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A Computationally Simple and Robust Method to Detect Determinism in a Time Series

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Abstract—**We present a new, simple, and fast computational technique, termed the incremental slope (IS), that can accurately distinguish between deterministic from stochastic systems even when the variance of noise is as large or greater than the signal, and remains robust for time-varying signals. The IS method is more accurate than the widely utilized Poincare plot analysis especially when the data are severely contaminated by noise. The efficacy of the IS is demonstrated with several simulated deterministic and stochastic signals.**

Keywords—**Incremental Slope, nonlinear determinism, phase map, surrogate technique, heart rate variability**

I. INTRODUCTION

Over the past few decades, different quantitative measures of invariant characteristics of the dynamics of a system have been developed to determine if the system is deterministic [1-4]. The largest Lyapunov exponent, for example, gives the rate of exponential divergence from initial conditions after a small perturbation; fractal dimension characterizes the geometrical structure of the phase map. However, the accuracy of these invariant classifiers is sensitive to noise corruption. Development of robust invariant classifiers that are less sensitive to noise has been a significant technical challenge for the last two decades. Some of the notable tests for determinism in a time series include Phase map based methods [5], entropy based algorithms [6], and nonlinear autoregressive (NAR) based algorithms [7-9], and recurrence plots method [10]. While these techniques have merit, their performance is still highly sensitive to signal-to-noise ratio (SNR). In this study, we introduce a novel invariant classifier, termed the incremental slope (IS) that remains robust despite severe noise corruption and is computationally efficient. The IS is based on the simple observation that for strange attractors of deterministic systems, the ratio of the number of points above and below the unit slope in a phase space is constant for every subset, thus resulting in a relatively straight line that is either increasing, decreasing or constant. This observation and a simple transformation of data results in the IS phenomenon for deterministic systems. The proposed method is validated on simulated deterministically chaotic time series. We determine that this method provides an accurate test for the presence of determinism in time series even with 0 dB additive Gaussian white noise and an equal amount of colored noise corruption.

II. METHOD

While it is not crucial to the accuracy of the IS, before applying the IS algorithm, appropriate embedding dimension (ED) parameter is need. For example, the discrete Mackey-Glass time series, which is described by the equation

$$x(t+1) = x(t) + \frac{0.2x(t-16)}{1+x^{10}(t-16)} - 0.1x(t)$$

The appropriate ED was selected to be 17. Proper choice of ED will ensure full unfolding of dynamics in the phase space. Widely recommended approach to determining ED is the false nearest neighbour method [11]. In the subsequent text, we will assume all time series have been appropriately embedded.

Realization of the IS method is simple and straightforward. First, we modify the time series with the following constraints:

$$\begin{cases} y(n) = y(n-1)+1 & \text{if} \quad x(n) > x(n-1) \\ y(n) = y(n-1)-1 & \text{if} \quad x(n) < x(n-1) \\ y(n) = y(n-1) & \text{if} \quad x(n) = x(n-1) \\ y(1) = x(1) \end{cases} \tag{1}$$

where *x(n)* and *y(n)* are the original and modified time series, respectively. Fig. 1 illustrates the phase map of the modified Henon, Mackey-Glass, Rossler time series, and Gaussian white noise (GWN), respectively. We observe in Fig.1 that all modified deterministic signals clearly shows an incrementing straight line while the modified GWN time series shows fluctuations that are characteristic of Brownian motion. For the modified Henon map, the correlation coefficient is 0.99 while it is significantly less (0.58) for the GWN time series.

Why does the modified deterministic time series exhibit a straight-line characteristic? Explanation of this behavior is illustrated in Fig. 2, where a diagonal line divides the Henon map. Because every subset of the time series shows similar strange attractors, the ratio between the number of points above and below the diagonal line is approximately a constant value for every subset, resulting in an approximately straight line that can be either linearly decreasing, increasing or constant when the time series is modified using Eq. (1). For the formal definition of the constant slope, we define the following: the number of points above and below the line y=x is denoted as N_1 and N_2, respectively, and the total number on the plane is denoted as $N = N_1 + N_2$. The IS value is then obtained by the following:

