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LARGE SCALE KALMAN FILTERING SOLUTIONS TO THE ELECTROPHYSIOLOGICAL SOURCE LOCALIZATION PROBLEM- A MEG CASE STUDY

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ABSTRACT

Computational solutions to the high-dimensional Kalman Filtering problem are described in the setting of the MEG inverse problem. The overall objective of the described work is to localize and estimate dynamic brain activity from observed extraneous magnetic fields recorded at an array of sensor positions on the scalp and to do so in a manner that takes advantage of the true underlying statistical continuity in the current sources. To this end, we outline inverse mapping procedures that combine models of current dipoles with dynamic state-space estimation algorithms. While these algorithms are eminently well-suited to this class of dynamic inverse problems, they possess computational limitations that need to be addressed either by approximation or through the use of high performance computational resources. In this work we describe such a High Performance Computing (HPC) solution to the Kalman filter and demonstrate its applicability to the Magnetoencephalography (MEG) inverse problem.

1. INTRODUCTION

Even in the context of large-scale data intensive applications, the Kalman filter has been historically proven as an effective means of minimising modelling and prediction error in fields such as weather forecasting [1] and oceanography [2], [3]. Kalman filtering (KF) and fixed-interval smoothing (FIS) approaches are in principle useful for these problems since under the appropriate assumptions of linearity and normality, they provide optimal recursive state estimates that extract the maximum amount of information from the observed data. In addition, these desirable properties hold across a wide variety of time-varying linear (and non-linear) potential candidate models. However, in its standard form the Kalman filter can be computationally prohibitive for such large-scale problems. In the case of the example applications listed above, the numerical calculations are often on systems of state dimension $N = O(10^7)$ with covariance matrices of size $N^2 = O(10^{14})$ [1]. The computationally intensive aspects of the Kalman algorithm stem from the prediction update on large time-dependent covariance matrices that require costly multiplications at each timestep. Since the dynamical error structure of these systems is often well-understood, many numerical solutions to these kinds of paradigm [3], [1] employ model reduction techniques to overcome the computational complexity inherent in such problems.

In MEG/EEG inverse problems, the motivation for exploring the use of (high-dimensional) Kalman filtering approaches is primarily a result of the limitation that most state-of the art EEG/MEG source localization techniques do not employ temporal continuity in their statistical formulations. That is, source estimates computed at time intervals separated by as little as a few milliseconds are not treated as related by rather as unrelated, independent estimation problems. The first approach which used multiple time points simultaneously in the estimation process was the time-varying multidipole approach first introduced to EEG analysis [4]. The same model is inherent in the MUSIC and RAP-MUSIC estimates [5], [6]. However, these methods still do not make use of the temporal sequence of the signals, i.e., if the original data points are first permuted and an inverse permutation is applied to the source estimates, the results are the same as without permutation. Recent studies by Galka, et al. [7] have used a random-walk dynamical model with Laplacian spatial relationships to represent the dynamics of EEG source currents, facilitating reduced-dimension Kalman filtering and recursive least-squares source localization computations. Inclusion of this temporal constraint improved not only the estimation of source time-series, but also resulted in a significant improvement in spatial localization. More generally, the transition matrix in the state-space dynamical model describes spatio-temporal relationships between different brain regions that preclude use of model reduction techniques, such as those devised for problems in the environmental sciences.

In the case of the MEG/EEG source localization problem, the source space normally contains approximately $N = O(10^3)$ potential sources leading to error covariances of dimension $N^2 = O(10^6)$. When performing Kalman filtering, the computations required at each time step involve three high dimension matrix multiplications leading to a total ap-

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proximate computational cost of $3 \times N^3$ as the system is full rank. When taking into account ancillary variables, required memory amounts to at least 24+ Gigabytes. In addition, the FIS requires one such multiply at each timestep followed by a large-scale matrix inversion. Furthermore, the FIS additionally requires approximately three Gigabytes of storage space would be required to accommodate both covariances in 32-bit precision storage format at each time step. The scale of these computations makes their calculation unfeasible on virtually any state-of the-art standalone computing resource. In order to address this limitation, we arranged the Kalman filtering computations such that the data-intensive aspects of the algorithm could be run in parallel on a distributed HPC system. We utilized the NSF Teragrid resource at the TACC (Texas Advanced Computing Center). This resource comprised 1024 CRAY-DELL nodes a 1024-processor Cray/Dell Xeon-based Linux cluster with 6.4 Teraflops of computing capacity.

