

# A 3D Collision Handling Algorithm for Surgery Simulation Based on Feedback Fuzzy Logic

Verónica García-Pérez, Emma Muñoz-Moreno, Rodrigo de Luis-García, Carlos Alberola-López

**Abstract**—A new algorithm for collision handling between 3D agents in a laparoscopic surgery simulator is proposed in this paper. Simulation in minimally invasive surgery pursues a trade off between real-time execution and fidelity in the virtual scene. In order to achieve visual realism, accurate deformations of graphical models are required when interactions between tools and organs occur. Specifically, during the simulation step, vertices of the organ detected as collided must be accurately shifted out of the tool to elude the interpenetration. Techniques reported in the literature usually resort to approaches grounded on physical properties that entail significant computational load. Consequently, it reverts on non-desirable simplifications in which the 3D character of tool models is typically avoided. Hence, the aim of this paper is to present an algorithm for collision handling between 3D deformable (organ) and non-deformable (surgical tool) objects involved in a non-structured interaction scene. The proposed approach obtains the new position of each collided vertex of the organ taking into account two key concepts. First, a parameter which embodies the ongoing state of the scene is obtained making use of both kinematic information of the surgical tool and geometric information of the organ surface that surrounds the vertex under analysis. Second, three parameters inferred from a feedback fuzzy logic system ponderate the nature of the tool motion with respect to the organ, modeled as penetration/extraction and sliding. Preliminary experimental results show that this solution is able to avoid the interpenetration among the multiple colliding points detected in each simulation step in an efficient, physically and spatially coherent manner.

## I. INTRODUCTION

**M**INIMALLY Invasive Surgery (MIS) is a widespread technique for reduced body invasion surgery which was introduced in 1987. Compared to traditional open surgery, the relevance of MIS lies on the clinical, physical [1] and physiological [2] benefits that have reported. Nevertheless, new special skills must be developed by the surgeon, such as accommodation to a limited field of view, hand-eye coordination and force feedback from the surgical tools [3]. Hence, in order to achieve the proficiency level required, is it common that novel surgeons train with costly and non-ethical procedures.

Such problems have triggered the development of tailorable and reproducible training solutions qualified to offer, as many times as necessary, a wide variety of proficiency levels [4]. Integrated in a training and educational curriculum [5], these

simulators have been validated in the acquisition of technical skills [6].

The main benefits of surgical simulation lie on the possibility of allowing students to get used to a similar environment, where the level of immersion depends on the fidelity in the real-time interaction, tactile feedback forces and visual perception of the models and deformation responses [7]. With regard to the visual realism, efficiency handling the collisions occurred during the interaction between tools and organs appears as a challenge in the literature. The purpose of collision handling is to find where the nodes of the deformable object detected as colliding must be moved to, so that they are taken out of the tool with realistic resemblance and no interpenetration is allowed.

Because of the time-constraints of the surgical simulation cycle, most approaches to collision handling introduce inadequate or over-simplified representations of the environment to be modeled, thus worsening the desired visual realism of the system.

Within this context, the main goal of this paper is to propose an algorithm that performs the collision response task in real-time and solves some pitfalls of collision handlers reported before in the literature. A procedure that achieves realistic results, in spite of being simple, is pursued. Based on geometric and kinematic information, the algorithm calculates a displacement vector for each of the colliding points, taking into account distinguished cases of collision, namely, penetration/extraction and sliding. In order to adjust the displacement, a degree of similarity to each of these cases is computed by means of a fuzzy logic system. In addition, local geometric information of the deformable model is taken into account through the system feedback. Hence, the core benefits of the proposed algorithm are: (1) The method is able to handle several colliding points simultaneously; (2) The algorithm contributes to a more spatially coherent displacement of each collided vertex; and (3) User interaction comes into sight in a more natural manner.

The remainder of the document is structured as follows: Section II introduces collision handling within the simulation cycle context and its own requirements. A synopsis of the solutions proposed in the literature and some of their limitations are presented in Section III. Section IV is devoted to explain the theoretical foundations of the proposed collision handling method. Experimental results are shown and discussed in Section V and finally, conclusions and future work are summarized in Section VI.

