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Estimating Arterial Stiffness using Transmission Line Model

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Abstract— **An arterial stiffness index based on the transmission line model is defined. The parameters of the transmission line model are initially estimated using measured pressure, flow and aortic root diameter. Pressure is measured at the carotid using applanation tonometry. Flow is measured using Doppler at the ascending aorta. Aortic root diameter is measured using 2-D echocardiography. The initial estimates are then refined using grey-box identification. The resulting estimate of the distributive compliance of the transmission line model is proposed to be an arterial stiffness index. Similar to the Windkessel compliance, this index describes the global stiffness. However, it is based on a more realistic 1-D model that can simulate wave propagation and wave reflection.**

I. INTRODUCTION

A. *Mechanism of Cardiovascular System*

The cardiovascular system consists of the heart and a network of blood vessels. Arteries are thick-walled, elastic vessels that provide a cushioning effect to the intermittent ejection of blood from the left ventricle. Veins are thin-walled, compliant vessels that serve as conduit for blood back to the right heart and provide extra storage at low pressure. Capillaries connect the arterial system and the venous system, and serve as an exchange site for oxygen, nutrient, waste and various other substances.

When blood travels from the heart to the capillaries, it flows through a network of arteries and arterioles. This network starts from a single artery, namely the aorta, then bifurcations occur along the network and result in numerous endings connected to the capillaries. The capillaries are very narrow, single-layered vessels where the blood velocity is close to zero. Therefore, the arterial network can be considered as close-ended or terminated by the very narrow capillary tubes. The dynamics of flow in this network is governed by the ejection pattern, the elasticity of the arteries, the viscosity and density of the blood, the geometric structure of the arteries, and the resistance of the terminating capillary tubes.

When blood is ejected into an artery, the increase in pressure causes the artery to distend to accommodate the increased blood volume. The artery then recoils due to its elasticity and pushes the increased blood volume forward and thereby causes flow. The force required to push the blood volume forward and the resulting acceleration depend on the viscosity and density of blood. In fact, the interrelation of pressure, flow, elasticity of artery, viscosity and density of blood comply with the law of conservation of mass and Newton's second law [1]. In addition, as blood propagates along the arterial network, wave reflection occurs at bifurcations, terminating capillaries and other sites of geometric discontinuity. Reflected waves are often a significant component of the measured pulse wave contour and therefore play an important role in the analysis of arterial system flow dynamics [2].

The arterial network can be considered as a system with pressure and flow respectively as system input and output. Elasticity of arteries, viscosity and density of blood and geometric structure of arteries are system parameters.

B. *Importance of Arterial Stiffness*

Arterial stiffness describes how the arteries respond when subjected to distending pressures. When subjected to pressures of equal magnitude, a stiff artery can accommodate a smaller blood volume change than a distensible artery. Compliant is another frequently used term for distensible. An artery that has a higher elastic modulus, will have higher arterial stiffness [2,4,5]. The pressure and flow pulses propagate along the artery at a finite velocity, which is termed the pulse wave velocity. It is known from elastic theory and fluid dynamics that flow in a stiffer tube will result in a higher pulse wave velocity [1]. Therefore, in a subject with stiff arteries, the reflected wave may return from the distal arterial sites during systole because of a higher pulse wave velocity [2,6,7,8]. Such early return of the reflected wave to the central arteries results in amplification of the systolic pressure, of the pulse pressure and in lowering of the diastolic pressure (fig. 1). It has been suggested that arterial stiffness is a major determinant for increased systolic pressure and pulse pressure, and therefore, the cause of several cardiovascular complications, such as left ventricular hypertrophy, aortic aneurysm formation and arterial rupture [6,7]. Therefore, measures of arterial stiffness can be clinically useful in the early detection and assessment of these pathological conditions.

