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Efficacy of Using Mean Arterial Blood Pressure Sequence for Three-Element Windkessel Model Estimation

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Abstract—**The three-element Windkessel model is widely used and accepted for analyzing blood flow and pressure in arterial system and cerebral circulation. In most studies, changes in mean arterial blood pressure data is used as input to estimate the model parameters. However, estimation of linear model parameters, using input-output data, requires that the input be persistently exciting. This study examined the efficacy of using mean arterial blood pressure (MABP) sequence as an input stimulus for estimating the parameters of the three-element Windkessel model. Additionally, the study explored the use of a shorter MABP data segment of 1.5 min as compared to the commonly used 6 min data. MABP data was obtained from 11 healthy subjects. One thousand three-element Windkessel models, with parameter values randomly selected to be within physiological range, were subjected to seven different input sequences. For each input sequence and model, the values of the model (target-parameters) were estimated. The seven input sequence were: 1) six minutes of MABP measured from subjects; 2-5) four 1.5 min of measured MABP obtained by dividing the measured six minutes of MABP into non-overlapping contiguous segments; 6) a six-minutes of pseudo random binary sequence (PRBS) with amplitudes comparable to the MABP sequence; and 7) a 1.5 min of PRBS sequence with amplitudes comparable to the MABP sequence. The MABP data used was randomly selected from the 11 subjects for each estimation run. The model parameter estimation method had two phases of optimization. In the first phase, the parameters were estimated and optimized using the frequency transform of the input and output. In the second phase, the values of the estimated parameters were used as initial estimates and time-domain optimization was carried out to further refine the estimates. Results from the study, comparing the estimated-parameters with the target-parameters, show that for the MABP data, there was no significant difference between using the six minutes or 1.5 min of data for estimating the target-parameters. Also, parameters estimated from the MABP data were either equivalent or superior to the PRBS results, suggesting that changes in MABP can be used as an effective sequence for linear model estimation.**

Keywords—**Windkessel Model, Mean Arterial Blood Pressure, Monte-Carlo Simulation, Linear Model Estimation**

I. INTRODUCTION

The dynamic nature of human cerebral circulation has been characterized by several studies [8, 9, 11, and 12] such as parametric and non-parametric modeling [3, 5, 10, and 13], non-linear and neural network modeling [6, 7]. Even though most of these studies aimed to analyze cerebral perfusion pressure-flow relationships to evaluate dynamic cerebral autoregulation, none of them validated the use of arterial blood pressure time sequence as an effective input stimulus for model identification. Also, the possibility of using a short blood pressure data sequence of less than 3 minutes for model identification has not been examined in previous studies [9, 10 and 11].

The three-element Windkessel model (fig. 1) is often used to represent the pressure-blood flow relationship of regional circulation and has been used to capture the dynamics of the cerebral circulation [3, 4, and 13]. This lumped parameter model is derived from electrical circuit analogies where current (i) represents blood flow (cm^3/s) and voltage (v) represents pressure (mmHg). Resistances (R, $mmHg.s/cm^3$) represent vascular resistances that occur as a result of viscous dissipation inside vessels, capacitor (C, $cm^3/mmHg$) represents volume compliance of the vessels that allows them to store significant amounts of blood. Windkessel models have advantages over other parametric models that they provide a simple model structure in terms of individual elements (R, C) of its electrically analogous circuit, facilitating the extraction and interpretation of physiological changes from the dynamic variation of each element. But the estimation of these element values is not straightforward. Generally the estimation depends on minimization of a cost-function (a function of time or frequency) to either fit the model time-domain output to the measured time-domain output, or match the model frequency response with the measured frequency response.

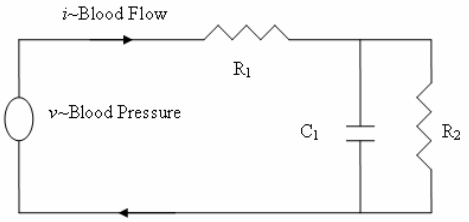

Fig. 1 The three-element Windkessel model with two resistors (R_1 and R_2) and one capacitor (C_1).

Previously we demonstrated the efficacy of beat-to-beat blood pressure time series with a data segment of duration 1.5 minutes for estimation of autoregressive (ARX) models [1]. Present study employs the three-element Windkessel model (fig. 1) to determine the efficacy of the use of mean arterial blood pressure (MABP) sequence in serving as an input stimulus for obtaining linear parametric model

estimation. In addition, this study will further explore the possibility of using a short data segment of 1.5 minute duration of changes in MABP for linear system identification.