$$\text{Incremental Slope}=\frac{N_1 - N_2}{N} \tag{2}$$

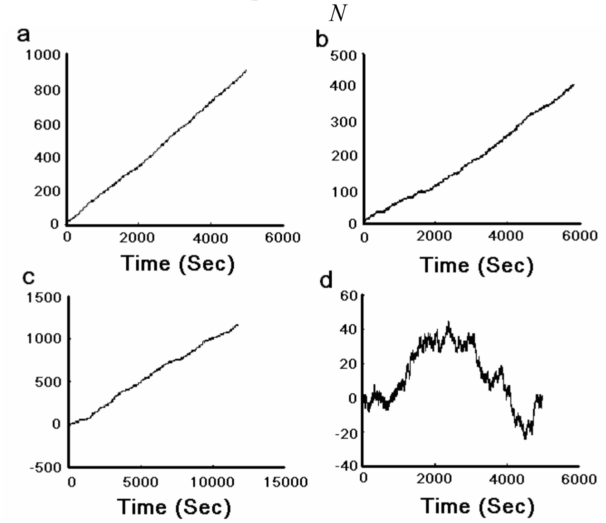

Fig. 1 Panel a, b and c show the modified Henon, Mackey-Glass and Rossler maps, respectively, which all exhibit incremental slope. Panel d shows modified Gaussian white noise, which does not exhibit incremental slope.

For three-dimensional attractors, which can only be divided into two parts by a plane instead of a line, the following can be defined:

$$\begin{cases} a(n) = a(n-1)+1 & \text{if} \quad z(n) > x(n)+y(n) \\ a(n) = a(n-1)-1 & \text{if} \quad z(n) < x(n)+y(n) \\ a(n) = a(n-1) & \text{if} \quad z(n) = x(n)+y(n) \\ a(1) = z(1) & \end{cases} \tag{3}$$

where *x(n), y(n)* and *z(n)* are the generated variables of a 3-dimensional system, with n=1…N, and *a(n)* is the modified time series. The IS is calculated the same as in equation (2), but N_1 is the number of points above the plane z=x+y, while N_2 is the number of points below the plane z=x+y. The bottom panels of Fig. 3 illustrate phase maps of the example of higher than 2-dimensional systems: Rossler systems of differential equations. These systems are expected to exhibit the IS characteristics as they are governed by deterministic dynamics, and indeed, both systems do.

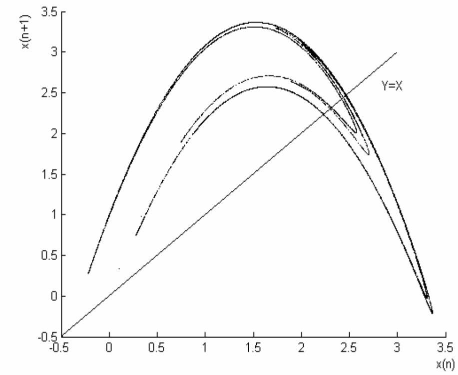

Fig. 2. The Henon map is divided into two parts by y=x. We denote the number of points above the line as N_1, the number of points below the line as N_2, and the increment slope as $\frac{N_1 - N_2}{N}$. $N = N_1 + N_2$.

III. SIMULATION

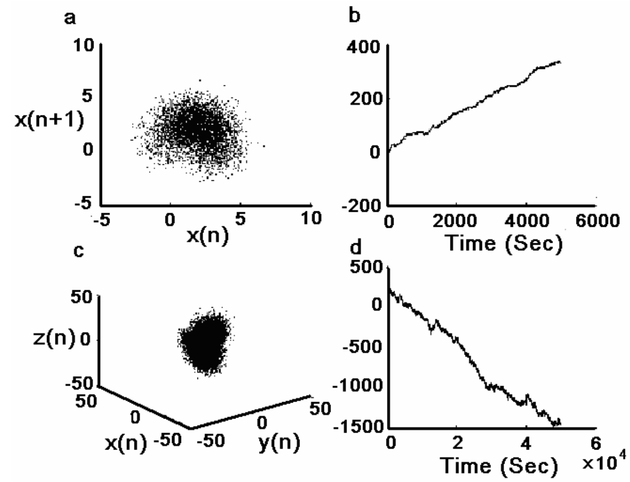

Fig. 3. The panel a and b are the 0 dB noise-corrupted Henon map and the modified Henon time series, which exhibits incremental slope. The panel c and d are 3 dB noise-corrupted Lorenz data and the modified Lorenz time series, which also exhibits incremental slope behavior.