Manuscript received June 30, 2006; revised 23 August, 2006. The authors acknowledge the Comisión Interministerial de Ciencia y Tecnología for research grant TEC2004-06647-C03-01, the Fondo de Investigaciones Sanitarias for grant PI-041483, the Junta de Castilla y León for grant VA075A05 and the European Commission for the funds associated to the Network of Excellence SIMILAR (FP6-507609).

V. García-Pérez, E. Muñoz-Moreno, R. Luis-García and C. Alberola-López are with the Image Processing Lab., University of Valladolid, Spain.

E-mail: {veronica, emunmor, rluigar, carlos}@lpi.tel.uva.es

## II. PROBLEM STATEMENT

In order to gain insight into the collision handling topic in laparoscopic simulation, the domain of the problem is examined here from two different perspectives: an explanation of the prime elements which comprise the whole system and surround the problem is given, and next a consideration about the required performance criteria is done. The arguments that justify assumptions made for each of these two points of view are also introduced here.

### A. Surgical Simulation Components

The collision handling method proposed here is involved in the development of a surgical simulator [8] whose simulation cycle can be seen in Figure 1. Its components are briefly described next.

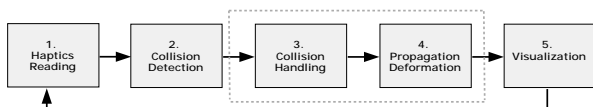


Fig. 1. Simulation cycle of the surgical simulator proposed in this paper.

Firstly, the input position of the tools is read from specialized devices able to provide the force feedback required to achieve tactile realism, which must also fit with the update rate demand [9]. It should be noted that, in a laparoscopic simulation, the most common interaction is, by far, that between rigid surgical tools and deformable organs. Likewise, visualization is carried out by means of the library WTK [10].

The module responsible for finding out the exact contact loci where agents in scene collide is the collision detection [11] module. In this case, two main preconditions have been beared in mind to choose the most suitable library: (1) It must consider topological changes in the structures involved in order to allow cutting or dissectioning practices, and (2) It must be able to detect collided and interpenetrated facets in order to manage 3D collisions. The last condition is very related with the selected geometrical models of tools and organs. For the surgical tools, several simplifications have often been made in the literature, such as considering them as a single point [12] or as a straight line [13]. As realism in this case is inadequate [14], a computationally manageable but more faithful approach (specifically, by means of prisms) has been considered as well [15]. As for the organs, there is a choice between volumetric or surface models. Even though the first-named model offers a more physical deformation accuracy, a surface model was been selected in order to gain computational efficiency [16]. The drawbacks of this approach will be compensated by means of the collision handling method proposed in this paper. Under the above assumptions, a detection library developed in [17] has been employed.

As explained in [18], the purpose of the collision handling module is to avoid the interpenetration between the objects involved in the collision by deciding where the colliding points detected of the deformable objects should be moved to out of the tool. Afterwards, a deformation is computed. This subject is the main topic of this paper so we will describe it in detail in following sections.

Finally, the deformation propagation module aims at calculating how the deformation of the colliding parts of the object propagates throughout the whole object. Substantial effort has been done in the literature related to the accomplishment of a physically realistic model [19], [20]. Here, an spring-mass based model has been selected [21].

As a concluding remark, even though several other classifications related with the collision topic have been proposed in which the whole process is jointly considered [22] or even left out [23], a more modular approach is preferred here.

To summarize, this paper faces the problem of handling multiple collisions between a surface deformable object and a surface rigid tool in an efficient manner.

### B. Performance Criteria

As mentioned above, one of the most important challenges in surgical simulation is to provide the level of realism necessary to allow an adequate interaction. Realism concerns the accuracy in the representation of the models involved in the scene and its physical response when interactions among them occur. Physicians require highly-realistic experiences in order to validate the usefulness of surgical simulators [7]. However, as this effort increases, so does time employed to process each simulation cycle. An optimal frame rate in surgical simulation must range the interval [30-40]Hz [22]. Hence, with regard to the performance, a balance between fidelity and real-time processing must be achieved by designers of surgical simulators [24].

