinograms of each time frame have been blurred using a spatially variant PSF and in order to simulate a more realistic measurement condition Poisson noise was also added. The dynamic images were simulated by using a two compartments model considering an input function  $C_p(t)$  and a set of kinetic rate constants ( $K_1 \dots k_4$ ) typical of a cardiac rat FDG scan. The FDG uptake  $C_t(t)$  in the myocardium was obtained using the following expression:

$$C_t(t) = (p_1 e^{-p_2 t} + p_3 e^{-p_4 t} \otimes) C_p(t)$$

where the coefficients  $p_1 \dots p_4$  are equal to:

$$p_1 = \frac{K_1}{\alpha_2 - \alpha_1} (k_3 + k_4 - \alpha_1)$$

$$p_2 = \alpha_1$$

$$p_3 = \frac{K_1}{\alpha_2 - \alpha_1} (\alpha_2 - k_3 - k_4)$$

$$p_4 = \alpha_2$$

$$\alpha_{1,2} = [(k_2 + k_3 + k_4) \pm \sqrt{(k_2 + k_3 + k_4)^2 - k_3 k_4}]$$

The input function  $C_p(t)$  was equal to:

$$C_p(t) = (A_1 \cdot t - A_2 - A_3) e^{\lambda_1 t} + A_2 e^{\lambda_2 t} + A_3 e^{\lambda_3 t}$$

where the coefficients  $A_1 \dots A_3$  and  $\lambda_1 \dots \lambda_3$  are chosen as in [5].

The algorithm has been also tested on true static PET images. To this purpose a mini Derenzo phantom was filled with 20 MBq of FDG and PET images were acquired for 30 minutes using a 100-700 keV energy window. The diameter of the phantom inserts was respectively equal to: 1.2, 1.6, 2.4, 3.2 and 4.8 mm. The original 3D data were rebinned into a stack (61 transaxial slices) of 2D sinograms using FORE rebinning.

All the code to perform image simulations and reconstruction was implemented using Interactive Data Language (IDL) 7.0.

## 3. Results

Figure 1 shows the last frame of the simulated MOBY phantom images. The images were reconstructed respectively using standard EM, EM with constant PSF, EM with variant PSF and with the inclusion of the anatomical prior. As one can see the quality of the image in the bottom right corner (obtained using EM with variant PSF and anatomical prior) is significantly better. Line profiles drawn across the heart region of the MOBY phantom images (see figure 1) are plotted in figure 2. In figure 4 a comparison between input obtained using different reconstruction methods is shown. The input function was measured by drawing a region of interest on the left ventricle. By looking at the plots shown in the figure the image reconstructed using EM with variant PSF and anatomical prior allows a better estimate of the true input function. Figures 5, 6 and 7 show images of the mini Derenzo phantom reconstructed respectively using standard EM, EM with constant PSF and EM with spatially variant PSF. In order to compare the different reconstruction methods, line profiles were drawn across the 1.6 mm hot rods insert. The profiles are shown in figure 8.

## 4. Discussion and conclusions

Comparison between reconstructed cardiac images of the MOBY phantom shows a significant improvement when using EM with MAP and PSF modeling with respect to standard EM or EM with constant PSF. The analysis of line profiles draw across the myocardium and the left ventricle shows that EM with MAP and PSF modeling provides a better estimate of the true radiotracer concentration. The use of spatially variant PSF is thus important in

