The influence of different bone remodeling equations on a 2-D vertebra model in the final bone density distribution

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Abstract—**The phenomenon of bone remodeling is a complex biological process which is dependent on genetic, hormonal, metabolic and age-related. Being familiar with the mechanisms of bone remodeling is of great importance for implant design and metabolic diseases such as osteoporosis, since it enables monitoring of bone tissue. The bone remodeling phenomenon can be described mathematically and simulated in a computer model, integrated with the finite element method. Important parameters in such a model are the geometry and the mechanical loading caused by everyday activities and the bone remodeling equation itself. Therefore, this paper deals with the biomechanical modeling of the bone remodeling process on a simplified geometry of a fifth lumbar vertebra (L5). A baseline bone remodeling equation, retrieved in the literature has been used, and the values of its coefficients have been varied in order to evaluate their effect on the description of the bone remodeling phenomenon. This way different bone remodeling conditions have been simulated. Likewise, two nonlinearities, i.e. the bone remodeling coefficient and the order of non-linear remodeling equation have been introduced. The influence of each non-linearity was investigated and its mechanical implications have been reported.**

I. Introduction

THE first researcher that observed the phenomenon of bone remodeling was Wolff when, in the late 1890's, observed that trabeculae in the proximal femur have the tendency to align with the principal stress trajectories. Formulating what is known today as "Wolff's law", he suggested that bone is created in areas that need to be strengthened and resorbed where bone is not needed.

Until now a lot of research is done on this hypothesis and two types of bone remodeling have been observed; "internal remodeling" that leads to alterations of the internal structure of bone in general and the "external" remodeling that leads to changes of the external geometry of bone. The internal remodeling that is defined as change of bone density could be described as modification in the value of the elastic modulus of the cancellous bone [1].

The exact mechanical stimulus of bone remodeling is still under investigation. In different theories it was assumed that the mechanical key stimulus initiating bone remodeling processes are either the mechanical stresses or the strains (i.e. normal and shear strains) [2-4]. Brown et al. [5] conducted a combined experimental-finite element modeling study to select the specific mechanical parameters responsible for initiating the adaptive responses of bone using an animal model. According to recent studies, however, bone remodeling is controlled predominantly by strains rather than stresses [6].

Bone remodeling equations have been treated as site specific and there are some studies that concern the specific site of the spine. One of these studies was conducted by Goel et al. [7] and demonstrated that the strain energy density in the vertebral cortex and cancellous bone induce the remodeling process.

In the present study internal bone remodeling is studied and strain energy density is considered to be the regulatory parameter. Different hypothesis for the bone remodeling equation are tested in two different loading conditions of a 2D finite element model simulating the fifth lumbar vertebra of a young and an old male, respectively.

II. Materials & Methods

A. Bone Remodeling Equation

The basic bone remodeling equation used, is the differential equation (1) proposed by Weinans in 1992 [8].

$$\frac{d\rho}{dt} = B \cdot \left[\frac{U}{\rho} - k \right] \tag{1}$$

where ρ is the bone mineral density, B is the bone remodeling coefficient, U is the strain energy density at the centroid of the finite element and k is the reference stimulus value. This value stands for the borderline quotient of U/ρ that below this, no bone remodeling takes place.

Bone remodeling in each finite element is considered to be converged when the equation $\frac{d\rho}{dt} = 0$ is satisfied, which happens when one or more of the following three conditions are satisfied: (i) reached preset reference stimulus value k of ratio between strain energy density and density of bone tissue, (ii) reached density of cortical bone tissue $\rho = \rho_{max}$, (iii) complete resorbtion of bone tissue from finite element $\rho = \rho_{min}$. Reaching one of the bone remodeling equilibrium conditions is achieved by the constant change in the density of bone tissue in each finite element. As density of the cortical bone the value of 1.74 g/cm^3 has been considered, while for the minimum value that stands for the lack of bone, the value 0.01 g/cm^3 has been considered.