In the sequel, we briefly review the Kalman and fixed interval smoother, its parallel implementation and culminate with a case study application to a dense MEG source localisation problem.

2. METHODS

2.1. A Dynamic Model for EEG/MEG

Assume that a MEG experiment is conducted in which a stimulus is applied at time zero and MEG activity is recorded in the interval $(0, T)$. For K large, assume that the measurements are recorded at times $k\Delta$ for $k = 1, \ldots, K$ and $\Delta = KT^{-1}$. We assume that there are S recording sites and that the stimulus is repeated R times. Let $y_{s,r}(k\Delta)$ denote the measurement at time $k\Delta$ at location s, on repetition r where $s = 1, \ldots, S$ and $r = 1, \ldots, R$. We take $y_s(k\Delta) = R^{-1} \sum_{r=1}^{R} y_{s,r}(k\Delta)$, and we define $y_k(k\Delta) = (y_1(k\Delta), \ldots, y_S(k\Delta))$ to be the $S \times 1$ vector of measurements recorded at time k. We assume that there are P sources and that the relation between the observations and the sources is defined by the observation model

$$y_k = Hx_k + \varepsilon_k \tag{1}$$

where $x_k = (x_{k,1}, \ldots, x_{k,p})$ is the $3P \times 1$ vector of source activities at time k, each $x_{k,\cdot}$ is a 3×1 vector, H is the $3 \times 3P$ lead field matrix defined as the quasi-static solution to Maxwell's equations, [8] and ε_k is zero mean Gaussian noise with covariance matrix which is defined by the background machine noise. We assume that because of the neural dynamics, the x_k obey the following spatio-temporal model [7],

$$x_k = x_{k-1} + n_k^{-1} \sum_{k \in N(k)} x_{k-1} + w_k \tag{2}$$

where $N(k)$ is the neighborhood of source k, n_k is the number of sources in the neighborhood $N(k)$ and w_k is a $3P \times 1$ vector of zero mean Gaussian noise with covariance matrix Σ_w. We can write Eq.(2) more compactly as a linear state-equation i.e.

$$x_k = Fx_{k-1} + w_k \tag{3}$$

where the elements of F encompass the neighborhood relation between all the sources at time k in terms of the sources at time $k-1$. Because the neighborhood of each source is assumed to be small, i.e. not more than 6 or 9 voxels, F is primarily a sparse matrix with zero elements everywhere except on the diagonal and the immediately adjacent off-diagonal rows. Equations (1) and (3) define a state-space model for MEG.

2.2. A Dynamic Solution to the MEG Source Localization Problem.

The state-space formulation of the MEG problem suggests two approaches to computing solutions to the inverse problem. The first is the well known Kalman filter (KF) estimate for each time k, $x_{k|k}$ defined for this problem by the recursion [9],

$$\begin{aligned} x_{k|k-1} &= Fx_{k-1|k-1} \\ \Sigma_{k|k-1} &= F\Sigma_{k-1|k-1}F^{'} + \Sigma_w \\ G_k &= \Sigma_{k|k-1}H^{'}\left[H\Sigma_{k|k-1}H^{'} + \Sigma_\varepsilon\right]^{-1} \\ x_{k|k} &= x_{k|k-1} + G_k\left[y_k - Hx_{k|k-1}\right] \\ \Sigma_{k|k} &= \left[I - G_kH\right]\Sigma_{k|k-1} \end{aligned} \tag{4}$$