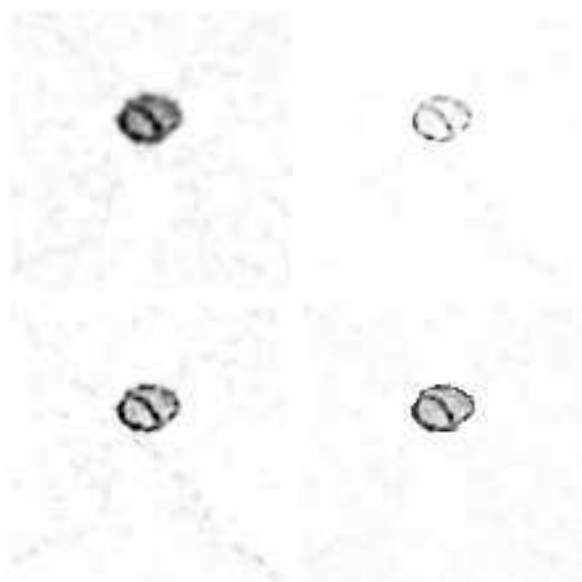


Figure 1. The figure show the last frame of the cardiac MOBY phantom. The images were reconstructed respectively using the standard EM algorithm (top left corner), EM with constant PSF (top right corner), EM with spatially variant PSF (bottom left corner) and by adding the prior (bottom right corner). As one can see the quality of image in the bottom right corner is significantly better.

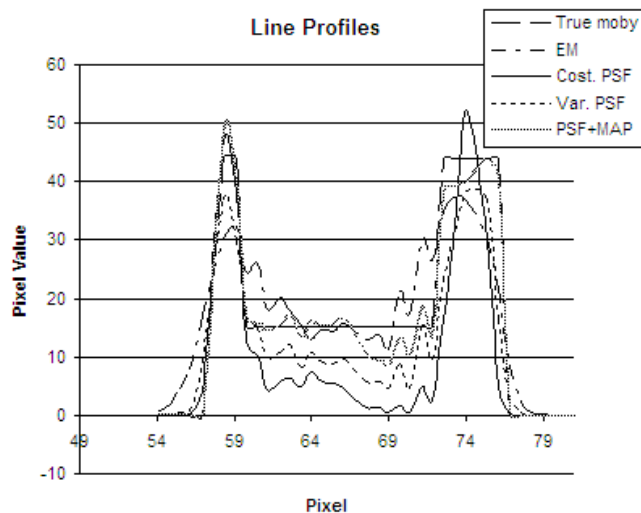


Figure 2. Line profiles drawn across the myocardium and left ventricle of the MOBY phantom images shown in figure 1.

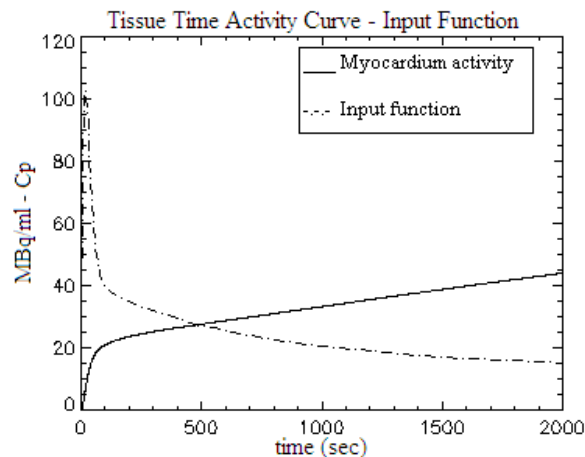


Figure 3. The figure show the (noise free) input function (dotted line) and the FDG myocardium uptake curve used in the simulations described in section 2.2.

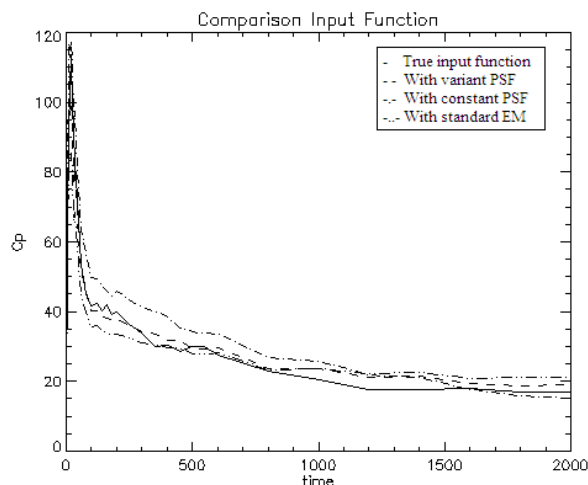


Figure 4. Comparison between input obtained using different reconstruction. By looking at the plots the image reconstructed using EM with variant PSF and anatomical prior allows a better estimate of the true input function.

order to achieve a good signal recovery. For example if a spatially invariant PSF is used, the reconstructed image looks sharper however the measured radiotracer concentration is significantly different from the true one.