According to Weinans [8] in (1) the coefficients k and B have the values of 0.25 J/g and $1 \frac{(g/cm^3)^2}{(MPa \cdot timeunit)}$, respectively.

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Investigating the influence of the bone remodeling coefficient the following equations are investigated

$$\frac{d\rho}{dt} = 1 \cdot \left[\frac{U}{\rho} - 0.25 \right] \tag{2}$$

and

$$\frac{d\rho}{dt} = 0.25 \cdot \left[\frac{U}{\rho} - 0.25 \right] \tag{3}$$

In the literature [9] the insertion of two kinds of non-linearities in the differential equation of bone remodeling has been proposed.

The first one, concerns the insertion of the exponent *a* on the term $\frac{U}{\rho \cdot k}$. McNamara proposed the value of 2.25 which indicated a satisfactory compromise with reality [10]. The equations that are going to be investigated having embodied the aforementioned nonlinearity are the following:

$$\frac{d\rho}{dt} = 1 \cdot \left[\frac{U}{0.25\rho} - 1 \right]^{2.25} \tag{4}$$

$$\frac{d\rho}{dt} = 0.25 \cdot \left[\frac{U}{0.25\rho} - 1 \right]^{2.25} \tag{5}$$

The second non-lineartity proposed, concerns the transformation of the constant bone remodeling coefficient *B* in (1) to an exponentially changing versus time, coefficient *B(t)*. The correlation between time *t* and the coefficient *B* proposed in (9) is in the form of:

$$B(t) = \left(B_0 - B_T\right) e^{0.002t} + B_T \tag{6}$$

where B_0 and B_T are constants. The equations that embed this nonlinearity and are going to be investigated are the following:

$$\frac{d\rho}{dt} = \left(\left(\left(1.0 - 0.01\right) / e^{0.02t} \right) + 0.01 \right) \cdot \left[\frac{U}{0.25\rho} - 1 \right]^{1} \tag{7}$$

$$\frac{d\rho}{dt} = \left(\left(\left(1.0 - 0.01\right) / e^{0.02t} \right) + 0.01 \right) \cdot \left[\frac{U}{0.25\rho} - 1 \right]^{2.25} \tag{8}$$

Finally a change in the correlation between time and the bone remodeling coefficient is proposed and the last two equations are investigated:

$$\frac{d\rho}{dt} = \left(\left(1.0 - 0.01\right) / \left(e^{0.02t} + 0.01 \right) \right) \cdot \left[\frac{U}{0.25\rho} - 1 \right]^{1} \tag{9}$$

$$\frac{d\rho}{dt} = \left(\left(1.0 - 0.01\right) / \left(e^{0.02t} + 0.01 \right) \right) \cdot \left[\frac{U}{0.25\rho} - 1 \right]^{2.25} \tag{10}$$

For the numerical solution of the differential equation the method of Adams-Bashforth of the fourth order has been used. Adams – Bashforth is not a self starting method so for the first three time steps Runge-Kutta of the fourth order has been used. The same numerical solution has been also proposed by Chen et al. [11].

Schematically the computational flow chart of bone remodeling is shown in Fig. 1.

Starting from the physical model, the geometrical model is extracted. Further discretization is applied with the use of the finite element (FE) method for the mechanical analysis of the vertebra, and the boundary conditions are applied. Then the mechanical properties of bone are set, i.e. Poisson ratio and Young's modulus, and the FE model is solved.

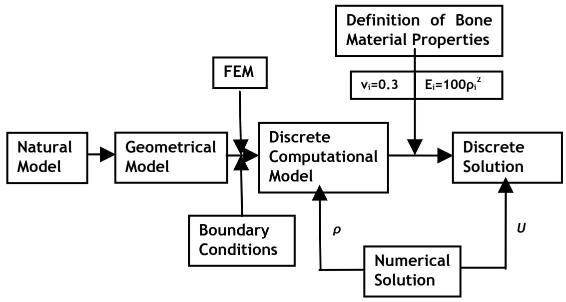


Fig. 1. Bone remodeling flow chart

The values of the strain energy density are inserted in the differential equation of the bone remodeling procedure and new bone density values are produced. These new values are used for the calculation of the new values of the elastic properties and so on until the limit of the iterations is reached.