for $k = 2, \ldots, K$ given the initial estimate, $\widehat{x}_{static,1}$ computed by the MNE method using y_1 and Σ_0 (the covariance estimate for the MEG background noise ε_k). The notation $x_{k|j}$ means the estimate at time k given the data up through time j. At each time k, the KF algorithm computes the estimate of the state given $y_1, \ldots, y_k$ - that is, all the data up to, and including timepoint k. A second source estimate that can be derived from the associated fixed interval smoothing (FIS) algorithm, defined as $x_{k|K}$ and computed with the following recursion from the KF algorithm as [9]

$$\begin{aligned} A_k &= \Sigma_{k|k}F\Sigma_{k+1|k}^{-1} \\ x_{k|K} &= x_{k|k} + A_k\left[x_{k+1|K} - x_{k+1|k}\right] \\ \Sigma_{k|K} &= \Sigma_{k|k} + A_k\left[\Sigma_{k+1|K} - \Sigma_{k+1|k}\right]A_k' \end{aligned} \tag{5}$$

for $k = K-1, \ldots, 1$ given the initial conditions $x_{K|K}$ and $\Sigma_{K|K}$. The FIS algorithm computes at each time k the estimate of the source given $y_1, \ldots, y_K$ - all the data in the experiment. The fixed interval smoothing algorithm computes the source estimates using all the data, whereas the Kalman filter uses less data from the MEG experiment as it computes the estimate of the source at each time k using only the data up through time k.

3. RESULTS AND DISCUSSION

3.1. Real Data Study Using High Dimensional Dynamic Solutions for MEG

To examine the effects of dynamic modeling solutions in the MEG inverse problem, data were acquired from a single subject using the 306 channel dc-squid Neuromag Vectorview MEG system at Massachusetts General Hospital [8]. The magnetic fields were recorded at 102 locations, each with 2 planar gradiometers and 1 magnetometer. The signals were recorded continuously with $601 Hz$ sampling rate and band-pass filtered (0.1-200Hz). The position of the electrodes and fiduciary points, e.g. the nose, nasion and preauricular locations, were digitized with the 3Space Isotrak II system to facilitate precise co-registration with MRI images. In a similar manner, the position of the head in the dewar was determined by digitizing the positions of four coils that are attached to the head and are subsequently recorded by the MEG sensors for co-registration. Ongoing magnetic activity was recorded in a single subject while 1) at rest with eyes closed, 2) at rest with eyes open, and 3) during sustained fingers movement with eyes open.

Fig. 1. Showing some representative MEG timecourses measured from SQUIDS in the proximity of the left parietal area.

This experiment was designed with the intent of studying alpha and mu rhythms in the occipital and somatosensory areas. We expect the current generators in these regions to produce synchronous oscillations approximately in the 10-20Hz range as shown in Figure 1.

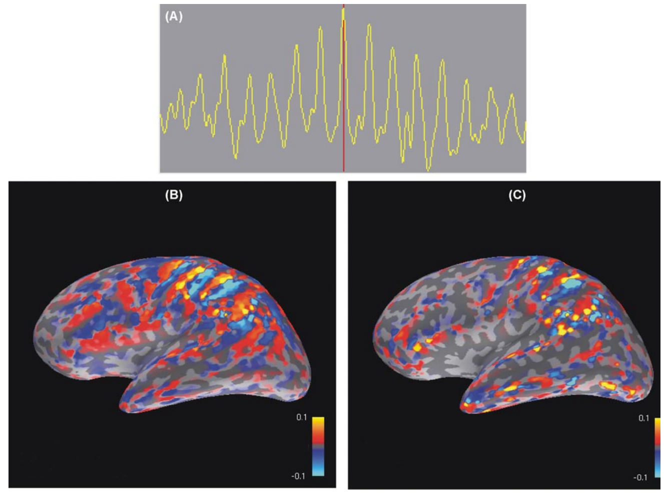

Fig. 2. (A) Example MEG timecourse taken from the left parietal region. (B) & (C) The MNE and Kalman inverse solutions for this task respectively. The maps shown represent the instantaneous timepoint activity at the peak timecourse as indicated in (A).