## III. STATE OF ART

In this section the state of the art in collision handling is briefly described. It is not restricted to the field of surgical simulation, but other application domains such as animation production and garment simulation are also considered.

For collision handling, two main approaches can be found in the literature, namely, those based on physical properties of the deformable object and those based on geometrical cues.

About the former, two main proposals deserve to be highlighted: constraint-based and penalty-based methods [25], [26]. Those approaches based on constraints aim at finding the precise time instant in which the objects would have collided so that the objects can be shifted back to their positions at that instant. Needless to say, these methods are characterized by a non-negligible computational burden [27] and their use in real time systems may be objectionable. Penalty-based methods apply a force to the surfaces in contact that is proportional to the penetration depth of the rigid object into the deformable one [28]. An important effort has been carried out to calculate this force fast [29] so that the overall computational load is acceptable even for real time applications. These methods, however, made no provision to respect the geometry in the contact and some interpenetration may be observed some time after the collision. Additionally, for rapid motions in which penetration depth may be large, discontinuities or instabilities in the biomechanical model have been observed [30].

Regarding those approaches based on geometrical cues, they aim at determining how the colliding surfaces must be

reallocated in order to avoid interpenetration between them. In [18] an additional care is taken not to create instabilities in the biomechanical models when some of its vertices have been moved. In the field of surgical simulators, some examples of geometrical cues can be found [27], [30], although the way the motion of these vertices is computed is not clearly described. The most intuitive solution is probably to shift the vertices of the deformable object to the closest rigid object surface in the direction of the outer normal of this surface. However, undesirable effects can be observed in this case as we will indicate in Section V.

On the other hand, real time response in surgical simulation is a must. Therefore, simplifications of the problem to be solved are usually needed to meet this requirement. As mentioned in section II, some methods limit the interaction to straight lines or the tip of the tool. In these cases, collision only takes place with one point of the tool. Needless to say, this may constitute a realistic approximation if the tool is a needle [31], but a realistic response may not be obtained when the volume of the tool is not negligible, as it is the case with scissors or tweezers. Nevertheless, other approaches deal with several colliding points and, in these cases, other simplifications are considered. In [27], [30] cylinders are used as a model for the tool. In the first work, vertices are moved orthogonally to the axis of the tool towards the closest tool surface, whereas in the second one vertices are moved to their projections on a plane defined by the tool axis and whose normal is the average of the normals of the colliding facets. These conditions may cause facets to cross if the contacting surface is not sufficiently regular. Additionally, the interpenetration between both objects is not totally avoided since, if two neighboring vertices must be shifted to different planes, the facet subtended by them might cross the volume of the rigid object.

A related problem also appeared in our previous work [15]. Here, collisions were efficiently solved taking into account several colliding points and the displacement was computed from different kinds of movement. However, the result was also dependent on tool facets normals. This could cause completely opposite displacement decisions for very similar situations represented by almost identical normal vectors. Taking into account these normals can also lead to ambiguous situations where possibly incorrect displacement decisions are made.

#### IV. COLLISION HANDLING

Let us define two main vectors which represent the state of the scene. First, the normalized motion vector  $\vec{v}_m$  represents the direction of the displacement of the tool towards the organ between two simulation steps<sup>1</sup>. Second, a vector that represents the organ surface. Collisions may affect several vertices of the deformable model. It was adopted in preliminary works to jointly deal with all colliding vertices, because facet crossings may be easily avoided this way. Nevertheless, if the

<sup>1</sup>Even though this displacement is the most common cause of collision, it can be the case that a collision occurs the other way round, i.e., because of the organ moves towards the tool. This situation is tackled by considering the tool motion vector as the reverse of the organ motion vector ( $\vec{v}_m = -\vec{v}_m$ ). In the case that the two of them move, vectors are added

collision occurs in two opposite facets of the tool, all vertices move together over one of them and the tool is crossed. This effect can appear in basic simulation scenes like cutting. Thus, an independent vertex handling is adopted. Each outer normal to the facet  $j$  that surrounds the vertex  $i$  is taken into account, say  $\vec{n}_{def_{i,j}}$ . Both these vectors are normalized.