B. Mechanical Properties of bone

Bone density that appears in the bone remodeling equation is directly connected to the Young's modulus of the bone. A lot of research is done on this field. In the present paper the relationship between bone density and Young's modulus is considered to be

$$E = 100 \cdot \rho^2 \tag{11}$$

as mentioned in [9] and [12]. As far as the Poisson's ratio is considered it is set to 0.3

C. Finite Element Model

The remodeling equations have been applied in a 2D FE model of a fifth lumbar vertebra. The geometry of the vertebra is the same as used in [9] and [12] and its dimensions come from the observation of the external shape and internal structure of the cross-section of a vertebra. The vertebral body height is 24.9mm and the width of its upper endplate is equal to 43.9mm while the width of its lower endplate is 49.0mm. For the FE analysis the ANSYS v11.0 software has been used. Both the FE analysis and the numerical solution of the differential equation have been programmed in APDL, the programming language of ANSYS software.

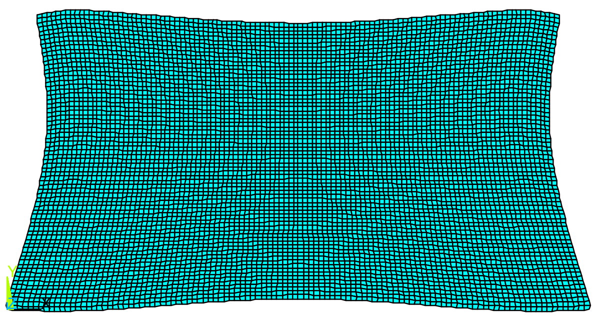


Fig. 2. 2D model used to simulate the crossection of a typical third lumbar vertebra

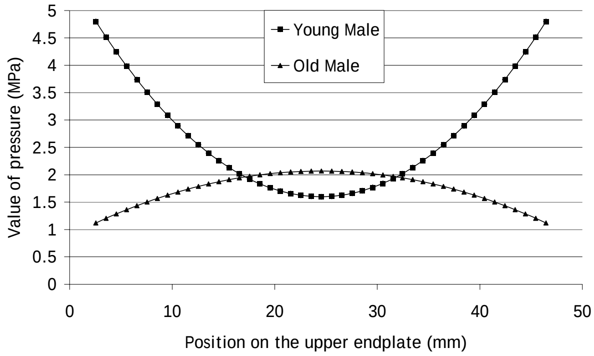
The FE model is shown in Fig. 2. For the discretization of the 2D model the 2D plane stress FE Plane42 from ANSYS FE library have been used. The final mesh consisted of 7700 finite elements and had 15762 degrees of freedom in total.

D. Boundary Conditions

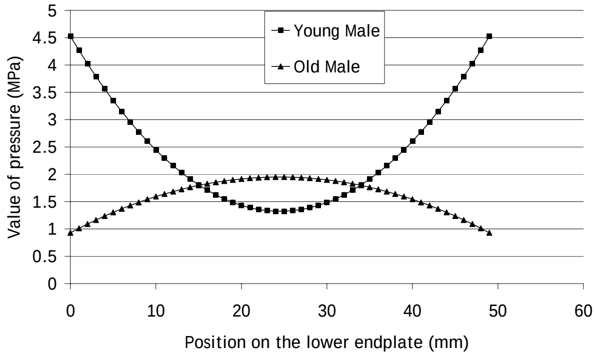
As far as the boundary conditions are concerned the two extreme nodes of the lower endplate are considered articulated, while on the upper endplate two typical loading has been considered.

For the young male the maximum pressure applied on the edges of the upper endplate is 4.8MPa and the minimum on the middle is 1.6MPa. Likewise the maximum pressure on the edges of the lower endplate is 4.5MPa while the minimum on the middle is 1.3MPa. The total magnitude of the force that is applied on the vertebra for both the upper and the lower endplate is equal to -117.3N.

