Bivariate Phase Space Divergence: A Measure for the Automatic Detection and Anticipation of Epileptic Seizures in ECoG Data

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Abstract— **To detect and predict epileptic seizures from brain electrical potentials, changes in the dynamics of the underlying nonlinear system can be utilized. Several approaches exist, where first a univariate signal is embedded into phase space and then features are extracted from the reconstructed states in phase space. In this paper we introduce a novel method that uses a bivariate approach in phase space, the Bivariate Phase Space Divergence (BPSD), to detect dynamical changes due to an epileptic seizure. To calculate BPSD, we embed two simultaneously recorded time series into phase space and define the divergence as the increase in the distance between two initially neighboring points on the two trajectories. The value of BPSD is large if two neighboring states diverge quickly in time, whereas small values are found for states that show a similar evolution in phase space. By directly using a bivariate approach of a channel-wise embedded bivariate signal our method is different from other measures that are used to detect changes in dynamics using only one single univariate embedding.**

When applying our measure to ECoG data we found high values of BPSD in the seizure free time and a clear drop of the feature during the ictal period. Our findings are in good agreement with the proposed neurophysiological mechanism that different areas of the brain synchronize during an epileptic seizure which should lead to a similar evolution of neighboring states in phase space. To compare our measure with results from univariate algorithms we calculate the mean of all BPSD combinations for one channel versus all other channels of one hemisphere and compare these results with the Short Term Lyapunov exponent.

I. Introduction

1% of the world's population suffer from epilepsy. In about 65% of these patients seizures can be controlled efficiently with anti-epileptic drugs. Another 8% can be treated with epilepsy surgery. For a successful surgical treatment the exact localization of the epileptic focus is required. To locate these focal regions, longtime electroencephalograms (EEG) and in ambiguous cases electrocorticograms (ECoG) are recorded. Due to the long recording period, which typically is in the order of 90 hours, the analysis of these data is a very time consuming and expensive process. Automatic online seizure detection during recording could reduce the analysis time significantly. In addition, if seizures could be reliably predicted, an automatic warning system could help those patients that can not be treated with drugs or surgery.

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Several features have been reported in the literature that can be applied to detect or predict epileptic seizures from electrical brain signals. These approaches include e.g. energy based measures [1], [2], [3], wavelet-based methods [4] or synchronization based methods [5], [6], [7], [8]. Methods from nonlinear time series analysis applied to ECoG recordings have proven to give important information about dynamical changes in the brain leading to the occurrence of seizures [9], [10]. One of these measures that has been applied to the analysis of EEG-data is the *Largest Short Term Lyapunov Exponent* (STLmax) [11], [12], [13], which is an indicator for chaoticity of the underlying system. For the estimation of the STLmax, the signal from one recording site is embedded into phase space and the feature is extracted from the embedded states. However, it does not exploit information about the spatio-temporal behavior of the underlying system that can be extracted from a multivariate signal recorded simultaneously at different recording sites unless the results obtained from signals from different channels are combined in a new feature for multivariate signals [12], [13].

In this paper we introduce a novel method for bivariate time series to calculate a feature in phase space that can be interpreted as a measure for predictability of one signal in phase space given a different signal. We use the conventional delay embedding introduced by Takens [14] for the reconstruction of the phase space for each signal and quantify the divergence of neighboring states in phase space coming from different recording sites. In contrast to measures like STLmax we directly use a bivariate approach to calculate the divergence feature from those different embedded signals.

II. Method

A. Bivariate Phase Space Divergence

For any finite time series, points in phase space of the underlying system can be reconstructed using the delay embedding method described by Takens [14]. With adequate choice of phase space dimension D and time delay τ this reconstructed phase space is equivalent to the phase space of the underlying system.

Given a bivariate finite time series X of length T as

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,T} \\ x_{2,1} & x_{2,2} & \dots & x_{2,T} \end{bmatrix},$$

the set of points $\{\mathbf{x}_{s,t} | s = 1, 2; t = 1, 2, \dots, T - (D-1)\tau\}$ in related phase space of dimension D can be reconstructed

by delay embedding with

$$\mathbf{x}_{s,t} = [x_{s,t}, x_{s,t+\tau}, x_{s,t+2\tau}, \ldots, x_{s,t+(D-1)\,\tau}].$$

Note that for such an embedding the length T of the time series must be sufficiently large, i.e. $T \gg (D-1)\,\tau$. Any sequence $[\mathbf{x}_{s,t_1}, \mathbf{x}_{s,t_1+1}, \ldots, \mathbf{x}_{s,t_2}]$ in phase space with $\{t_1, t_2\} \in \mathbb{N}$ and $t_1 < t_2 < T - (D-1)\,\tau$ is called a trajectory.