Images of the Derenzo phantom show that the EM algorithm with spatially variant PSF provides a better signal recovery with respect to standard EM or EM using a spatially invariant PSF. Line profiles drawn on the 1.6 mm hot rods showed that a better contrast can be achieved when using EM algorithm with spatially variant PSF.

We can conclude that the proposed reconstruction method based on accurate modeling on the system PSF and

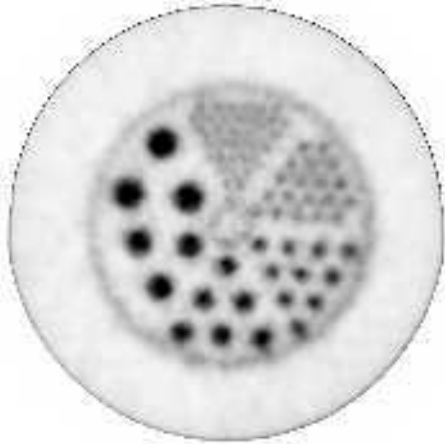


Figure 5. Derenzo phantom reconstructed using standard EM.

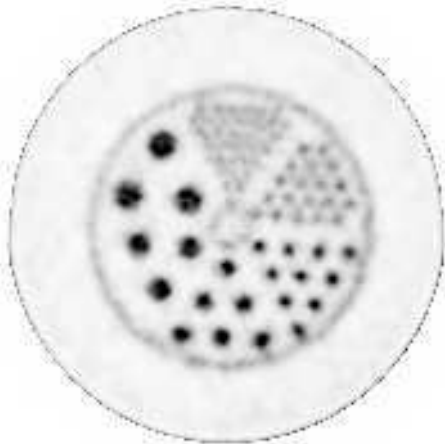


Figure 6. Derenzo phantom reconstructed using EM with constant PSF modeling.

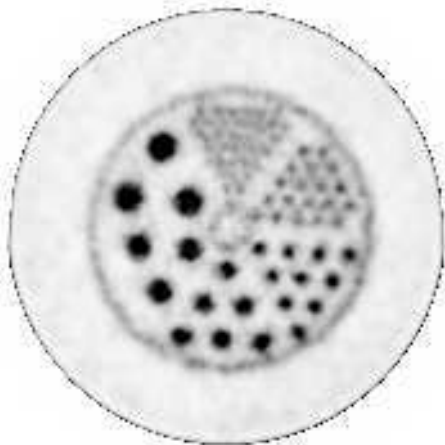


Figure 7. Derenzo phantom reconstructed using EM with spatially variant PSF modeling.

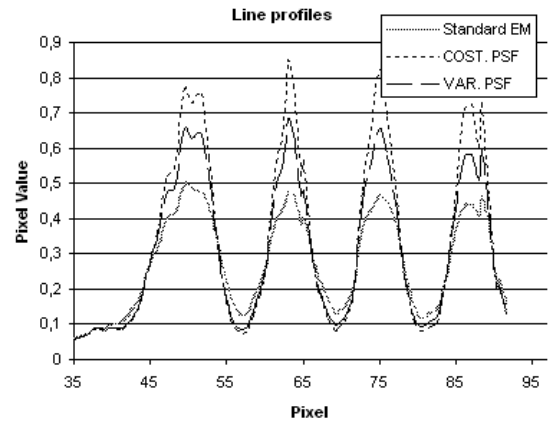


Figure 8. Line profiles drawn across the 1.6 mm hot rods insert of the mini Derenzo phantom.

anatomical prior provides significant better results with respect to the standard EM algorithm.

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Address for correspondence:

Antonello E Spinelli, Ph.D.  
 Servizio di Fisica Sanitaria,  
 Policlinico S. Orsola - Malpighi,  
 Via Massarenti, N. 9,  
 40138, Bologna, Italy  
 E-mail address: antonello.spinelli@aosp.bo.it