On the other hand, three cases of interaction can be distinguished, named: (1) Penetration, when  $\vec{v}_m$  and  $\vec{n}_{def_{i,j}}$  tend to be parallel but opposite ( $\pi$  radians); (2) Sliding, in which  $\vec{v}_m$ ; and  $\vec{n}_{def_{i,j}}$  tend to be perpendicular; and (3) Extraction, if  $\vec{v}_m$  and  $\vec{n}_{def_{i,j}}$  tend to be parallel (0 radians). A fuzzy system is here introduced. It takes into account a collision index  $I_C$  calculated as:

$$I_C = \vec{v}_m \cdot \vec{n}_{def_{i,j}} \quad (1)$$

And in order to bear in mind all the surrounding facets of a vertex, the fuzzy system is feedbacked. The system outputs the tuple  $\mathbf{I} = \{I_P, I_S, I_E\}$ , which weigh the degree of similarity of the situation to each interaction case and it is also used as an input for the next iteration. Hence, the vector displacement of each collided vertex of the deformable model,  $\vec{v}_{displ_i}$ , is obtained by:

$$\vec{v}_{displ_i} = I_P \vec{v}_m - I_S \vec{n}_{def_{i,j}} - I_E \vec{v}_m \quad (2)$$

Finally, Figure 2 shows the respective membership functions of linguistic variables  $I_C$  and the tuple  $\mathbf{I}$ . It can be seen in Figure 2.a that  $\epsilon$  symbolizes a threshold among the interaction cases. These cases are considered to be equally probable so, with regard to (1), the limit of the angle between  $\vec{v}_m$  and  $\vec{n}_{def_{i,j}}$ , in order to distinguish each case, will be 45. Therefore,  $\epsilon$  has been set to 0.7071.

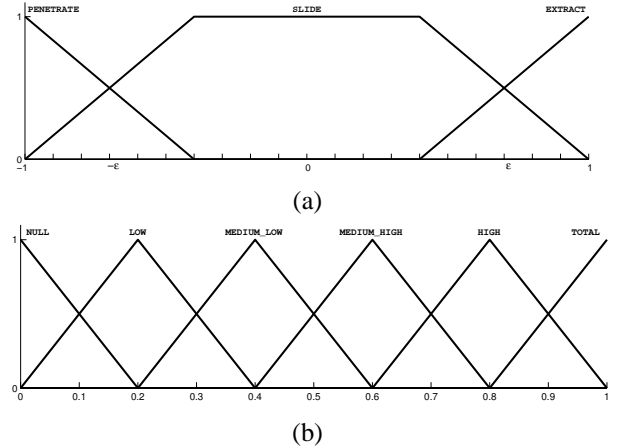


Fig. 2. Membership functions: a) For linguistic variable  $I_C$ ; b) For linguistic variables  $I_P, I_S, I_E$ .

The fuzzy rule base is described in Table I. The first three rules govern the displacement of vertex  $i$  in the first iteration, considering  $\vec{v}_m$  and  $\vec{n}_{def_{i,j}}$ . Rules 4 and 5 determine whether the controller is in the first execution cycle or in feedback cycles, through the fuzzy control variable  $C_1$ , which can adopt the discrete values 0 or 1 converted to the linguistic terms YES

TABLE I  
PROPOSED INFERENCE RULES

Number	Rule
1	If $I_C = \text{PENETRATE}$ then $I_P = \text{TOTAL}$ and $I_S = \text{NULL}$ and $I_E = \text{NULL}$
2	If $I_C = \text{SLIDE}$ then $I_P = \text{TOTAL}$ and $I_S = \text{NULL}$ and $I_E = \text{NULL}$
3	If $I_C = \text{EXTRATE}$ then $I_P = \text{TOTAL}$ and $I_S = \text{NULL}$ and $I_E = \text{NULL}$
4	If $C_1 = \text{YES}$ then $C_1 = \text{NO}$
5	If $C_1 = \text{NO}$ then $C_1 = \text{YES}$
6	If $C_1 = \text{NO}$ and $I_P = X$ and $I_S = Y$ and $I_E = Z$ then $I_P = X$ and $I_S = Y$ and $I_E = Z$

and NO. The set of rules that define the feedback is stated in rule 6, where X, Y and Z correspond to the linguistic terms in Figure 2.b.