For the elderly male the maximum pressure that is applied on the middle of the upper and lower endplate has the value of 2.07MPa and 1.95MPa respectively, while the minimum pressure applied on the edges of both endplates has the value of 1.12MPa and 0.93MPa respectively. The total pressure applied in the form of perpendicular force on the vertebra equals to -41.87N.



(a)



(b)

Fig. 3. Loading of the (a) upper and (b) lower endplate for both the young and the elderly male

In Fig. 3 (a) the loading profile for the upper endplate is shown for both the young and the old male, while the same thing for the lower endplate is shown in Fig. 3. (b).

III. Results

First of all time step magnitude has been investigated. Time steps between 0.03 that is proposed in the literature and 0.06 were tested. The results in both cases were the same so the value of 0.06 was adopted. For every bone remodeling equation 1400 iterations were performed. This corresponds to time equal to 84 time units. In all the runs the starting value for bone density was equal to 0.8 g/cm^3.

A. Final Bone Density Distribution

In Fig. 4. (a) that follows, the final distribution of bone density is shown for bone remodeling equations (2-5) and (7-10) for the young male, while in Fig. 4 (b) the

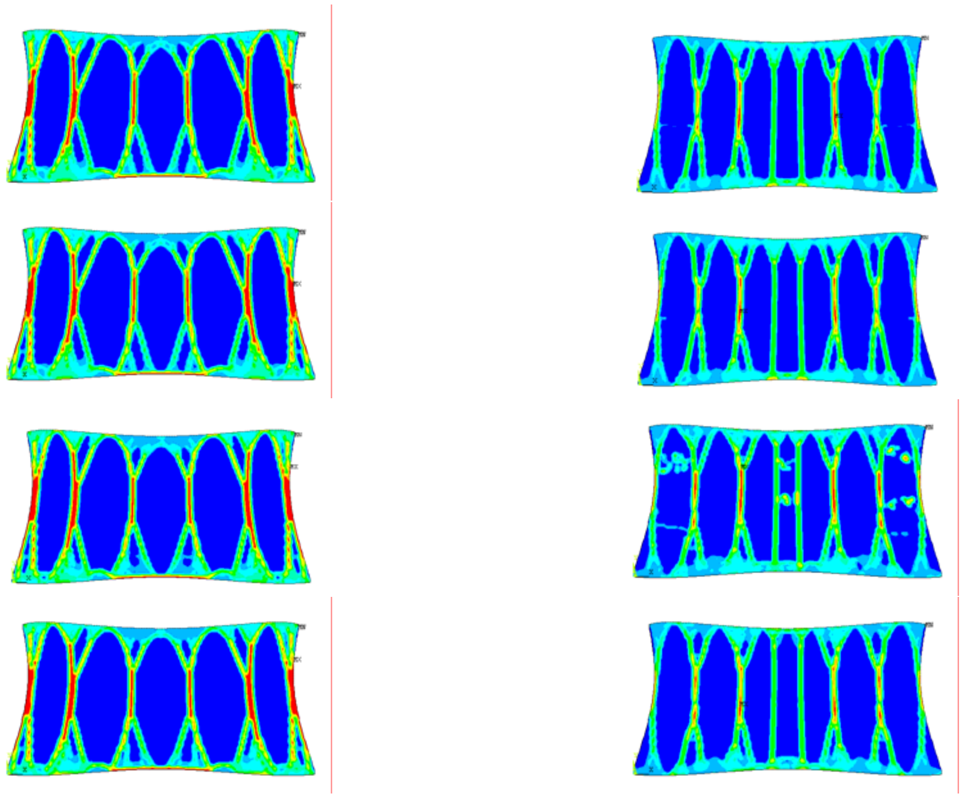
Eq. No (a) (b)

(2)

(3)

(4)

(5)