II. METHODOLOGY

Data for the present study was obtained from 11 healthy subjects, 9 men and 2 women (aged 29 ± 6 years) in supine resting position. No subject smoked or had known medical problems. Subjects were screened carefully with a medical history and a physical examination with 12-lead ECG. All subjects signed an informed consent form approved by the Institutional Review Boards of the University of Texas Southwestern Medical Center and Presbyterian Hospital of Dallas. The beat-to-beat MABP was obtained by photo-plethysmography (Finapres, Ohmeda). Six minutes of data for all the 11 subjects was obtained and resampled at 2 Hz using cubic spline interpolation. Prior to model estimation, all data sequences were first detrended and then normalized by the range of time series (maximum value of sequence minus minimum value of the sequence).

Block diagram in Fig. 2 illustrates the applied methodology for the present study, which was based on performing 1000 trials Monte-Carlo simulation involving the estimation of the three-element Windkessel model parameters (element values) in each trial. First a target Windkessel model (M_t) was created with its parameters (R_1, R_2 and C_1), referred as target-parameters. For each trial, the parameters of M_t were randomly chosen from the range 7.0 to 14.0 for R_1, 3.0 to 8.0 for R_2 and 1.0 to 5.0 for C_1. These ranges were selected to be consistent with the values reported in the literature that were derived from a cerebral blood flow regulation study [13]. M_t was subjected to input (v) and the computed output (i) was calculated. Then using v and i, a Windkessel estimation of M_t was done. The estimation technique involved parameter extraction by a two phase optimization process, and approximation of resistance parameter R_1 [13]. The two phase optimization process resulted in better parameter estimates and lower mean squared error (MSE) value between the computed output (i) and predicted model output as compared to single phase optimization. The optimization method used was a constrained optimization [2], which implemented a sequential quadratic programming (SQP) technique along with cubic polynomial search procedure. This type of optimization minimizes a multivariable objective function starting with some initial estimates of the function parameters and resulting into final parameter values that are subject to constraints in terms of lower and upper bounds that they can attain.

For the first phase of optimization, referred as frequency-domain optimization, first an impedance curve from v and i was generated. This was done by finding the cross-correlation (r_{vi}) of v and i sequences, and auto-correlation (r_{vv}) of v. Then by Welch's averaged, modified periodogram method, a fast Fourier transform (FFT) of r_{vi} and r_{vv} was calculated to be S_{vi} and S_{vv} respectively (512 point FFT for 1.5 minute data sequences, 2048 point FFT for 6 minute data sequences) with 50 percent overlap, using a Hanning window and a sampling frequency of 2 Hz [2]. The measured impedance curve Z_m was computed by dividing the absolute value (modulus) of S_{vi} by the absolute value of S_{vv}.

$$Z_m = \frac{|S_{vi}(\omega)|}{|S_{vv}(\omega)|} \ . \tag{1}$$

The transfer function of the three-element Windkessel model in Fig. 1 (considering current (i) or blood flow as output of the model and voltage (v) or pressure as input to the model) using Laplace transform (s-domain) was obtained as

$$g_1(s) = \frac{i(s)}{v(s)} = \frac{[sC_1R_2 + 1]}{[sC_1R_1R_2 + (R_1 + R_2)]} \ . \tag{2}$$

In (2), the complex variable s was replaced by $j\omega$, where j is imaginary number ($\sqrt[2]{-1}$) and ω is frequency (rad/sec), and the absolute value of the resulting transfer function (function of ω with unknown parameters) was taken to be the target (i.e. desired) impedance Z_t as

$$g_1(\omega) = \frac{[j\omega C_1R_2 + 1]}{[j\omega C_1R_1R_2 + (R_1 + R_2)]} \ , \tag{3}$$