## V. EXPERIMENTAL RESULTS

In order to show the performance of our algorithm in a real laparoscopic surgery simulator, the one developed in the project SINERGIA [8] is employed. Boundary conditions reported and justified in Section II are taken into account. We will next depict different situations that help describe the performance of the proposed method and its advantages over other related approaches.

First, Figure 3 illustrates the importance of the distinction between cases. Specifically, both penetration and sliding are shown in Figures 3.a and 3.b. The effect of managing a sliding as if it were a penetration can be shown in Figure 3.c. Clearly, the vertices of the deformable object are put together, pulled by the tool motion, causing an unnatural resemblance.

Next, Figure 4 depicts the situation created if the displacement of all vertices is jointly computed. As mentioned in Section IV, this approach is straightforward but it has two major drawbacks. The displacement causes less natural results than the method proposed here, and it may causes an incorrect tool crossing in some simulation scenes.

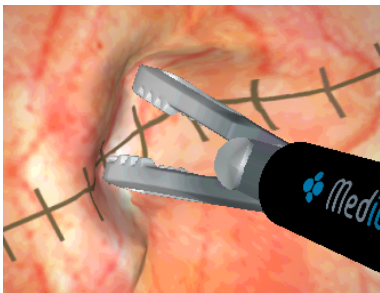


Fig. 4. Displacement of all vertices is jointly computed.

Once the distinction between different collision cases and the computation of an individual displacement for each vertex have been shown to overcome some important problems in the surgical simulation scenarios, we will compare the proposed method to the one proposed in [30]. This solution moves each vertex to the nearest face of the tool, according to the outer normal of this face. Figure 5.a depicts the situation when the movement is slow (in this case, the interpenetration is avoided). Figure 5.b shows that, if rapid motions are performed with the tool, the nearest face of the colliding vertices would never be the tool tip, but a side face. This causes a dramatic penetration of the tool into the organ. Figures 5.c and 5.d show the wire model for the described situations in Figures 5.a and 5.b, respectively, where the interaction between the tool and the organ can be better understood.

Finally, in Figure 6.a, it can be seen the effect of taking into account the faces of the tool in the computation of the displacement, which was described in Section III. Some of the computed displacement vectors cross each other, which is not correct. Figure 6.b, depicts how the proposed approach overcomes this problem almost completely.

## VI. CONCLUSIONS AND FUTURE WORK

In this paper, a simple algorithm has been proposed that provides collision responses in real time for an actual laparoscopic surgical simulator. We have chosen a geometric and kinematic approach, as opposed to a biomechanical approach, and operations have been carried out by a simple scalar product and a fuzzy logic system. The algorithm can deal with several colliding points simultaneously as well as with commonly-used surgical tools. Also, the approach displaces each vertex independently and without taking into account tool-facets. This yields more natural results and avoids some vertex-crossing problems, as has been graphically shown. Even though, some problems of vertex-crossing can still occur. Future work will focus on addressing this problem by means of fuzzy control and adopting fractional simulation steps.