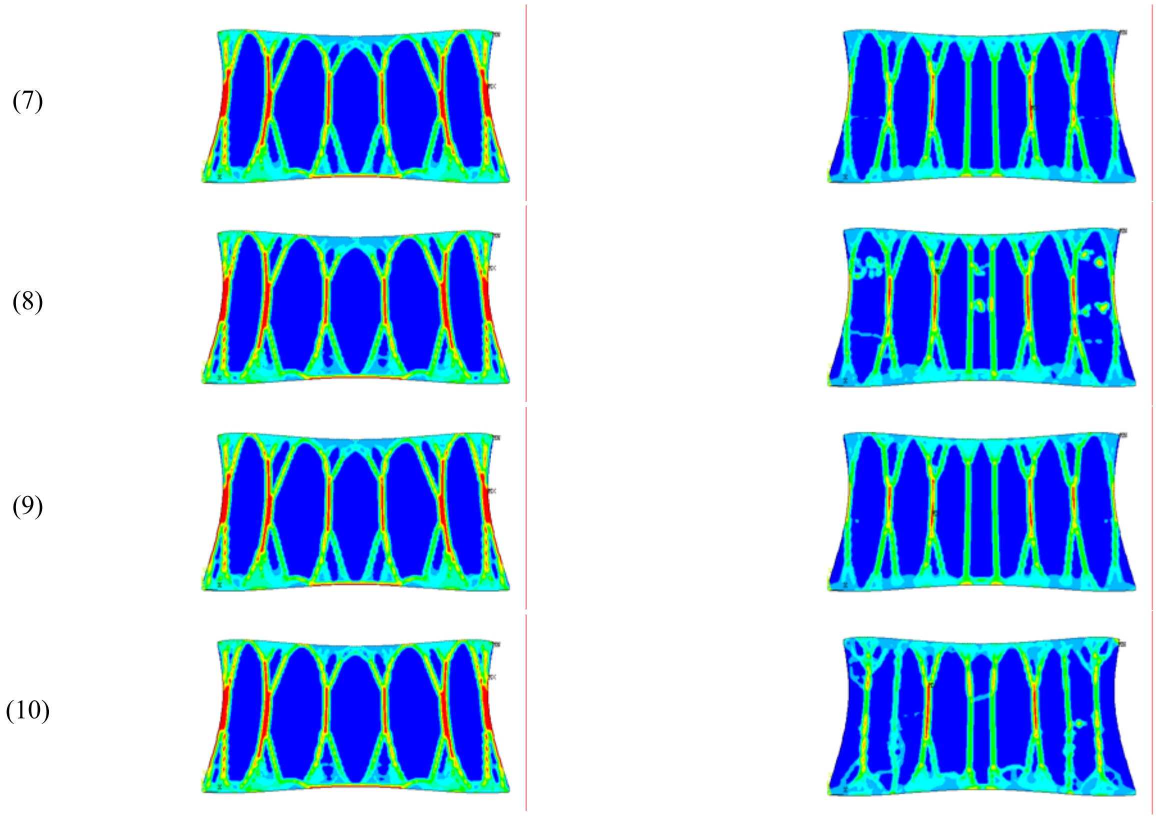


Fig. 4. Final bone density distribution for the (a) young and (b) old male

bone density distributions for the old male, are shown.

In all the cases, vertical columns, that look like trabeculae are formed, and bone material has almost a continuum distribution on the upper and lower endplate. On the other hand the numbers of the vertical columns, as well as the bone density distributions are different depending on the loading condition that corresponds to age difference.

Besides the final distribution of the bone density inside the 2D vertebra in the literature some performance criteria for the bone remodeling procedure have been proposed. Therefore, for further evaluation of the different bone remodeling equations these criteria have been examined.

B. *Mass Criterion*

Using the FE method, and according to the bone remodeling equation, every finite element can only influence its own mass, and can result to its change. The mass criterion investigates whether the final outcome is close to real mass distribution of the vertebra and whether it is close to the optimized result of bone mass minimization.