C. *Models of Arterial System*

Numerous cardiovascular models have been put forward. The most accepted model is the Windkessel model [2,3,8]. It

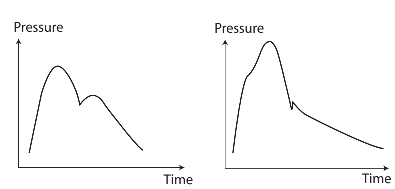

Fig. 1. reflected wave returns at diastole (left); at systole (right)

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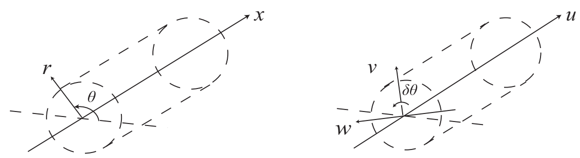

Fig. 2. Position and velocity coordinate systems of the Navier-stokes equations

considers the entire arterial system as a compliant chamber terminated by a very narrow nozzle. The compliance of the chamber corresponds to the distensibility of the arteries, and the resistance of the very narrow nozzle corresponds to the peripheral resistance. In electrical analogy, the Windkessel model is similar to a RC network.

Although conceptually useful, this model is unrealistic because the compliance is not localized at just one site but is distributed along the aorta and the major arteries [4]. Since this is a zero-dimensional model, it cannot model the effects of wave propagation and wave reflection. The arterial system can be modeled as an uniform elastic tube terminated by a very narrow nozzle. The pressure and flow relation inside the tube is governed by the Navier-Stokes equations. Assuming the blood is incompressible and the flow is axisymmetric, the Navier-Stokes equations can be simplified to:

$$\frac{\partial p}{\partial x} + \underbrace{\frac{\rho}{A}\frac{\partial q}{\partial t}}_{\text{acceleration term}} = \underbrace{\frac{2\pi a\mu}{A}\left(\frac{\partial u}{\partial r}\right)_{r=a}}_{\text{resistive term}} \quad (1)$$

$$\frac{\partial q}{\partial x} + \underbrace{\frac{\partial A}{\partial p}\frac{\partial p}{\partial t}}_{\text{compliance term}} = 0 \quad (2)$$

where p is pressure, q is volumetric flow, ρ is the density of blood, μ is blood viscosity, a is the tube radius and A is the cross-sectional area of the tube [1]. The coordinate system in which the Navier-Stokes equations are derived is shown in figure 2. The volumetric flow q is related to the axial velocity u by:

$$q = 2\pi \int_0^a rudr \quad (3)$$

These equations can be further simplified by making assumption of the velocity profile. For laminar flow in slightly tapered vessel, the velocity profile is rather flat [9]. Therefore, the velocity profile can be assumed to be flat with a boundary layer of thickness δ:

$$u = \begin{cases} u_x, & \text{for} \quad r \leq a-\delta \\ u_x(a-r)/\delta & \text{for} \quad a-\delta < r \leq a \end{cases} \quad (4)$$

And the Navier-Stokes equations can be reduced to:

$$\frac{\partial p}{\partial x} + \frac{\rho}{A}\frac{\partial q}{\partial t} = -R'q \quad (5)$$

$$\frac{\partial q}{\partial x} + \frac{\partial A}{\partial p}\frac{\partial p}{\partial t} = 0 \quad (6)$$

where R' is termed distributive resistance. This set of one-dimensional, linearized Navier-Stokes equations is analogous to the transmission line equations with pressure corresponding to voltage and volumetric flow corresponding to current. The term $\frac{\partial A}{\partial p}$ describes how the tube cross-sectional area changes when there is a change in pressure. Therefore, this term is essentially describing compliance and is termed distributive compliance C'. The term $\frac{\rho}{A}$ describe the inertia of the blood and is termed distributive inertance L'. Since the tube is terminated by a single resistance R, the boundary condition is given by:

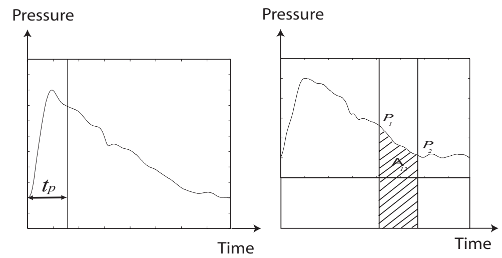

Fig. 3. Determination of t_p (left); and area method (right)

$$p(Z,t) = Rq(Z,t) \quad (7)$$

where Z is the tube length.

The transmission line model is a more realistic model since it can model wave propagation and wave reflection.

