Proceedings of the 28th IEEE
EMBS Annual International Conference
New York City, USA, Aug 30-Sept 3, 2006

Meshless Methods in Potential Inverse Electrocardiography

Yong Wang, and Yoram Rudy, *Fellow, IEEE*

***Abstract*—Potential Inverse Electrocardiography (PIE) is a method for reconstruction of epicardial potentials from measured body surface electrocardiograms and heart-torso geometry. The method of choice for computing epicardial potentials has been either the Boundary Element Method (BEM) or the Finite Element Method (FEM). These methods require time-consuming meshing of the heart and torso surfaces or the volume between the two surfaces. Moreover, meshing can introduce artifacts in the reconstructed epicardial images if optimization is not carefully done. Here we introduce the application of a meshless method, the Method of Fundamental Solutions (MFS) to PIE in the hope of overcoming such meshing-related problems. This study shows that MFS is a promising meshless alternative to BEM and FEM in PIE.**

I. INTRODUCTION

Computation of potentials on the epicardial surface of the heart from potentials measured on the body surface involves solving Laplace's equation in the source-free volume between the torso and epicardial surfaces. Several mathematical and computational approaches have been introduced to solve this problem, known as Potential Inverse Electrocardiography (PIE). [1-4] Traditionally, the Boundary Element Method (BEM) [5,6]' or Finite Element Method (FEM)[5] were used to solve Laplace's equation. [7-10] These numerical methods require discretizing the torso and epicardial surfaces into continuous non-overlapping mesh elements, a procedure called meshing. Meshing is difficult to apply to irregular surfaces [11] and can introduce mesh-related artifacts, especially in the ill-posed inverse computation of PIE[12], if mesh optimization is not carefully done. These difficulties led us to explore the possibility of applying a meshless method in PIE. PIE is a 3D Cauchy problem for the Laplace operator which has very well behaved, analytic fundamental solutions.[13] This property suggests the Method of Fundamental Solutions (MFS) [14] as the method of choice among the family of meshless methods. In this conference proceedings paper, we summarize a recently published article that describes MFS application in PIE.[15]

Manuscript received May 25, 2006. This work was supported by NIH-NHLBI Merit Award R37-HL-33343 and Grant R01-HL-49054 (to Yoram Rudy) and by a Whitaker Foundation Development Award.

Yong Wang is with the Department of Biomedical Engineering, Washington Unviersity in St. Louis, St. Louis, MO, 63130 USA (phone: 314-935-8163; e-mail: wangyong@wustl.edu).

Yoram Rudy is with the Department of Biomedical Engineering, Washington Unviersity in St. Louis, St. Louis, MO, 63130 USA (phone: 314-935-8160; fax: 314-935-8168; e-mail: rudy@wustl.edu).

II. METHODS

The MFS is one of the most intuitive approaches to solving Laplace's equation numerically. In MFS, an approximate solution is represented in the form of a linear superposition of source functions (fundamental solutions) located on a set of points (fictitious points or virtual sources) over an auxiliary surface $\hat{\Gamma}$ ($\hat{\Gamma}$ encloses the auxiliary domain $\hat{\Omega}$, which contains the domain of interest Ω). As the fundamental solutions satisfy Laplace's equation everywhere except at source points, this representation satisfies Laplace's equation in the domain Ω. In addition, the specified boundary conditions are imposed at a set of boundary points (collocation points) on the domain boundary Γ. Since the fundamental solutions do not have singularities at points on the boundary Γ, standard quadrature rules can be used to approximate the surface potential and its normal gradient when computed on the boundary. The Tikhonov regularization method[12] with a CRESO-determined regularization parameter[8] is employed to stabilize the inverse procedure and obtain the coefficients of fundamental solutions, which are used to calculate the epicardial potentials.

III. RESULTS

MFS is evaluated on data from animal experiments and human studies, and compared to BEM. Data sets included (i) Single-site pacing in an isolated canine heart suspended in a human torso-shaped tank; data were obtained during pacing from a right ventricular (RV) anterior epicardial location; [16] (ii) RV endocardial pacing in a patient undergoing bi-ventricular pacing for cardiac resynchronization therapy (CRT); (iii) Simultaneous RV endocardial pacing and left ventricular (LV) epicardial pacing in a patient undergoing bi-ventricular pacing for CRT; (iv) Normal atrial activation in a healthy human subject.[17] Results demonstrate similar accuracy, with the following advantages: 1. Elimination of meshing and manual mesh optimization processes. 2. Elimination of mesh-induced artifacts. 3. Elimination of complex singular integrals that must be carefully computed in BEM. 4. Simpler implementation.

IV. DISCUSSION

Implementation of MFS requires choosing a fictitious boundary for placement of the virtual source points. There are two widely used approaches for doing so, the dynamic and static methods.[11] In the dynamic configuration, the fictitious boundary is determined together with the solution[18] via a complex, time-consuming nonlinear optimization procedure which does not always guarantee global convergence. In the static configuration, the fictitious boundary is pre-selected to correspond with the real boundary based on some fixed criteria, such as the inflation-deflation rule used in this study. The static method is not the optimal implementation of MFS, but it is very easy to implement and highly suitable for practical engineering applications. Very good accuracy has been obtained using the static method in engineering and industrial applications of MFS,[19] including the application presented here. Although optimal placement of the virtual source points poses a difficulty for MFS implementation, much faster and more efficient nonlinear optimization schemes[20] are under development and could facilitate use of the dynamic method in PIE.

