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# Unbiased Identification of Finite Impulse Response Linear Systems Operating in Closed-Loop

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*Abstract*— **The force and position data used to construct models of joint dynamics are often obtained from closed-loop experiments, where the joint position is perturbed using an actuator configured as a position servo. If the position servo is orders of magnitude stiffer than the joint, as is often the case, it is possible to treat the data as if they were obtained in open loop. It may be more relevant to study joint dynamics in compliant environments. This can be accomplished by adding an admittance controller, programmed to simulate a compliant environment, into the servo. Under these conditions, the presence of feedback cannot be ignored. Unbiased estimates of a system can be directly obtained from closed-loop data using the prediction error method. However, this is not true, in general, when linear regression or correlation-based analysis are used to fit nonparametric time- or frequency domain models. We develop a prediction error minimization based identification method for a nonparametric time-domain model, augmented with a parametric noise model. Simulations suggest that the method produces unbiased estimates of the dynamics of a system operating inside a feedback loop, even though linear regression results in substantial biases.**

*Index Terms*— **System Identification, Joint Dynamics, Compliant Environment, Separable Least Squares, Noise Model, ARMA.**

## I. Introduction

System identification, the construction of mathematical models of dynamic systems from input/output measurements, has been extensively used to characterize joint mechanics. Typically, an actuator is used to control the position of a single joint, or perhaps of the end of a limb. The actuator moves the joint through a prescribed series of motions, and the position of the joint and resulting forces are recorded. System identification techniques are then used to fit dynamical models between the measured force(s) and position(s).

The actuator is often configured as a position servo. It is supplied with a position command, and uses the difference between the position command and the measured position to compute the force that is applied to the limb to make it track the desired trajectory. Thus, the limb is an integral part of the feedback loop comprising the position servo. The dynamics of the limb must therefore be estimated using data that were gathered in closed loop.

One approach to this problem involves using an actuator that is much stiffer than the joint that is being studied [1]. If the position of the joint is considered to be the system input, and the servo is stiff enough that the forces produced by the limb have only a negligible effect on the position, then the feedback has been effectively broken. If one estimates a stiffness model, where position is the input and force is the output, the data can be treated as if they were gathered in open-loop conditions. While this approach has been used successfully, it places the limb in an unrealistic environment. More recent experiments have sought to characterize limbs as they interact with compliant environments [2], [3], [4], those that can be displaced when the limb applies a force. Under these conditions, the feedback through the actuator can no longer be ignored. Voluntary contraction from the subject result in significant displacements, which then enter the feedback loop. Thus, the identification must be performed using techniques that are appropriate for closed-loop data.

System identification using closed-loop data has been extensively studied in the control systems literature [5], [6]. One of the main conclusions in [5] is that unbiased, efficient estimates of a system operating in feedback may be obtained directly from closed-loop data, provided that the prediction error method [7] is used with a suitably chosen (parametric) model structure. The model structure includes two subsystems: the deterministic dynamics, which map the controlled input to the measured output, and the noise model, which explains the noise in the measured output as the result of filtering an unmeasured white noise sequence through a stable, invertible filter.

In essence, the inclusion of a noise-model in the closed-loop identification allows one to separate the contributions due to the external input from those due to noise propagating around the feedback loop. Unfortunately, nonparametric models are often used to describe joint dynamics, as they can more readily handle systems that have either long delays, or multiple delayed components (either of which can occur if reflexes are significant), than is possible using a low-order parametric model. In this paper, we will develop an algorithm for identifying a system consisting of a finite impulse response (FIR) model of the deterministic dynamics, and an Auto-Regressive Moving Average (ARMA) noise model. The prediction error is linear in the tap-weights that describe the FIR filter, but nonlinear in the parameters of the ARMA noise model. Thus, a separable least squares algorithm will be used to identify the optimal parameters of both the deterministic and noise systems.

## II. THEORY

### A. Notation

Capital and lower-case bold-faced letters will be used to denote matrices and vectors, respectively. Lower case, regular type-faced letters will either denote signals, $u(t)$, if they are parameterized by the time, $t$, or summation indexes. Since the analysis is carried out in discrete time, $t$ will be an integer. The forward shift operator will be denoted $q$, so that $qu(t) = u(t+1)$.