Collecting measurements from the MEG scanner in the absence of a subject yields an approximation to the observation noise structure, while the state covariance matrix was estimated by generating a static inverse MEG solution employing unregularized least-squares and carrying out a subsequent first-order differencing on the state-space realizations (Eq.(3)). We then computed dynamic inverse solution estimates to the current dipole components using the KF update equations given in Eq.(4).

To examine the initial hypothesis that the high dimensional Kalman filter can provide significant benefit over state-of -the-art MEG/EEG inverse procedures, we developed and ran a parallel HPC Kalman filtering program on a short window (~2s) of MEG data to simultaneously estimate the dipole strength from a dense source space that covered the entire cortex. To simplify the computations, we assumed that all dipole sources were oriented normal to the cortical surface. This resulted in a filtering problem of dimension $N = 6e3$. While not worst case, this problem still represents a considerable-sized computing challenge. When applied to the window of MEG data, this task took an average of approximately six hours for each forward Kalman sweep when using 16 CRAY-DELL nodes applied to the MEG data. Invocation of the Fixed-Interval Smoother (not shown) took approximately 20 hours on 24 processors and required about 110 Gigabytes of storage space. As expected from this mu-rhythm task, we observe strong current amplitudes in Figure(2) in an area close to the central sulcus. On first inspection, comparing the MNE and Kalman solutions, these results suggest that the dynamic approach improves localization of the inverse solution. A statistical interpretation of the apparently better localisation of the Kalman solution relates to the fact that its estimates appear heavy-tailed. That is to say, at each time instant there are many insignificant values compared to just a few large significant components. This apparent improvement may be a consequence of the KF's ability to track slow trends in the data and also the fact that the temporal continuity constraint of the state-space model allows more data to be used in estimating the sources at each time point.

4. CONCLUSIONS AND FUTURE WORK

Our findings indicate that state-space filtering methods are computationally feasible for solving source localization in MEG / EEG localization problems. These results also support our hypothesis that the dynamic source localization methods offer a more informative method of tracking brain activity than the static methods.

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6. REFERENCES

[1] B.F. Farrell and P.J. Ioannou, "State estimation using a reduced order kalman filter," *J. Atmos. Sci.*, vol. 58, pp. 3666–3680, 2001.

[2] I. Fukumori, J. Benvensite, C. Wunsch, and D.B. Haidvogel, "Assimilation of sea surface topography into an ocean circulation model using a steady-state smoother.," *J. Phys. Oceanogr.*, vol. 23, pp. 2162–2181, 1993.

[3] I. Fukumori and P. Malanotte-Rizzoli, "An approximate kalman filter for ocean data assimilation: An example with an idealized gulf stream model," *" J. Geophys. Res.*, pp. 6777–6793, 1995.

[4] Scherg M and Von Cramon D, "Two bilateral sources of the late AEP as identified by a spatio-temporal dipole model," *Electroencephalogr Clin Neurophysiol.*, vol. 62, no. 1, pp. 32–44, 1985.

[5] Mosher JC, Lewis PS, and Leahy RM, "Multiple dipole modeling and localization from spatio-temporal MEG data.," *IEEE Trans Biomed Eng*, vol. 39, pp. 541–557, 1992.

[6] Mosher JC and Leahy RM., "Recursive MUSIC: A framework for EEG and MEG source localization.," *IEEE Trans Biomed Eng.*, vol. 45, no. 11, pp. 1342–1354, 1998.

[7] Galka A, Yamashita O, Ozaki T, Biscay R, and Valdes-Sosa P., "A solution to the dynamical inverse problem of EEG generation using spatiotemporal kalman filtering.," *NeuroImage*, vol. 23, pp. 435–453, 2004.

[8] Hämäläinen MS, Hari R, Ilmoniemi RJ, Knuutila J, and Lounasmaa OV, "Magnetoencephalography - theory, instrumentation, and application to noninvasive studies of the working human brain," *Review of Modern Physics*, vol. 65, pp. 413–497, 1993.

[9] G. Kitagawa and W. Gersch, *Smoothness Priors Analysis of Time Series*, Springer-Verlag, New York, 1996.