ACKNOWLEDGMENT

We thank Professor C.S. Chen from University of Southern Mississippi for very useful advice regarding MFS theory and implementation. We thank Dr. Bruno Taccardi for the torso-tank experiments, conducted in his laboratory at the University of Utah. We would also like to acknowledge the assistance of L. Ciancibello in acquiring and transferring the CT imaging data and K. Ryu for his assistance in body surface potential mapping. Special thanks go to Dr. Ping Jia for helpful discussions. This study was supported by NIH-NHLBI Merit Award R37-HL-33343 and Grant R01-HL-49054 (to Yoram Rudy) and by a Whitaker Foundation Development Award. Yoram Rudy is the Fred Saigh distinguished professor at Washington University in St Louis.

REFERENCES

[1] R. C. Barr and M. S. Spach, "Inverse solutions directly in terms of potentials," in *The Theoretical Basis of Electrocardiography*, C. V. Nelson and D. B. Geselowitz, Eds. Oxford: Clarendon Press, 1976, pp. 294-304.

[2] P. Franzone, B. Taccardi, and C. Viganotti, "An approach to inverse calculation of epicardial potentials from body surface maps," *Adv Cardiol*, vol. 21, pp. 50-4, 1978.

[3] Y. Rudy and B. J. Messinger-Rapport, "The inverse problem in electrocardiography: solutions in terms of epicardial potentials," *Crit Rev Biomed Eng*, vol. 16, pp. 215-68, 1988.

[4] Y. Yamashita, "Inverse solution in electrocadiography: determining epicardial from body surface maps by using the finite element method," *Jpn Circ J.*, vol. 45, pp. 1312-1322, 1981.

[5] G. Beer and J. O. Watson, *Introduction to Finite and Boundary Element Methods for Engineers*. West Sussex: Wiley, 1992.

[6] C. A. Brebbia, J. C. F. Telles, and L. C. Wrobel, *Boundary element techniques: Theory and applications in engineering*. Berlin, Germany: Springer Verlag, 1984.

[7] R. C. Barr and M. S. Spach, "Inverse calculation of QRS-T epicardial potentials from body surface potential distributions for normal and ectopic beats in the intact dog," *Circ Res*, vol. 42, pp. 661-75, 1978.

[8] P. Colli-Franzone, L. Guerri, S. Tentoni, C. Viganotti, S. Baruffi, S. Spaggiari, and B. Taccardi, "Mathematical procedure for solving the inverse problem of electrocardiography," *Math Biosci*, vol. 77, pp. 353-396, 1985.

[9] P. Jia, C. Ramanathan, R. N. Ghanem, K. Ryu, N. Varma, and Y. Rudy, "Electrocardiographic imaging of cardiac resynchronization therapy in heart failure: Observation of variable electrophysiologic responses," *Heart Rhythm*, vol. 3, pp. 296-310, 2006.

[10] Y. Rudy and J. E. Burnes, "Noninvasive electrocardiographic imaging (ECGI)," *Annals of Noninvasive electrocardiol.*, vol. 4, pp. 340-359, 1999.

[11] M. A. Golberg and C. S. Chen, "The method of fundamental solutions for potential, Helmholtz and diffusion problems.," in *Boundary Integral Methods - Numerical and Mathematical Aspects*, M. A. Golberg, Ed. Southampton-Boston: Computational Mechanics Publications, 1998, pp. 103-176.

[12] A. N. Tikhonov and V. Y. Arsenin, *Solution of Ill-Posed Problems*. Washington DC: VH Winston & Sons, 1977.

[13] Y. C. Hon and T. Wei, "Solving Cauchy problems of elliptic equations by the method of fundamental solutions," in *Boundary Elements XXVII*, A. Kassab, C. A. Brebbia, E. Divo, and D. Poljak, Eds. Southampton-Boston: WIT press, 2005, pp. 57-65.

[14] G. Fairweather and A. Karageorghis, "The method of fundamental solutions for elliptic boundary value problems," *Adv Comput Math.*, vol. 9, pp. 69-95, 1998.

[15] Y. Wang and Y. Rudy, "Application of the Method of Fundamental Solutions to Potential-based Inverse Electrocardiography," *Ann Biomed Eng*, vol. In press, 2006.

[16] H. S. Oster, B. Taccardi, R. L. Lux, P. R. Ershler, and Y. Rudy, "Noninvasive electrocardiographic imaging: reconstruction of epicardial potentials, electrograms, and isochrones and localization of single and multiple electrocardiac events," *Circulation*, vol. 96, pp. 1012-1024, 1997.

[17] C. Ramanathan, R. N. Ghanem, P. Jia, K. Ryu, and Y. Rudy, "Noninvasive electrocardiographic imaging for cardiac electrophysiology and arrhythmia," *Nat Med*, vol. 10, pp. 422-8, 2004.

[18] A. Karageorghis and G. Fairweather, "The method of fundamental solutions for the numerical solution of the biharmonic equation," *Journal of Computational Physics*, vol. 69, pp. 435-459, 1987.

[19] Y. C. Hon and T. Wei, "A fundamental solution method for inverse heat conduction problem," *Engineering Analysis with Boundary Elements*, vol. 28, pp. 489-495, 2004.

[20] G. Fischer, B. Pfeifer, M. Seger, C. Hintermuller, F. Hanser, R. Modre, B. Tilg, T. Trieb, C. Kremser, F. X. Roithinger, and F. Hintringer, "Computationally efficient noninvasive cardiac activation time imaging," *Methods Inf Med*, vol. 44, pp. 674-86, 2005.