$$Z_t = |g_1(\omega)| \ . \tag{4}$$

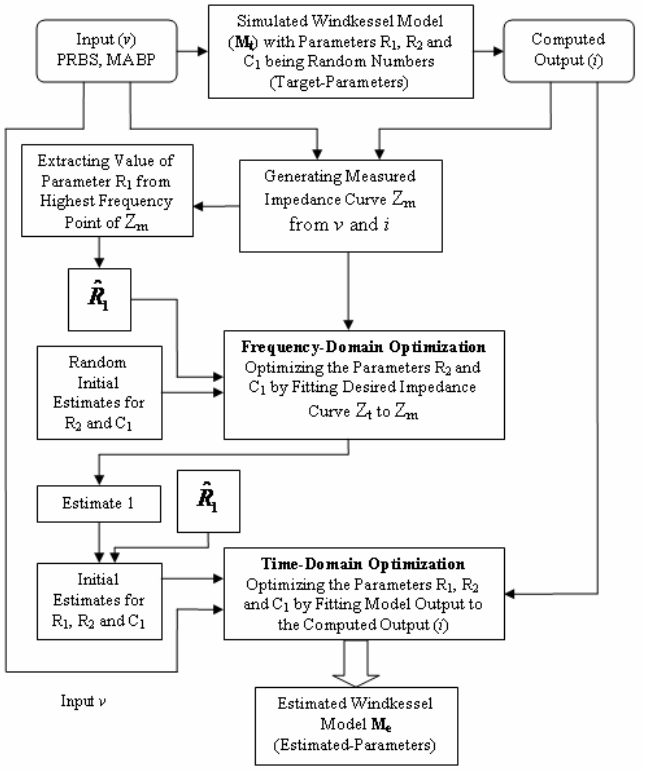

Fig. 2 Methodology used for parameter estimation of the three-element Windkessel model. Each trial of the 1000 trials Monte-Carlo simulation involved this type of estimation.

Equation (3) can be written as

$$g_1(\omega) = \frac{[C_1R_2 + 1/j\omega]}{[C_1R_1R_2 + (R_1/j\omega + R_2/j\omega)]} \quad . \tag{5}$$

Applying the limit of ω tending to infinity to modulus of $g_1(\omega)$ in (5) would give

$$\lim_{\omega\to\infty}|g_1(\omega)| = \lim_{\omega\to\infty} Z_t = 1/R_1 \, . \tag{6}$$

Hence in order to find the R_1, R_2 and C_1 that produce Z_t that matches Z_m, in the frequency-domain optimization phase the value of resistance parameter R_1 was extracted from the highest frequency point of the measured impedance curve Z_m, as shown in (6). This value is referred as $\hat{R}_1$. Essentially, the frequency-domain optimization is selection of parameters R_2 and C_1 of the model, such that MSE between Z_m and Z_t is minimized. The initial estimates for parameters R_2 and C_1 in the frequency-domain optimization were generated from a different set of uniformly distributed random numbers in the range 3.0 to 8.0 and 1.0 to 5.0 respectively. The lower and the upper bounds for R_2 and C_1 were 3.0, 8.0 and 1.0, 5.0 respectively.

The second phase of optimization, referred as time-domain optimization, involved fitting the predicted model output to the computed output *i* by selection of model parameters R_1, R_2 and C_1 in order to minimize the MSE between *i* and the predicted model output (predicted model excited with *v*). The initial estimate for parameter R_1 in time-domain optimization was taken from the value $\hat{R}_1$, and the initial estimates for parameters R_2 and C_1 were taken from the final estimates (Estimate 1 in Fig .2) of the frequency-domain optimization. The lower and the upper bounds for R_1, R_2 and C_1 were 7.0, 14.0; 3.0, 8.0 and 1.0, 5.0 respectively. The final estimates of parameters for the three-element Windkessel model from the time-domain optimization were considered as the estimated-parameters (model M_e) of M_t.

The 1000 models constituted Monte-Carlo simulation trials of the above mentioned two phase optimization process and estimation technique in each trial. The target-parameters (M_t) and initial estimates for parameters R_2 and C_1 in the frequency-domain optimization were random and different for each trial. The 1000 trials simulation was done first with *v* as 6 minutes MABP data, and then for all the four non-overlapping contiguous 1.5 minute sections of the 6 minutes MABP data. For each trial, the subject MABP data was selected randomly from the group of 11 subjects, as shown in Fig. 3. The simulation was then performed with *v* as 6 minutes and 1.5 minute pseudo random binary signal (PRBS) with minimum value of -1, maximum value of +1, initial seed equal to zero and sample time equal to 0.5 s.

III. RESULTS

From the present study, seven sets (each set containing 1000 values corresponding to 1000 trials of the Monte-Carlo simulations) of model estimated-parameter (M_e) values (R_1, R_2 and C_1) were generated, one for each type of the data used as input to the model (one 6 minutes MABP, four 1.5 minute MABP, one 6 minutes PRBS, one 1.5 minute PRBS). Table 1 shows the average target-parameter values (M_t) of the 1000 trials (M_t was same for each of the seven sets of M_e). Table 2 shows the average values of the 1000 trials for all the seven sets of model estimated-parameters (M_e). Table 3 gives the t-test results (P_{val}) for the comparison between models M_t and M_e, for all the seven sets of M_e. The statistical level of significance for the t-tests was 0.05, the null-hypothesis was that there is no difference in the models M_t and M_e, and the alternative-hypothesis was that the models M_t and M_e are different in terms of their parameter values.