To calculate a measure for divergence in phase space using two different time series $\{x_{s,t} | s = 1, 2; t = 1, 2, \ldots, T\}$ of a system, for every reconstructed point in phase space $\mathbf{x}_{s_1,t_1}$ the nearest point reconstructed from the other time series $\mathbf{x}_{s_2,t(t_1)}$, with $s_1 \neq s_2$, t_1, $t(t_1) < T - (D-1)\tau - \Delta t$ and $t(t_1) \notin [t_1 - \theta, t_1 + \theta]$ with $\theta \ll T$ is taken. Let $d(\mathbf{x}_{s_1,t_1}, \mathbf{x}_{s_2,t(t_1)})$ denote the euclidean distance of these two points at time t_1. Comparing this distance with the distance $d(\mathbf{x}_{s_1,t_1+\Delta t}, \mathbf{x}_{s_2,t(t_1)+\Delta t})$ of the trajectories at time $t_1 + \Delta t$ and $t(t_1) + \Delta t$, respectively, the divergence of the trajectories can be quantified. If the trajectories diverge from each other, such that the state $\mathbf{x}_{s_1,t_1+\Delta t}$ is not in the vicinity of $\mathbf{x}_{s_2,t(t_1)+\Delta t}$, it is impossible to draw any conclusions from $\mathbf{x}_{s_2,t(t_1)}$ to the future state $\mathbf{x}_{s_2,t(t_1)+\Delta t}$ by following the trend of a neighboring trajectory in phase space starting at $\mathbf{x}_{s_1,t}$. In the other case, if neighboring trajectories continue to run close to each other, future states of the reconstructed trajectories $\mathbf{x}_{s_1,t_1+\Delta t}$ of one time series will help to forecast future states $\mathbf{x}_{s_2,t(t_1)+\Delta t}$ of the other trajectory.

For exponential divergence the distance between succeeding points on these trajectories can be approximated by an exponential function

$$d(\mathbf{x}_{s_1,t_1+\Delta t}, \mathbf{x}_{s_2,t(t_1)+\Delta t}) \approx d(\mathbf{x}_{s_1,t_1}, \mathbf{x}_{s_2,t(t_1)})\, e^{\lambda \Delta t}. \quad (1)$$

We now introduce the *mean divergence function* depending on Δt as

$$P(\Delta t) = \frac{1}{T - \Delta t} \sum_{t_1=1}^{T-\Delta t} log(d(\mathbf{x}_{s_1,t_1+\Delta t}, \mathbf{x}_{s_2,t(t_1)+\Delta t})).$$

With (1) the slope of this function is approximately linear. To quantify the increase of the distance between points $\mathbf{x}_{s_1,t_1+\Delta t}$ and $\mathbf{x}_{s_2,t(t_1)+\Delta t}$ on the two trajectories, we take the first derivative of $P(\Delta t)$ as our measure for BPSD.

B. ECoG Data

Electrocorticograms (ECoG) were recorded from three patients suffering from temporal lobe epilepsy. For presurgical evaluation four to five stripes with six or eight electrodes have been surgically implanted subcranially over the temporal lobe. For one patient the schematic electrode placement is depicted in Fig. 1B. On each electrode contact, electrical potentials were recorded with a sampling frequency of 256 Hertz. ECoGs were measured over a period of at least 90 hours. If possible, for our analysis we used sequences of about 2.5 hours before and after the ictal phase that did not contain any further seizures (for details see table I). The unequivocal seizure onset for each patient, i.e., the time where first epileptic patterns occur in the ECoG, and the focus electrodes, where these patterns can be seen at first, were defined by clinical experts.

TABLE I

CHARACTERISTICS OF ANALYZED ECOG-DATA: PATIENTS ARE NUMBERED FROM 1-3, LOCATION OF THE FOCUS WAS EITHER LEFT OR RIGHT TEMPORAL WITH GENERALIZED EPILEPTIC ACTIVITY IN PATIENT 3

Patient	Location of epileptogenic focus	Nr. of electrodes	Duration	Nr. of seizures recorded
1	right temporal	32	~11h	3
2	1st seizure left temporal, others right temporal	28	~11h	4
3	starts right temporal, later generalized	32	~3h	3
Total			~25h	10

To calculate BPSD on ECoG data, we first reduced recording artifacts by excluding sequences which show extremely high or low amplitudes or variance. 50Hz line noise was removed using a notch filter and the normalized signal (zero mean, unit variance) was embedded into phase space. For the embedding dimension D and time delay τ we chose constant values of $D = 7$ and $\tau = 15$ ms [12]. A moving average filter of length 10 was applied after the calculation of BPSD within a moving window.

