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# Improved Signal-to-Noise Ratio in Parallel Coronary Artery Magnetic Resonance Angiography using Graph Cuts based Bayesian Reconstruction

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## Abstract

*High resolution 3D coronary artery MR angiography is time-consuming and can benefit from accelerated data acquisition provided by parallel imaging techniques without sacrificing spatial resolution. Currently, popular maximum likelihood based parallel imaging reconstruction techniques such as the SENSE algorithm offer this advantage at the cost of reduced signal-to-noise ratio (SNR). Maximum a posteriori (MAP) reconstruction techniques that incorporate globally smooth priors have been developed to recover this SNR loss, but they tend to blur sharp edges in the target image. The objective of this study is to demonstrate the feasibility of employing edge-preserving Markov Random Field priors in a MAP reconstruction framework, which can be solved efficiently using a graph cuts based optimization algorithm. The preliminary human study shows that our reconstruction provides significantly better SNR than the SENSE reconstruction performed by a commercially available scanner for navigator gated steady state free precession 3D coronary magnetic resonance angiography images (n = 4).*

## 1. INTRODUCTION

Over the past decade parallel imaging techniques such as SENSE [1,2], SMASH [3] and GRAPPA [4] have revolutionized the development of rapid magnetic resonance (MR) imaging. These techniques employ multiple receiver coils to acquire under-sampled data, which causes folding artifacts in the images, and then use the coil sensitivities to unfold these artifacts in either k-space or image space [5,6]. This paper focuses on the image space approach where the SENSE algorithm is considered the standard parallel imaging reconstruction technique.

Given a coil configuration, all reconstruction techniques suffer from noise amplification, particularly at high reduction factor R. Maximum likelihood based reconstruction techniques such as the SENSE algorithm are the most prone to this phenomenon and tend to yield low signal-to-noise ratio (SNR) images, which compromises the diagnostic accuracy. Maximum a posteriori (MAP) based reconstruction techniques have been developed to improve the SNR at the cost of global smoothing and edge blurring [7,8,9]. Here we propose a graph cuts based MAP reconstruction framework using piecewise smooth edge-preserving priors (EPP) which assume that the voxel intensity varies slowly over small regions but can change significantly at object boundaries [10,11]. EPP has been widely used in statistics [12], computer vision [13,14,15], and image processing [16]. For energy functions arising from MAP detection with EPP, graph cuts based methods have been demonstrated to yield promising results [17,18]. We show that the proposed reconstruction significantly improves the SNR and overall image quality over the standard SENSE reconstruction for high-resolution navigator gated steady state free precession (SSFP) 3D coronary artery MR angiography.

## 2. THEORY

### 2.1. Maximum Likelihood Reconstruction

The Cartesian SENSE reconstruction as described in [1] can be written as a linear inverse problem:

$$y = Ex + n \tag{1}$$

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where $x$ is a discrete representation of the target image, $y$ represents the aliased images collected by multiple receiver coils, $n$ is the observation noise and $E$ is the coil sensitivity and sampling matrix. Assuming additive white Gaussian noise [19], the maximum likelihood or least-squares estimate of $x$ can be obtained via pseudo-inverse of $E$ as follows:

$$x_{\text{SENSE}} = (E^H E)^{-1} E^H y \tag{2}$$

If E is ill-posed, this solution suffers from noise amplification. Tikhonov-type regularization can be used to stabilize the solution as suggested in [7,8,9]:

$$x_{\text{tikhSENSE}} = \arg\min_x (\|Ex - y\|^2 + \mu^2 \|A(x - x_r)\|^2) \tag{3}$$

where the first term represent the least-squares error and the second term imposes a penalty for non-smooth solutions through a choice of matrix $A$, prior reference image $x_r$ and penalty factor $\mu$ [9].

### 2.2. Bayesian Reconstruction

Given the imaging process (1), observation $y$, and the prior probability distribution $P(x)$ of the target $x$, Bayesian methods maximize a posteriori probability

$$P(x|y) = P(y|x)P(x) \tag{4}$$

where $P(y|x)$ is the likelihood function. A posteriori probability is maximized by

$$x_{\text{MAP}} = \arg\min_x (\|y - Ex\|^2 + G(x)) \tag{5}$$

where $G(x)$ is the choice of prior distribution. One can choose the traditional Gaussian to impose smoothness, or more complicated but powerful Gibbsian priors modeled by Markov Random Fields (MRFs) [14,15]. Traditionally, smoothness functions of the kind $G(x) = \|Ax\|^2$, commonly known as Tikhonov regularization [9], are used. This choice imposes global smoothness that leads to excessive edge blurring since images have discontinuities at object boundaries. This leads us to choose EPPs that impose piecewise smoothness as