In Table I the final mass for each case is shown

TABLE I
FINAL MASS

	Young Male	Old Male
(2)	365.38	262.98
(3)	366.19	262.94
(4)	368.85	285.72
(5)	368.17	264.04
(7)	365.38	262.98
(8)	368.85	285.72
(9)	365.38	263.23
(10)	369.21	285.72

In Fig. 5 (a) that follows the graph of the ratio of the final mass to the initial one is presented for all the equations with constant bone remodeling coefficient for the vertebra belonging to the young male, while in Fig. 5. (b) is shown the graph of the same ratio for the bone remodeling equations with exponential bone remodeling coefficient.

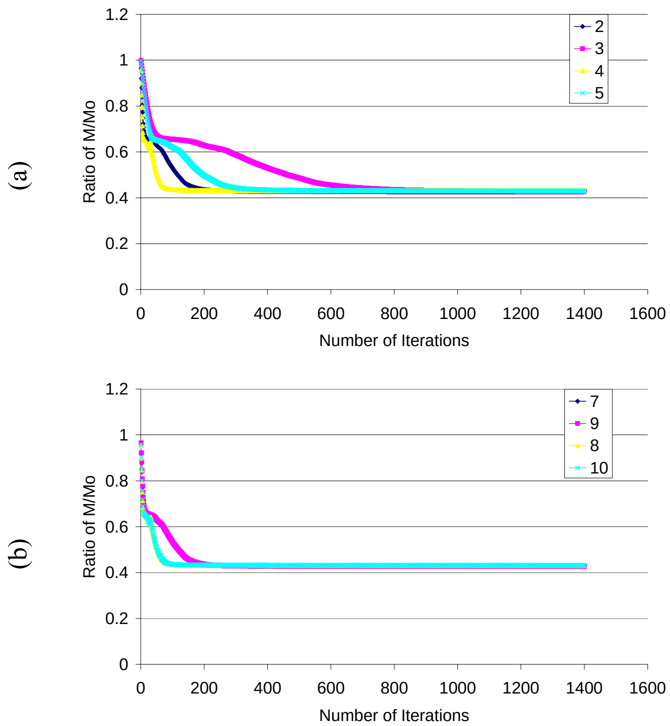


Fig. 5. Ratio of the mass of each time step per initial mass for the vertebra that belong to the young male

The fact that the initial mass for all equations is the same, since bone density is the same, needs to be stressed out.
In Fig. 6 are shown the same ratios for the old aged vertebra for all bone remodeling equations.

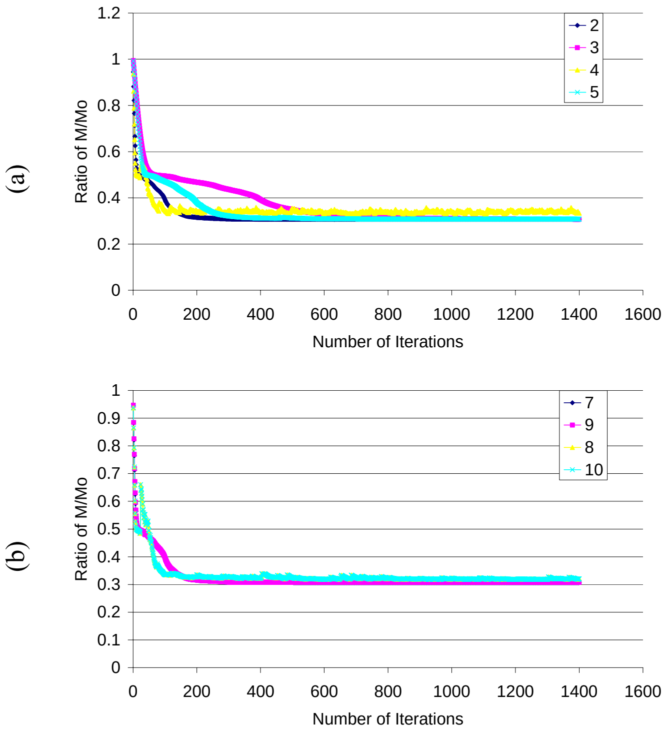


Fig. 6. Ratio of the mass of each time step per initial mass for the vertebra that belong to the old male