C. Latency

An important parameter for automatic seizure detection is the latency. It is defined as the time between the unequivocal seizure onset and the first drop of the signal below a given threshold. The threshold was determined as the minimum value of BPSD after application of a moving average filter of length 10 over a period of 20 minutes before seizure onset.

III. RESULTS

A. BPSD of ECoG data from epilepsy patients

BPSD was calculated for all seizures of the three patients for all possible channel combinations. Figure 1A shows typical time courses of five hours of BPSD calculated from data of patient one (see table I). The feature was calculated within non-overlapping moving windows of five seconds. The focal channels for this seizure were C2, D1 and D2. The example shows the BPSD between channel C2 and C1, C3-C5, D1-D4, all lying within the focal area of the patient. The inset (Fig. 1B) gives the anatomical position of these eight electrodes highlighted by a gray polygon.

An ictal decrease of BPSD is clearly visible for all combinations shown. The strongest drop is found for those combinations that contain two of the three focus electrodes (C2/D1 and C2/D2). This drop is less pronounced for combinations that contain electrodes located more distant from the focal area (C2/C5, C2/D4). In addition to the ictal drop, an increase of BPSD is found during the postictal phase. This increase lasts for about 60 minutes.

To determine the latency of the ictal drop, Fig. 1C and 1D show magnifications of BPSD five minutes before and after seizure onset for the two combinations C2/C3 and C2/D2. The unequivocal seizure onset is marked with a dot-dashed line. The first data point that drops below threshold is marked with