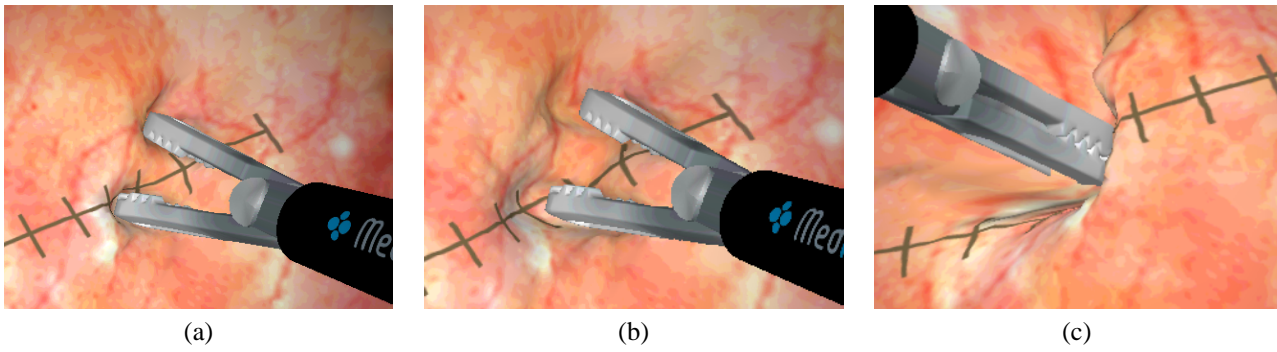


Fig. 3. a) Penetration case using the proposed approach; b) Sliding case with the proposed approach; c) Sliding considered only as penetration.

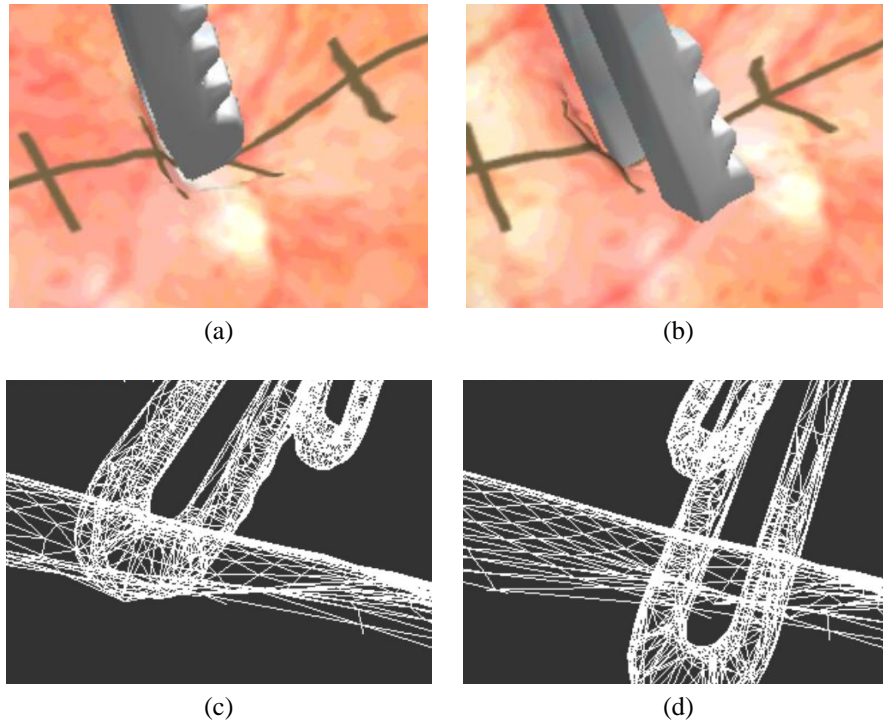


Fig. 5. a) Slow movement of the tool; b) Dramatically penetration of the tool in the organ due to a rapid movement; c) and d) Wire models of the same situation.

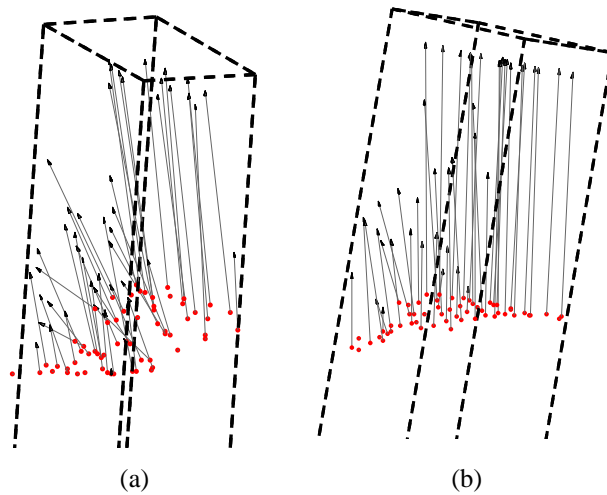


Fig. 6. a) Computed displacement vectors using the method proposed in [15]; b) Displacement of vertices using the method proposed in this paper.