### B. Deterministic and Noise Models

Given $N$ measurements of a system's input and output, $u(t)$ and $y(t)$, respectively, the objective of a system identification is to construct a model that predicts $y(t)$, based on the previous and present values of $u(t)$, and perhaps the previous values of the output. If the output is expected to be a possibly noise corrupted but otherwise linear, function of the input then the model structure shown in Fig 1 can be used to describe the system,

$$y(t) = G(q)u(t) + H(q)e(t) \tag{1}$$

where $G(q)$ and $H(q)$ are (possibly infinite degree) polynomials in the backward shift operator, $q^{-1}$. The noise in the output measurement, $v(t) = H(q)e(t)$, is obtained by filtering an IID sequence of random variables, $e(t)$, with a stable and inversely stable, monic filter, $H(q)$.

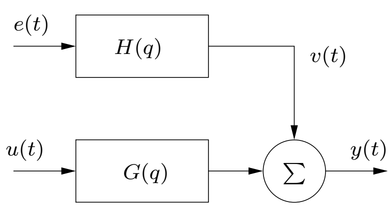


Fig. 1. Complete model of a linear system showing both the deterministic subsystem, and the noise model. The experimenter is assumed to have access to $u(t)$ and $y(t)$. The innovation input, $e(t)$ is an unmeasurable sequence of independent, identically distributed random variables.

If $u(t)$ is uncorrelated with $e(t)$, then an unbiased estimate of the deterministic part of the system, $G(q)$, can be obtained by least squares regression. One creates a matrix whose columns contain delayed versions of the input,

$$\mathbf{U}(i,j) = \begin{cases} u(i-j+1) & j \le i \\ 0 & j > i \end{cases}$$

and solves

$$\hat{\mathbf{g}} = (\mathbf{U}^T\mathbf{U})^{-1}\mathbf{U}^T\mathbf{y} \tag{2}$$

where $\mathbf{y}$ is a vector containing $y(t)$, and $\hat{\mathbf{g}}$ is a vector containing an estimate of the impulse response of the deterministic filter $G(q)$.

### C. Closed Loop Identification

If the system is in a feedback loop, such as that shown in Fig. 2, then the assumption that $u(t)$ and $e(t)$ are uncorrelated no longer holds. Indeed, the input to $G(q)$ is given by:

$$u(t) = \frac{F_c(q)}{1+F_c(q)G(q)}r(t) + \frac{H(q)F_c(q)}{1+F_c(q)G(q)}e(t)$$

Consider the cross-correlation between the deterministic input, and the output of the noise model:

$$\begin{aligned}\phi_{uv}(\tau) &= E(u(t-\tau)v(t)) \\ &= E\left(\frac{H(q)F_c(q)}{1+F_c(q)G(q)}e(t-\tau)\cdot H(q)e(t)\right)\end{aligned}$$

since $e(t)$ is assumed to be independent of the reference input, $r(t)$. Let

$$\frac{F_c(q)}{1+F_c(q)G(q)}v(t) = \sum_{k=0}^{\infty} h_{eu}(k)v(t-k)$$

then,

$$\phi_{uv}(\tau) = \sum_{k=0}^{\infty} h_{eu}(k)\phi_{vv}(\tau+k)$$

to guarantee that $\phi_{uv}(\tau) = 0$ for $\tau \ge 0$, $v(t)$ must be white noise, thus $H(q) = 1$. In which case

$$\phi_{uv}(\tau) = \sum_{k=0}^{\infty} h_{eu}(k)\phi_{ee}(\tau+k)$$

which will be zero for all $\tau > 0$. For the case $\tau = 0$, this can only be true if $h_{eu}(0) = 0$. Thus, there must be at least a one sample delay in the controller. Otherwise, the estimated impulse response function (IRF) will be biased.

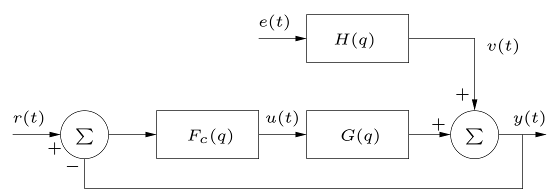


Fig. 2. Block diagram of a linear feedback system.