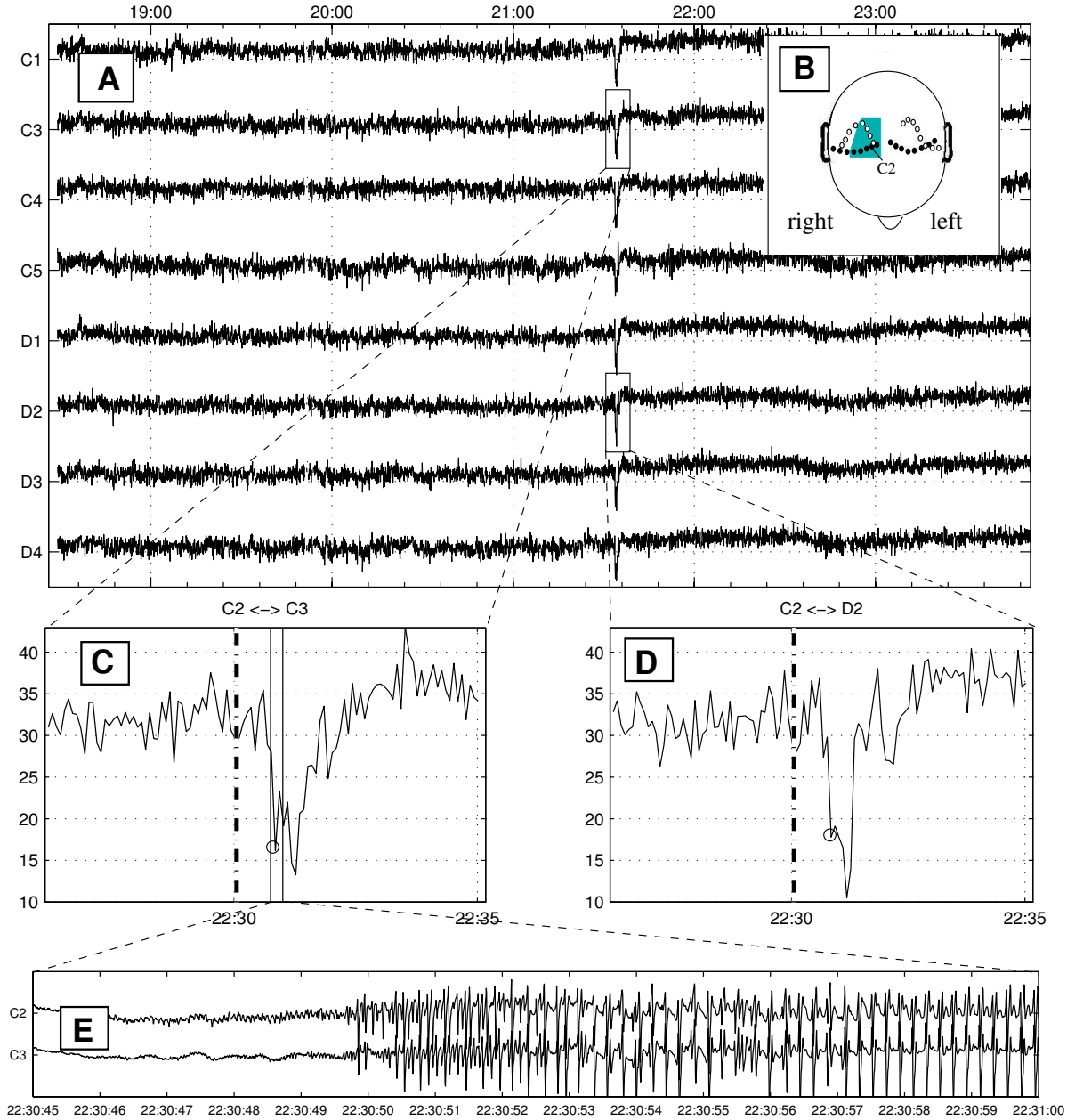

Fig. 1. **A:** Five hours of BPSD calculated between data from focus channel C2 and channels in the vicinity of the focal area as labeled on the y-axis. **B:** Schematic view of the electrode placement for patient one. Filled and open circles in the right hemisphere are referred to as channels C1 to C8 and D1 to D8, respectively, on the left hemisphere as A1 to A8 and B1 to B8, respectively. Electrodes located more medial have smaller numbers. The polygonal area marks the electrodes used for the calculation in A. **C and D:** Magnification of the BPSD for channels C2/C3 and C2/D2, respectively. The dot-dashed line marks the unequivocal seizure onset. The first data point below threshold is marked with a circle. The latency relative to seizure onset was 45 seconds. **E:** Raw ECoG-sequence recorded on electrodes C2 and C3 in the time window indicated by solid vertical lines in C.

a circle. For this example we found a latency of 45 seconds for both channel combinations. In order to determine which property of the ECoG-signal leads to the drop in the BPSD, we compared the time course of the feature with the raw ECoG-Signal. Figure 1E shows these ECoG-sequences recorded at electrodes C2 and C3. The time period of the raw signal is indicated by solid vertical lines in Fig. 1C. The circle marks the first drop of BPSD below threshold and corresponds to a time window between 22:30:51 and 22:30:56. Comparing raw signal and BPSD shows that the initial large drop in BPSD is due to strong oscillations starting at 22:30:51.

Analysis of all 25 hours of ECoG data from three different patients containing ten temporal lobe epileptic seizures revealed a clear drop of the BPSD for several channel combinations during the ictal period for each seizure. This drop was most clearly seen in combinations containing at least one channel placed in the vicinity of the focal area. In contrast, BPSD calculated between channels far away from the focal region showed no drop or only a small decrease during the ictal period. We found an increase of BPSD lasting for

15 minutes to one hour during the postictal phase for all seizures. However, the strength of this increase showed a high variability within as well as between patients.

Analysis of the latency of BPSD between unequivocal seizure onset and the time point where it drops below a given threshold revealed results that varied significantly between patients. Whereas patient one showed latencies of approximately 45 seconds, patients two and three showed smaller latencies of about 10 to 20 seconds. In some channel combinations even smaller latencies could be observed and for two seizures of patient two BPSD dropped below threshold slightly before unequivocal seizure onset.

B. Comparison of BPSD with STLmax

A univariate measure related to BPSD and applied frequently to ECoG-data is the STLmax [11], [12]. It determines the increase of the distance between initially neighboring points of one trajectory obtained by embedding only one time series whereas BPSD quantifies the increase of the distance between points on two trajectories obtained by embedding two different signals. In order to compare the two measures we calculated the mean of BPSD for the combination of all channels of the hemisphere where the epileptic focus was located with a given channel. STLmax was calculated using the same embedding parameters as for BPSD.

When comparing STLmax computed for a focus channel with the mean BPSD for the same focus channel versus all other channels of one hemisphere we found comparable drops for BPSD and STLmax for all seizures studied. An example taken from patient one is shown in Fig. 2. In addition, we found several seizures where the drop in BPSD was more pronounced. During some seizures BPSD shows significant drops over a longer period of time. In addition, the variance of the signal outside the ictal period was often lower for the mean BPSD than for STLmax.