## REFERENCES

- [1] K. Fuchs, "Minimally Invasive Surgery," *Endoscopy*, vol. 34, no. 2, pp. 154–159, 2002.
- [2] T. Tang, S. Dolan, B. Robinson, and L. Delbridge, "Does the surgical approach affect quality of life outcomes? A comparison of minimally invasive parathyroidectomy with open parathyroidectomy," *International Journal of Surgery, In Press, Corrected Proof*, 2006.
- [3] R. Aggarwal, K. Moorthy, and A. Darzi, "Laparoscopic skills training and assessment," *British Journal of Surgery*, vol. 91, no. 12, pp. 1549–1558, December 2004.
- [4] A. Park, D. Witzke, and M. Donnelly, "Ongoing deficits in resident training for minimally invasive surgery," *Journal of Gastrointestinal Surgery*, vol. 6, no. 3, pp. 501–507, 2002.
- [5] A. G. Gallagher, E. M. Ritter, H. Champion, G. Higgins, M. P. Fried, G. Moses, C. D. Smith, and R. M. Satava, "Virtual reality simulation for the operating room: proficiency-based training as a paradigm shift in surgical skills training," *Annals of Surgery*, vol. 241, no. 2, pp. 364–72, 2005.
- [6] N. E. Seymour, A. G. Gallagher, S. A. Roman, M. K. O'Brien, V. K. Bansal, D. K. Andersen, and R. M. Satava, "Virtual reality training improves operating room performance: results of a randomized, double-blinded study," *Annals of Surgery*, vol. 236, no. 4, pp. 458–63, 2002.
- [7] S. L. Dawson and J. A. Kaufman, "The imperative for medical simulation," *Proceedings of the IEEE*, vol. 86, no. 3, pp. 479–483, 1998.
- [8] F. Sánchez, E. J. Gómez, C. Monserrat, S. Pascual, M. Alcañiz, F. del Pozo, and J. Usón, "Sinergia: technologies for surgical simulation and planification in minimally invasive surgery," in *XXI Congreso Anual de la Sociedad Española de Ingeniería Biomédica (CASEIB'03)*, November 2003.
- [9] C. Basdogan, S. Kim, J. Kim, and H. Srinivasan, "Haptics in minimally invasive surgical simulation and training," *Computer Graphics and Applications, IEEE*, vol. 24, no. 2, pp. 56–64, 2004.
- [10] "Wtk library;" <http://sese8.sierraweb.net/products/wtk8>.
- [11] P. Jiménez, F. Thomas, and C. Torras, "3D Collision Detection: A Survey," *Computers and Graphics*, vol. 25, no. 2, pp. 269–285, April 2001.
- [12] C. Zilles and J. Salisbury, "A constraint-based god-object method for haptic display," in *IEEE International Conference Intelligent Robots Systems, Human Robot Interaction, and Co-operational Robots*, vol. 3, 1995, pp. 146–151.
- [13] C.-H. Ho, C. Basdogan, and M. A. Srinivasan, "Ray-based haptic rendering: Force and torque interactions between a line probe and 3d objects in virtual environments," *The International Journal of Robotics Research*, vol. 19, no. 7, pp. 668–683, 2000.
- [14] C. Forest, H. Delingette, and N. Ayache, "Surface contact and reaction force models for laparoscopic simulation," in *International Symposium on Medical Simulation*, June 2004.
- [15] V. García Pérez, E. Muñoz Moreno, S. Rodríguez Bescós, S. Aja Fernández, and C. Alberola López, "Respuesta Geométrica a Colisiones Basada en Lógica Borrosa," in *XXIII Congreso Anual de la Sociedad Española de Ingeniería Biomédica (CASEIB'05)*, Madrid, Spain, 10-12 November 2005, pp. 73–76.
- [16] H. Delingette, "Toward realistic soft-tissue modeling in medical simulation," *Proceedings of the IEEE*, vol. 86, no. 3, pp. 512–523, 1998.