II. INITIAL ESTIMATES OF PARAMETERS

The transmission line model has five parameters: C', L', R', R and Z. These parameters are initially estimated as follows:

A. Terminating Resistance

The terminating resistance is estimated as the peripheral resistance. The steady components of the arterial pressure and flow depend on the resistance of the arterioles and capillaries. Therefore the peripheral resistance is calculated as mean aortic pressure $\bar{P}$ divided by mean aortic flow $\bar{Q}$.

$$R = \frac{\bar{P}}{\bar{Q}} \quad (8)$$

B. Distributive Inertance

The distributive inertance is estimated as the blood density divided by the aortic root area estimated from echocardiography. The standard value of 1060 kg/m^3 for whole blood density is used.

C. Tube length

In the arterial system, wave reflections occur at numerous sites. The transmission line model simplifies the situation by assuming that the total wave reflection occurs at an effective reflecting site [2]. This effective reflecting site is an imaginary site defined for modeling purpose. The tube length is estimated has the distance to the effective reflecting site.

The tube length is then calculated from the aortic pressure waveform using the following equation:

$$Z = c_0 t_p / 2 \quad (9)$$

where t_p is the time from the initial upstroke of the pressure wave to the inflection point (fig. 3) and c_0 is the pulse wave velocity. The inflection point of the pressure waveform indicates the initial upstroke of the reflected wave [2]. Therefore, t_p represents the time of travel of the pressure pulse from the measuring point to the effective reflecting site and back.

D. Distributive Compliance

The distributive compliance is estimated as the total arterial compliance divided by the tube length. The total arterial compliance is based on the Windkessel model and can be calculated by the area method (fig. 3). Two points on the diastolic portion of the aortic pressure curve are selected and the total arterial compliance C is calculated as:

$$C = \frac{\mathsf{A}_{12}}{R(P_2 - P_1)} \tag{10}$$

where A_{12} is the area under the pressure curve.

E. Distributive resistance

The distributive resistance is found by best-fitting the model output to the measured data while keeping all other parameters constant at their initial estimates. It is identifiable if there exists a unique solution such that the least-squares error is minimized.

III. REFINING PARAMETER ESTIMATES

Grey-box identification is then applied to refine the parameter estimates. The basic idea is to define a nest of model structures with increasing generality. A falsification technique is chosen and the most unfalsified model in each model structure is found in sequence.

The least-squares method is used as the falsification technique and the model with the minimum least-squares error is the most unfalsified model. The nest of model structures is defined by converting constant parameters to variable parameters during identification. In the original transmission line model, only the distributive resistance R' is considered as variable and identified. It is considered as the least general model structure and assigned as $M(R')$. The next more general model structure can be $M(R', L')$, in which both the distributive resistance and distributive inertance are identified. A nest of model structures with increasing generality is then defined:

$$\begin{aligned} M(R') &\subseteq M(R', L') \\ &\subseteq M(R', L', C') \\ &\subseteq M(R', L', C', Z) \end{aligned} \tag{11}$$

When there are two or more variable parameters, the gradient-descent method is used to find the minimum least-squares error model. The solution from the previous, less general model is used as initial guess and therefore the new solution will have at least the same, if not smaller, least-squares error.

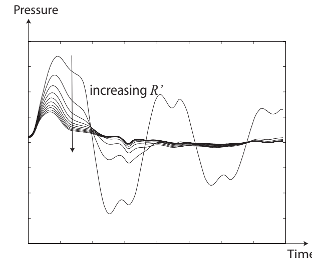

Fig. 4. Effect of distributive resistance on output flow

IV. EXPERIMENTS AND METHODS

Following Ethics approval three children subjects participated in this study. While a subject was in a supine position, ascending aortic blood velocity was obtained using Doppler with the transducer positioned at the suprasternal notch. Carotid pressure was obtained simultaneously using applanation tomometry. Descending aortic blood velocity was then obtained using Doppler. Aortic annular area and distance between two recording sites were measured using 2D echocardiography from the suprasternal notch view of the aortic arch. Carotid pressure was used as a surrogate of ascending aortic pressure. Diastolic blood pressure and systolic blood pressure were measured using cuff sphygmomanometer and used to calibrate carotid pressure.

Pulse wave velocity was calculated from the distance between the two recording sites and the delay between corresponding points on the measured waveform.