Forssell and Ljung [5] demonstrated that unbiased estimates of $G(q)$ could be obtained using the prediction error method (PEM), provided the controller contains a delay. In PEM [7], both the $G(q)$ and $H(q)$ are fitted, such that the error in the one step ahead prediction,

$$\epsilon(t) = H^{-1}(q)\left(G(q)u(t) - y(t)\right) \tag{3}$$

is minimized, in the mean squared error sense. Thus, transfer functions $G(q)$ and $H(q)$ are chosen to minimize

$$V_N = \frac{1}{2N}\sum_{t=1}^{N}\epsilon^2(t) \tag{4}$$

Notice that computing the prediction error (3) involves filtering both the input and output by the inverse of the noise model. This has the effect of whitening the measurement noise (in the output). Provided the controller contains a delay, this will also eliminate the correlation between the input and measurement noise, and hence eliminate any bias in the estimate of $G(q)$.

The usual practice in the control systems literature, is to model the two sub-systems, $G(q)$ and $H(q)$, with recursive digital filters. While we will use a parametric description for $H(q)$, the deterministic dynamics will be modeled as a FIR filter. Thus, the deterministic dynamics are given by:

$$G(q)u(t) = \sum_{k=0}^{T} g(k)u(t-k)$$

for some finite memory length $T$. The measurement noise, however, will be an Auto-Regressive Moving-Average (ARMA) process,

$$\begin{aligned} v(t) &= \frac{C(q)}{D(q)}e(t) \\ &= \sum_{k=1}^{n_c} c_k e(t-k) - \sum_{j=1}^{n_d} d_j v(t-j) + e(t) \end{aligned}$$

and the prediction errors will be given by:

$$\epsilon(t) = \frac{D(q)}{C(q)}\left(G(q)u(t) - y(t)\right)$$

The objective is to find $C(q)$, $D(q)$ and $G(q)$, that minimize the sum of squared prediction errors. In general, this is a nonlinear least squares optimization, and can only be solved using some form of iterative optimization.

Notice, however, that for any given choice of $C(q)$ and $D(q)$, that that optimal $G(q)$ can be found in closed form by fitting a FIR filter in the usual way (i.e. by solving (2)), between the filtered input and output,

$$\begin{aligned} u_f(t) &= \frac{D(q)}{C(q)}u(t) \\ y_f(t) &= \frac{D(q)}{C(q)}y(t) \end{aligned}$$

Thus, minimizing (4) is a separable least squares problem [8], and we need only search over the parameters that define the noise model. This is significant, since the number of parameters in the noise model will likely be at least order of magnitude smaller than the number of tap-weights in the FIR filter.

## III. Simulations

The feedback system in Fig 2 was simulated in MATLAB. The transfer functions were as follows

$$\begin{aligned} F_c(z) &= \frac{100}{z-0.5} \\ G(z) &= 10^{-5}\cdot\frac{4.83z+4.67}{z^2-1.89z+0.90} \\ H(z) &= 10^{-3}\frac{2.91z^3+3.44z^2-1.89z+2.38}{z^4-3.36z^3+4.27z^2-2.44z+0.53} \end{aligned}$$

The reference input was a low-pass filtered sequence of white Gaussian noise, while the innovation was an IID sequence of Gaussian random variables. A 100 trial Monte-Carlo simulation was performed. Fig. 3 summarizes the results obtained by fitting the deterministic model using least squares regression. The identified IRFs were very noisy. Even the mean of the 100 estimated IRFs was so noisy that is was not possible to determine whether or not there was significant bias in the estimate. However, given the estimation variance, the estimate was all but useless.

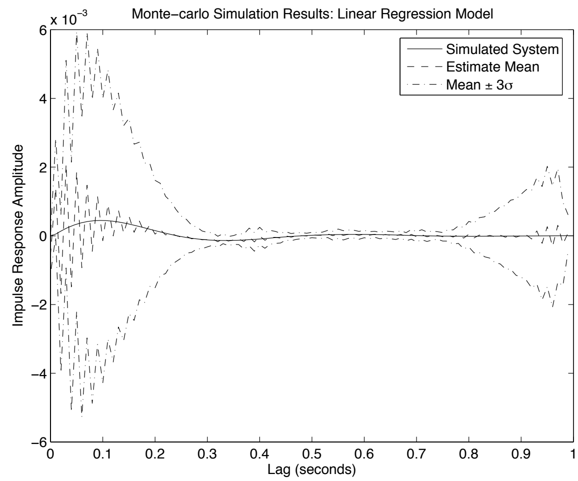


Fig. 3. Results of the Monte-Carlo simulation. The solid line shows the IRF of the simulated system. The dashed and dash-dotted lines show the mean ± three standard deviations of the IRF estimates obtained using least squares regression on the deterministic model alone. Clearly, the estimates are noisy. Given the noise in the estimates, it is unclear whether or not they are biased