$$G_{\text{EP}} = \sum_{(p,q)\varepsilon N} V(x_p, x_q) \tag{6}$$

where $N$ defines the neighborhood system (usually 8 connected neighbors), and $V(x_p, x_q)$ defines the separation cost. This form can be derived from MRFs [14]. Typically $V$ is non-convex, which limits the use of computationally efficient convex optimization methods such as steepest descent or conjugate gradients. We use the truncated linear function for $V(x_p, x_q) = \lambda * \min(|x_p - x_q|, K)$, where $K$ and $\lambda$ are scalar parameters. We can write first term in (5) as

$$\|y - Ex\|^2 = a^2 + \sum_p b(p)x_p^2 - \sum_p 2c(p)x_p + \sum_{(p,p')\varepsilon N_a} 2d(p,p')x_p x_{p'}$$

where $a, b(p), c(p), d(p,p')$ are functions of signals observed by multiple coils and coil sensitivity profiles. Substituting the two terms in (5) gives the following objective function

$$E(x) = a^2 + \sum_p b(p)x_p^2 - \sum_p 2c(p)x_p + \sum_{(p,p')\varepsilon N_a} 2d(p,p')x_p x_{p'} + \sum_{(p,q)\varepsilon N} \lambda * min(|x_p - x_q|, K)$$

We minimize this function using graph cuts with the $\alpha$-expansion method. The method assigns a discrete set of labels, $L = \{1, 2, \ldots, N_{\text{labels}}\}$ to pixels in the image. The $\alpha$-expansion method constructs the MAP solution $x$ by giving some of the pixels the label $\alpha$ that minimizes energy $E(x)$, where $\alpha$ takes values on the label set $L$. Like [10,11], we compute a high-quality $\alpha$ expansion using a single graph cut by relying on the construction of Hammer *et al.* [20] (we thank Vladimir Kolmogorov for directing us to Hammer's work).The method converges to a labeling where there is no $\alpha$-expansion that reduces the value of the objective function for any $\alpha$. For details on image reconstruction using discrete graph cuts optimization, please refer to [11].

## 3. METHODS

The human study was approved by our Institutional Review Board and written consent was obtained from all subjects. Imaging experiments were performed in 4 healthy male volunteers (mean age of $27 \pm 3$ years, mean body weight of $65 \pm 10$ kg) using a 1.5 T EXCITE 12 MR system (GE Medical Systems, Waukesha, WI). A commercially available eight-element phased-array cardiac coil (4 anterior and 4 posterior elements) was used for signal reception and parallel imaging reconstruction. Subjects were imaged supine with vector electrocardiography gating. For the purposes of navigator positioning and coronary artery localization, real-time gradient echo scout scans were performed to localize the diaphragm and the heart. A pencil-beam navigator was placed through the dome of the right hemidiaphragm. The right coronary artery (RCA) was imaged in a volume containing the atrioventricular sulcus. The optimal delay time between the cardiac trigger and the period of minimal cardiac contraction was determined from a cine scout scan.

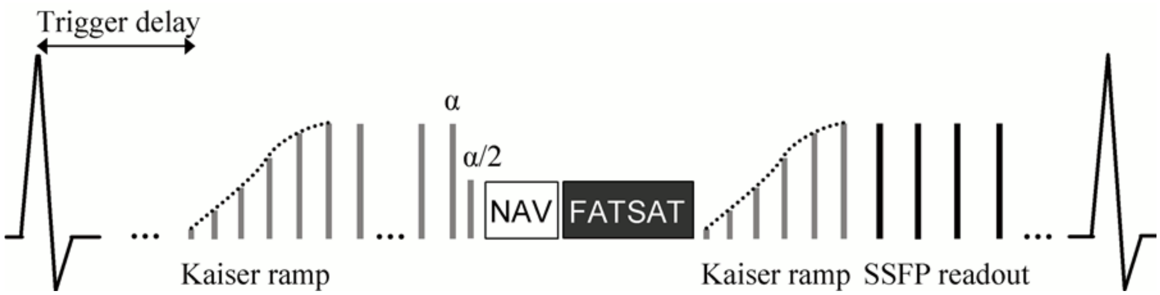
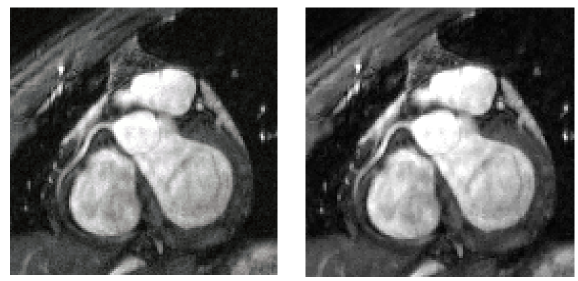