- [17] "Centro de investigación e innovación en bioingeniería," <http://www.mediclab.upv.es/>.
- [18] D. Baraff and A. Witkin, "Large steps in cloth simulation," in *SIGGRAPH '98: Proceedings of the 25th annual conference on Computer graphics and interactive techniques*. New York, NY, USA: ACM Press, 1998, pp. 43–54.
- [19] U. Meier, O. López, C. Monserrat, M. Juan, and M. Alcañiz, "Real-time deformable models for surgery simulation: a survey," *Computer Methods and Programs in Biomedicine*, no. 77, pp. 183–197, March 2005.
- [20] "Epidaure: a research project in medical image analysis, simulation and robotics at INRIA," *IEEE Transactions on Medical Imaging*, vol. 22, no. 10, pp. 1185–1201, October 2003.
- [21] C. Monserrat, "Modelos Deformables de Tejidos Elásticos en Tiempo Real," Ph.D. dissertation, Universidad Politécnica de Valencia, September 1999.
- [22] J. Kim, S. De, and M. Srinivasan, "Computationally efficient techniques for real time surgical simulation with force feedback," in *HAPTICS '02: Proceedings of the 10th Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems*. Washington, DC, USA: IEEE Computer Society, 2002, p. 51.
- [23] A. Liu, F. Tendick, K. Cleary, and C. Kaufmann, "A Survey of Surgical Simulation: Applications, Technology, and Education," *Presence: Teleoperators and Virtual Environments*, vol. 12, no. 6, pp. 599–614, 2003.
- [24] M. Slater, A. Steed, and Y. Chrysanthou, *Computer Graphics and Virtual Environments: from realism to real-time*, 1st ed., Addison-Wesley, Ed. Edinburgh Gate, Harlow, Essex CM20 2JE: Pearson Education Limited, 2002.
- [25] J. C. Platt and A. H. Barr, "Constraints methods for flexible models," in *SIGGRAPH '88: Proceedings of the 15th annual conference on Computer graphics and interactive techniques*. New York, NY, USA: ACM Press, 1988, pp. 279–288.
- [26] A. Deguet, A. Joukhadar, and C. Laugier, "Models and algorithms for the collision of rigid and deformable bodies," in *WAFR '98: Proceedings of the third workshop on the algorithmic foundations of robotics on Robotics: the algorithmic perspective*. Natick, MA, USA: A. K. Peters, Ltd., 1998, pp. 327–338.
- [27] C. Forest, H. Delingette, and N. Ayache, "Surface contact and reaction force models for laparoscopic simulation," in *International Symposium on Medical Simulation*, June 2004.
- [28] L. Raghupathi, L. Grisoni, F. Faure, D. Marchal, M.-P. Cani, and C. Chaillou, "An intestinal surgery simulator: Real-time collision processing and visualization," *IEEE Transactions on Visualization and Computer Graphics*, vol. 10, no. 6, pp. 708–718, November-December 2004.
- [29] G. Hirota, S. Fisher, and M. Lin, "Simulation of non-penetrating elastic bodies using distance fields," Chapel Hill, NC, USA, Tech. Rep., 2000.
- [30] G. Picinbono, J.-C. Lombardo, H. Delingette, and N. Ayache, "Improving realism of a surgery simulator: linear anisotropic elasticity, complex interactions and force extrapolation," *Journal of Visualization and Computer Animation*, vol. 13, no. 3, pp. 147–167, July 2002.
- [31] W. Chou and T. Wang, "Human-Computer interactive simulation for the training of minimally invasive neurosurgery," in *IEEE International Conference on Systems, Man and Cybernetics*, vol. 2, 5-8 October 2003, pp. 1110–1115.