C. *Stiffness Criterion*

Zhao and Hornby in 1998 [9] introduced the idea of a material efficiency indicator for the evaluation of a structure under the aspect of the structure with the maximum stiffness. They suggested that the average stiffness per unit volume could be used in order to conclude on how effectively the material is used. Extending this idea to biological materials this criterion could also be used in bone. Note that bone density is a value between $0.01 \,{}^{g}/_{cm^3}$ (lack of bone) and $1.74 \,{}^{g}/_{cm^3}$ (cortical bone). The total stiffness of a structure is proportional to the total work produced by all the external forces, consequently the stiffness indicator can be defined as:

$$\overline{K} = \frac{1}{W_E^s \cdot M^s} \qquad (12)$$

where W_E^s is the total work produced from all the external forces acting on the structure and M^s is the current mass of the structure.

In Table II the value of the stiffness indicator is shown.

TABLE II
STIFFNESS INDICATOR

	Young Male	Old Male
(2)	3.35E-06	7.03E-06
(3)	3.35E-06	7.05E-06
(4)	3.39E-06	6.60E-06
(5)	3.39E-06	6.99E-06
(7)	3.35E-06	7.03E-06
(8)	3.39E-06	6.60E-06
(9)	3.35E-06	7.07E-06
(10)	3.41E-06	6.38E-06

D. *Cost Function*

The final criterion used for the evaluation of the bone remodeling equation is the value of the cost function. Function F is defined as:

$$F = \frac{1}{m} \cdot \sum_{i=1}^{m} \left(\frac{U_i}{\rho_i} - k \right) \qquad (11)$$

where m is the number of the finite elements where the bone remodeling process still goes on.

In Fig. 7 that follows the graph of the cost function for all the equations with (a) constant bone remodeling coefficient and (b) exponential bone remodeling coefficient is presented for the vertebra belonging to the young male.

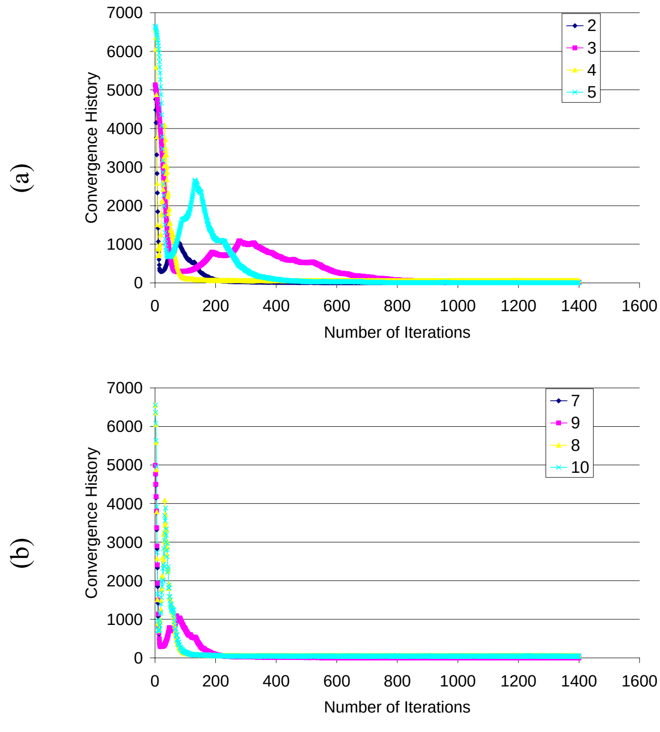
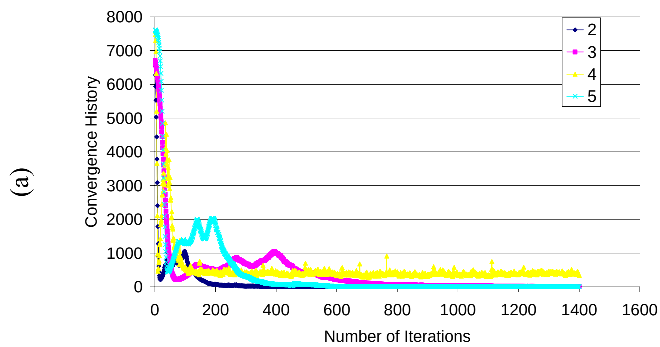