Mean blood velocity was calculated from the Doppler image. Wavelet noise removal was then applied to smooth the waveform. Volumetric flow was then calculated by multiplying mean blood velocity and aortic annular area. Approximately 20 beats of pressure and flow were collected in each subject. For each subject, three or four consecutive beats were averaged and used in the identification of model parameters. Two set of parameters were obtained. One was the initial estimates calculated from direct measurements and identification of the distributive resistance. The other was the refined estimates calculated using grey-box identification. Twelve other beats were then used in validation of these two sets of parameters. Simulations of the transmission line model were done in Pspice using the lossy transmission line

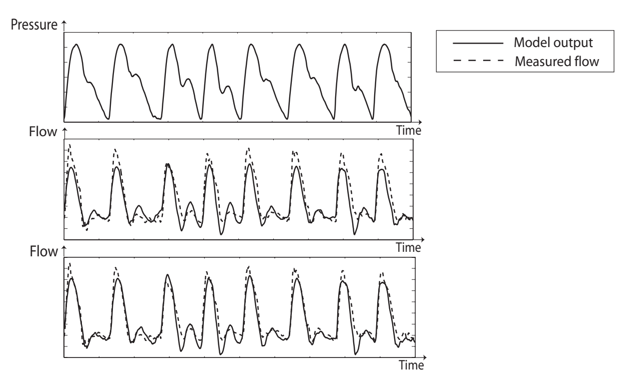

Fig. 5. Model input (upper); model output of initial estimates and measured flow (middle); model output of refined estimates and measured flow(lower)

TABLE I

CLINICAL DETAILS OF SUBJECTS

	Age	Aortic root diameter (cm)	Systolic BP (mmHg)	Diastolic BP (mmHg)	Pulse wave velocity (cm/s)
Subject A	6	1.47	102	52	442
Subject B	13	1.61	112	64	581
Subject C	8	1.39	104	60	796

TABLE II

INITIAL PARAMETER ESTIMATES

	Distributive Resistance ($mmHgs/cm^4$)	Distributive Inertance ($cm^2/mmHg$)	Distributive Compliance ($mmHgs^2/cm^4$)	Peripheral Resistance ($mmHgs/cm^3$)	Tube Length (cm)
Subject A	0.0140	0.000470	0.00455	2.72	47.9
Subject B	0.00800	0.000391	0.00842	3.08	54.2
Subject C	0.00900	0.000525	0.00629	3.00	49.8

library.

V. RESULTS

As seen on fig. 4, simulations show that if the distributive resistance increases while other parameters are kept constant, the amplitude of the output flow decreases. This is expected since current amplitude decreases as resistance increases. Therefore, one can expect a unique solution when estimating the distributive resistance alone.

For each subject, 3 or 4 beats of consecutively measured pressure and flow were used for parameter identification. The two sets of parameter estimates are shown in table II and III.

To validate the parameters for each subject, 12 beats of measured pressure were applied as input to the transmission line model which was identified previously. The model output was then compared to the measured volumetric flow (fig. 5). The least-squares error was calculated for each beat. The difference in least-squares error for each beat when using the different sets of parameter estimates was then calculated. The average in least-squares error reduction was found for each subject. It was shown that the refined estimates can reduce least-squares error by as much as 29% (table IV).

VI. CONCLUSIONS

The refined estimate of the distributive compliance is a better arterial stiffness index than the Windkessel compliance in the sense that it can produce a more validated output when applying the transmission line model.

The clinical significance of the proposed arterial stiffness index will be investigated. It will be measured in normal children and children with Marfan syndrome, a connective tissue disease characterized by increase in aorta stiffness. This index is measured non-invasively and will cause none or minimal risk and discomfort.

Although the refined estimates do improve the accuracy of the model output, it is not known whether they are the global solution to the error-minimization problem. The nature of the error function and the influence of the accuracy of the initial estimates will thus be further investigated.

VII. ACKNOWLEDGEMENT

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TABLE III

REFINED PARAMETER ESTIMATES

	Distributive Resistance ($mmHgs/cm^4$)	Distributive Inertance ($cm^2/mmHg$)	Distributive Compliance ($mmHgs^2/cm^4$)
Subject A	0.0128	0.000891	0.00498
Subject B	0.00700	0.000664	0.00775
Subject C	0.00893	0.000703	0.00650

TABLE IV

VALIDATION

	Least-Squares Error (Initial Parameters)	Least-Squares Error (Refined Parameters)	% Reduced in Least-Squares Error
Subject A	8.43e4	6.11e4	28.79
Subject B	3.20e5	2.91e5	8.82
Subject C	1.01e5	9.23e4	8.35