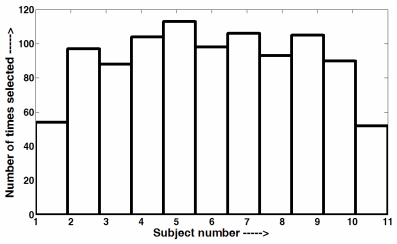

Fig. 3 Histogram of data selection process for 1000 trials Monte-Carlo simulation of the three-element Windkessel model estimation with MABP data of 11 subjects.

TABLE 1
AVERAGE TARGET-PARAMETER (M_t) VALUES AND STANDARD DEVIATIONS (STD) OF THE THREE-ELEMENT WINDKESSEL MODEL FOR 1000 TRIALS MONTE-CARLO SIMULATION

	R_1	R_2	C_1
Average M_t	10.5069	5.5285	2.9799
STD M_t	2.0503	1.4220	1.1811

TABLE 2
AVERAGE ESTIMATED-PARAMETER (M_e) VALUES AND STANDARD DEVIATIONS (STD) OF THE THREE-ELEMENT WINDKESSEL MODEL FOR 1000 TRIALS MONTE-CARLO SIMULATION

	R_1	R_2	C_1
6 min MABP Average M_e	10.5080	5.5331	2.9874
6 min MABP STD M_e	2.0511	1.4268	1.1904
1st 1.5 min MABP Average M_e	10.5164	5.5010	3.0306
1st 1.5 min MABP STD M_e	2.0537	1.5865	1.2182
2nd 1.5 min MABP Average M_e	10.5120	5.4427	3.0002
2nd 1.5 min MABP STD M_e	2.0540	1.5136	1.1882

TABLE 2 *continued*

3rd 1.5 min MABP Average M_e	10.5077	5.4536	2.9868
3rd 1.5 min MABP STD M_e	2.0507	1.4704	1.1861
4th 1.5 min MABP Average M_e	10.5208	5.5374	3.0252
4th 1.5 min MABP STD M_e	2.0533	1.5868	1.2053
6 min PRBS Average M_e	10.5068	5.5127	2.9919
6 min PRBS STD M_e	2.0503	1.4299	1.1960
1.5 min PRBS Average M_e	10.5063	5.1859	2.9727
1.5 min PRBS STD M_e	2.0499	1.4076	1.1799

TABLE 3
COMPARISON BETWEEN TARGET (M_t) AND ESTIMATED (M_e) PARAMETER VALUES OF THE THREE-ELEMENT WINDKESSEL MODEL FOR 1000 TRIALS MONTE-CARLO SIMULATION

		R_1	R_2	C_1
P_{val}	6 min MABP	0.9909	0.9427	0.8873
	1st 1.5 min MABP	0.9174	0.6830	0.3449
	2nd 1.5 min MABP	0.9557	0.1914	0.7021
	3rd 1.5 min MABP	0.9927	0.2469	0.8969
	4th 1.5 min MABP	0.8794	0.8950	0.3956
	6 min PRBS	0.9993	0.8042	0.8217
	1.5 min PRBS	0.9953	6.9E-08	0.8916

IV. DISCUSSION

Results from the present study (Table 1 and Table 2) show that for each of the five types of the MABP data used as input to the model (one 6 minutes MABP, four 1.5 minute MABP), the estimated-parameters (M_e) were fairly close to the target-parameters (M_t). This is also reflected from the results in Table 3 where there was no significant difference between the parameters of the models M_t and M_e for the five types of the MABP data used, regardless of the data length (i.e. 6 or 1.5 minutes). This of course, is based on the observation that for all the cases P_{val} is greater than the statistical level of significance (0.05), rejecting the alternative-hypothesis. It can be noted clearly from these results that model M_e was a fairly good estimate of M_t, irrespective of which one of the four 1.5 minute sections is taken from the 6 minutes MABP data.

Even though PRBS is a very wide bandwidth signal and is considered persistently exciting, results from Table 2 show that for 1.5 minute PRBS as input to the model, the estimation of resistance parameter R_2 in model M_e was poor as compared to model M_t (the explanation for this result is being further investigated). This is also reflected in Table 3 where the P_{val} of 1.5 minute PRBS data for parameter R_2 is less than the statistical level of significance (0.05), supporting the alternative-hypothesis. Also, comparing the five MABP data set results with respective PRBS results in Tables 2 and 3, MABP data results were either equivalent or superior to the PRBS results. Hence it can be concluded that the MABP sequence can be used as effectively as the PRBS sequence for linear model estimation.

V. CONCLUSION

Input stimulus of 1.5 minute duration of beat-to-beat blood pressure sequence is effective and adequate for linear system identification of the three-element Windkessel model which can be used to describe dynamic characteristics of the cerebral circulation.

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