Figure 1: Schematics for ECG-triggered navigator gated SSFP 3D coronary MRA

Coronary artery imaging was performed using an ECG-triggered segmented k-space navigator gated steady state free precession 3D coronary MRA pulse sequence (Fig 1). The navigator and fat saturation pulses were executed during steady state after the preparatory RF pulses (6 Kaiser ramp and 30 constant pulses). Note that an $\alpha/2$ pulse and 6 Kaiser ramp pulses were used to conserve and restore the steady state, respectively. The imaging parameters were: TR = 4.0 ms, TE = 1.2 ms, flip angle = 60 degrees, readout bandwidth = $\pm$ 62.5 kHz, slice thickness = 3 mm, 16 slices, FOV = 26 cm, in-plane resolution = $1.0 \times 1.0$ mm$^2$, 32 partial echoes per heartbeat (corresponding to an acquisition window of 128 ms), centric view ordering along kz. A real-time navigator gating program was implemented on the scanner's computer that collected navigator data, extracted motion, and controlled data acquisition using the efficient phase ordering with automatic window selection (PAWS) gating algorithm [21]. A gating window of 3 mm was used.

On the scanner, coronary artery imaging was performed without parallel imaging (R = 1). The resulting full raw data set was captured to a disk file, reduced in size according to the desired reduction factor (R = 2) and transferred to a remote cluster for off-line parallel computing reconstruction using our algorithm. The cluster consisted of 32 nodes equipped with dual 3.2 GHz EM64T Xeon processors (1 MB of cache each) and 2 GB of RAM memory [22]. The sensitivity profile of each coil was generated using 32 center phase encoding lines of the fully sampled data. Target images were then reconstructed using the graph cuts based method described in [11] and standard SENSE method [1]. In our implementation, 512 labels and 3-5 iterations were used. Model parameter $K$ was set to $N_{\text{labels}}/7$ where $N_{\text{labels}}$ is the total number of labels, with an initial guess solution chosen to be a zero image. For analysis, signal to noise ratio was defined as the ratio between the blood signal and the noise standard deviation measured from the aorta and the background air, respectively.



(a) (b)

Figure 2: RCA images obtained with a) Standard SENSE reconstruction (R = 2) and b) Graph Cuts reconstruction (R = 2, $\lambda$ = 20)

## 4. RESULTS AND DISCUSSION

Human coronary MR angiograms were obtained successfully from all subjects. Fig 2(a) & (b) are reconstruction using standard SENSE (R = 2) and Graph Cuts with the $\alpha$ expansion method (R = 2, $\lambda$ = 20). Graph cuts demonstrates excellent SNR without excessive vessel blurring as compared to standard SENSE. Table 1 summarizes the blood SNR differences between the reconstruction from standard SENSE reconstruction (R = 2) and our graph cuts based MAP reconstruction (R = 2). Over the four subjects, our reconstruction improved the SNR over the standard SENSE reconstruction by 27% $\pm$ 13.9%.

| Subject | SENSE | Graph Cuts |
|---|---|---|
| 1 | 47.5 | 69.6 |
| 2 | 45.0 | 51.0 |
| 3 | 30.0 | 37.4 |
| 4 | 30.8 | 38.3 |

Table 1: SNR comparison for SENSE (R = 2) and Graph Cuts (R = 2, $\lambda$ = 20) reconstructions

In Graph Cuts reconstruction the parameter $\lambda$ provides a trade-off between the degrees of noise reduction and edge blurring during the reconstruction process. To the best of our knowledge, previous graph cuts implementations chose the value of $\lambda$ heuristically. In this preliminary study, the choice of optimal $\lambda$ was determined experimentally on the first subject and then fixed for the subsequent subjects. Graph cuts optimization is an iterative process repeated over a large set of labels, which makes it rather slow in practice. In our implementation, the reconstruction time for an iteration with 512 labels was about 5 minutes. In present form, current method is useful for application where real time reconstruction is not a necessity.

## 5. CONCLUSIONS

We have presented a Bayesian parallel imaging reconstruction technique for coronary artery MR angiography. Our preliminary results indicate that the proposed reconstruction is better than the standard SENSE reconstruction both visually and quantified in terms of SNR. For coronary artery imaging, choosing the truncated linear function (non-convex) as the prior distribution imposes piecewise smoothness and preserves boundaries. High quality reconstruction can be achieved using the graph cuts energy minimization method.

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