Fig. 7. Convergence history for the vertebra that belongs to the young male

In Fig. 8 are shown the same ratios for the vertebra belonging to the old male for all bone remodeling equations.



(b)

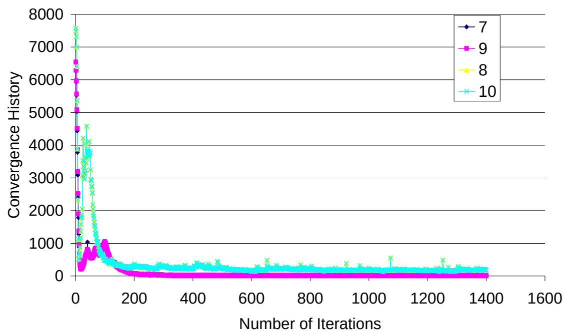


Fig. 8. Convergence history for the vertebra that belongs to the old male

IV. Discussion

The discussion of the results is going to be on two levels. The first one deals with each loading condition alone and the second deals with the differences caused on the final outcome due to the different bone remodeling equation.

As far as final bone density distribution on the young vertebra is concerned, is obvious that the use of different bone remodeling equation does not alter gravely the final bone density distribution on the same number of time units. The only alterations that can be observed are close to the lower endplate and on the lower level of the center columns. The same thing does not stand for the old vertebra. The use of equations (4), (8) and (10) leads to the development of parts with middle bone density in the space between the columns. The common characteristic of these equations is the use of the nonlinearity induced by the exponent α. The rest of the final distributions of bone density are the same with slight differences on the lower part of the columns.

The use of mass criterion for both the young and the old vertebra proves that in all cases the ratio of the final mass to the initial one is of the same order. The difference lies on the iteration that this is reached. On the young vertebra, this value is reached quicker with the use of the equation (4) for the case of constant B and (8) for exponentially changing B versus time. It is important that the use of exponentially changing B leads to the same rate of decrease in mass regardless the nonlinearities. For the old male's vertebra the equations with the quickest rate of decrease in mass are again (4) and (8). This leads to the conclusion that this nonlinearity affects the rate of mass decrease.

The stiffness criterion proclaims that for the majority of equations the material is best used. The stiffness value is higher for the use of (10) for the young vertebra and for (9) for the old one meaning that in these cases the material is utilized in the best possible way. The common characteristic of these two equations is that B is exponentially changing versus time with a new scheme. Finally as far as the convergence criterion is concerned, it reveals that the use of the exponentially changing versus time B leads to the equilibrium after 200 iterations, while if B is constant the equilibrium is reached after 800 iterations, in all cases.

V. Conclusions

An important conclusion of this study is that the nonlinearity of the exponent a gravely affects the behavior of a bone remodeling equation. Also it is important to observe that the change of the dependency of the bone remodeling coefficient versus time, in both cases leads to the distribution of maximum stiffness.

Gathering all the above information is obvious that computational results alone are not enough to reach to a conclusion over which is the right bone remodeling equation, but on the other hand a computational work can be conclusive on how each coefficient affects a bone remodeling equation. Loading conditions are another factor that gravelly affects the way an equation influences the final outcome, since starting from a common geometry, the change of the loading alone leads to a completely different outcome in terms of distribution.

An investigation of the above equations on a real three dimensional geometry of a vertebra with all the possible loading conditions, could lead to more conclusive results about the way an equation of such a type should be built.

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