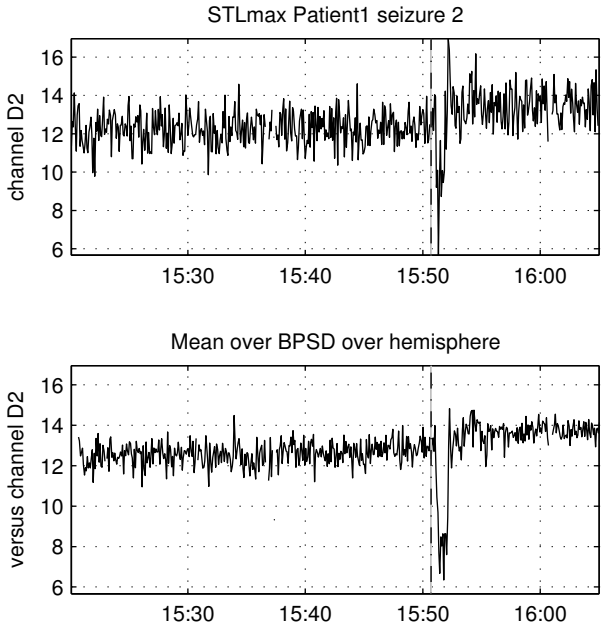

Fig. 2. Comparison of STLmax calculated for electrode D2 (top) and the mean of BPSD for channel D2 and all other channels of the same hemisphere (bottom). The dot-dashed line marks the unequivocal seizure onset.

IV. Discussion

Our results of the calculation of BPSD showed that it can be used for a reliable detection of epileptic seizures from ECoG-Data. The decrease of the feature during ictal periods is strongest for combinations containing at least one focal channel and less pronounced if channels more distant from the focal region are involved. Comparing the mean of BPSD with estimation of STLmax showed a comparable decrease during seizures. However, smaller variance of the mean BPSD outside the ictal periods might make this measure a more promising candidate for epileptic seizure detection.

In contrast to STLmax, BPSD can not be interpreted as a measure for chaoticity. Whereas small values of BPSD are an indicator for less chaoticity of the underlying systems, increasing values can not be interpreted as an increase of chaoticity because high values can also be found for non chaotic but independent systems. The decrease of BPSD can instead be interpreted as an increase in predictability of one channel given a second channel. Two points on two different trajectories that are close in state space reflect similar states for the two underlying systems. If BPSD shows small values, these two neighboring states on the two different trajectories depart slowly in time. Hence, information about the time course of the state of one channel could help to predict the future evolution of the state of a second channel. If, in turn, BPSD shows large values, i.e. the two states depart quickly in time, it becomes increasingly difficult to predict the future evolution of one state given information about the second. Due to this interpretation it would be interesting to study BPSD in the context of *Granger causality* (GC). GC quantifies the directed causal dependence between two time series. [15], [16]. A common approach is to compare the regression error of two separate univariate signals with the regression error of the combined bivariate signal. Further research should be done to investigate the relationship of BPSD and GC.

A disadvantage of BPSD in the context of epileptic seizure detection is the time consuming computation needed to calculate this measure. In order to reconstruct a trajectory that reflects the dynamics of the system reasonable well, a sliding window has to have a minimal size. A nearest neighbor search has to be done for each channel combination, which for larger sliding windows is a very expensive function. Given signals from n different channels and using the symmetry of BPSD, i.e. BPSD for channel combination A/B is equal to channel combination B/A, one has to calculate $\frac{(n+1)\cdot n}{2}$ channel combinations for each sliding window.

Extending the bivariate version described within this paper, BPSD can be formulated for the multivariate case involving more than two channels. While it is fairly straight forward to extend the computation of nearest neighbor search to a multivariate signal, finding a divergence function that allows a meaningful interpretation is not obvious for more then two

signals. Furthermore, a multivariate version of BPSD will most likely result in an even more time consuming computation.

V. Conclusions

In this paper we introduced a novel measure for the analysis of bivariate time series in phase space that can be applied for a reliable detection of epileptic seizures. Performance as defined by an ictal drop relative to baseline is comparable with STLmax but the mean BPSD shows a smaller variance outside the ictal period. It can be interpreted as a measure for phase space predictability. Further research will be necessary to compare this measure with other prediction measures such as GC and to further explore the spatio-temporal information contained in BPSD.

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