Next, we used a robust impulse response estimation technique [9], that uses a pseudo-inverse to improve the conditioning of the estimation problem (at the cost of introducing a slight bias). These estimates are shown in Fig 4. The estimation variance is greatly reduced, however, a slight bias is evident, especially surrounding the first peak in the IRF.

Figure 5 shows the mean and 3 standard deviation confidence bounds obtained using the PEM based approach described in this paper. Note that the simulated IRF is virtually identical to the ensemble mean of the identified IRFs, and that any discrepancies between the two are at least an order of magnitude smaller than the $3\sigma$ bounds, plotted as a dash-dotted line. Thus, if there is any bias in the estimates, it was not detected in this simulation. The addition of the noise model appears to have had two effects. First, it dramatically reduced the variance in the estimates of the deterministic subsystem. Secondly, as expected theoretically, the use of PEM appears to have resulted in unbiased estimates of the deterministic subsystem.

## IV. Conclusions

A new algorithm for the identification of linear FIR systems from closed-loop data has been developed. The bias due to the effects of feedback has been removed by fitting an ARMA noise model, using a separable least squares optimization. Simulation results demonstrate the performance of the algorithm.

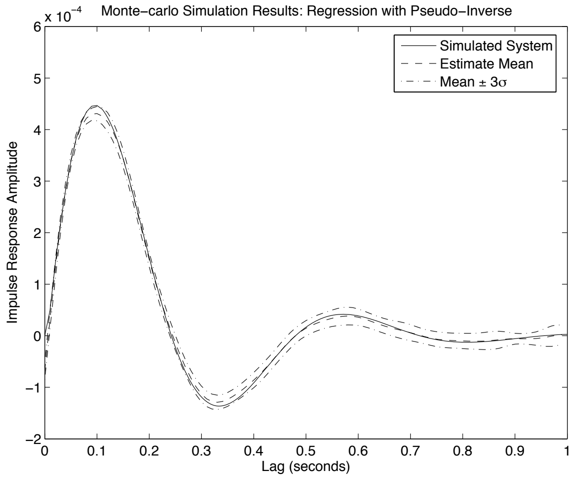


Fig. 4. Results of the Monte-Carlo simulation. The solid line shows the IRF of the simulated system. The dashed and dash- dotted lines show the mean ± three standard deviations of the IRF estimates obtained using a robust pseudo-inverse based technique on the deterministic model alone. Note the bias in the estimates.

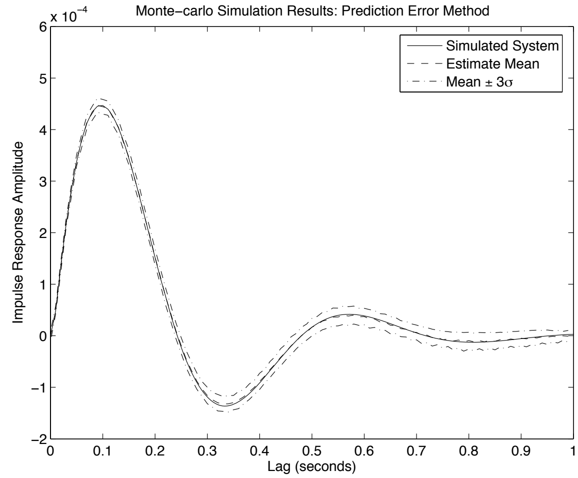


Fig. 5. Results of the Monte-Carlo simulation. The solid line shows the IRF of the simulated system. The dashed and dash-dotted lines show the mean ± three standard deviations of the IRF estimates obtained using the prediction error method with a FIR deterministic system and an ARMA noise model. The results suggest that this approach produced unbiased estimates of deterministic dynamics, with relatively little variance.

Although it was implemented using a causal, linear FIR model, for the dynamics, it could easily be extended to include either a two-sided impulse response, a Hammerstein model, or even a parallel cascade reflex model.

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