The IS, as designed, is inherently less affected by noise contamination than prior methods. In the phase space, only significant noise contamination can cause datapoints to cross over the unity line. Thus, even for low signal-to-noise ratios, only a small number of points close to the unity line in the phase space are affected by noise corruption. To test this assertion, 0 dB and 3dB GWN signals were added respectively to the Henon map and Lorenz's equations, and the results are shown in the top and bottom right panels of Fig. 3, respectively. The top and bottom left panels represent phase maps of the two systems corrupted by 0 dB and 3 dB GWN, respectively. For both the discrete and continuous system represented in Fig. 3, we observe consistent IS behavior despite severe noise contamination. Comparing the noise-free and noise-corrupted (0 dB additive GWN and 0 dB colored noise) Henon, Mackey-Glass, Lorenz and Rossler's systems of equations, the correlation coefficient values associated with 0 dB GWN and 0 dB colored noise corrupted signals are close to the value of 1 (>0.98), as is also the case with the noise-free signals.

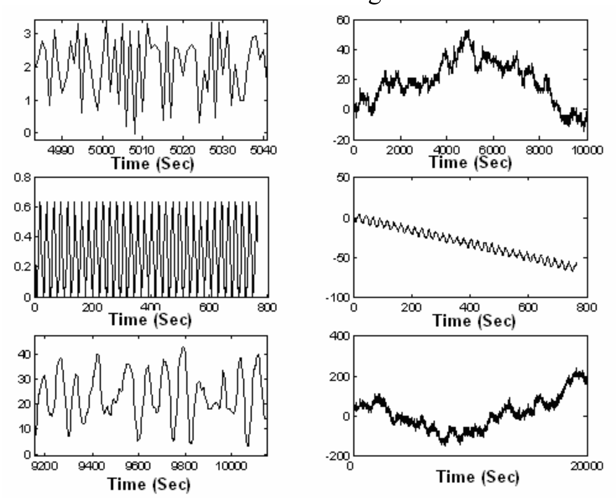

Fig. 4. The top left panel is the surrogated Henon time series; the top right panel is the modified surrogated Henon time series. The middle left panel is the surrogated capacitor charge and discharge time series; the middle right panel is the modified surrogated capacitor charge and discharge time series. The bottom left panel is the z vector of the Lorenz time series, and the bottom right panel is the modified surrogated Lorenz time series, even when the surrogate of the linear dynamics is modified.

From the definition of incremental slope, it can be seen that all time series whose phase maps have specific geometrical trajectories (e.g. fixed point, limit cycle, and strange attractor) will exhibit IS characteristics. The fixed point process is the simplest and can be represented by a periodic oscillator. The next logical question is how do we distinguish between nonlinear deterministic systems and linear systems since linear periodic oscillators can also exhibit IS behavior? This question can be answered by the use of the surrogate data technique (SDT) pioneered by Theiler [12]. The surrogate data will destroy any nonlinear determinism in the signal via phase randomization. Thus, the surrogate data from a true nonlinear deterministic signal should not exhibit IS behavior even when the surrogate data is modified using Eq. (1), whereas linear oscillators should retain IS behavior, This is indeed what we observe in the right panels of Fig. 4 for the Henon map (top left), a capacitor (middle left) and the Lorenz system (bottom left). Only the modified surrogate data of a capacitor retains the IS characteristics. Therefore, with the use of SDT, differentiation between linear and nonlinear deterministic signals can be efficiently made. The correlation coefficients of the modified noise free and 10 dB GWN perturbed Henon map and Mackey-Glass system before and after surrogate data was compared. For all deterministic systems examined, there is a statistically-significant difference ($p < 0.001$) in the correlation coefficients between pre- and post-surrogate data, for both the noise-free and noise-contaminated cases.

Another application of the IS is identification of switching dynamics. Changes in dynamics are represented by different incremental slopes. To represent changes in dynamics we consider the Henon map where the parameter "a" in Eq. (4) is changed from 3.18 to 2.93 at time index 4000:

$$y(n) = ay(n-1) + 0.3y(n-2) - y^2(n-1) \qquad (4)$$

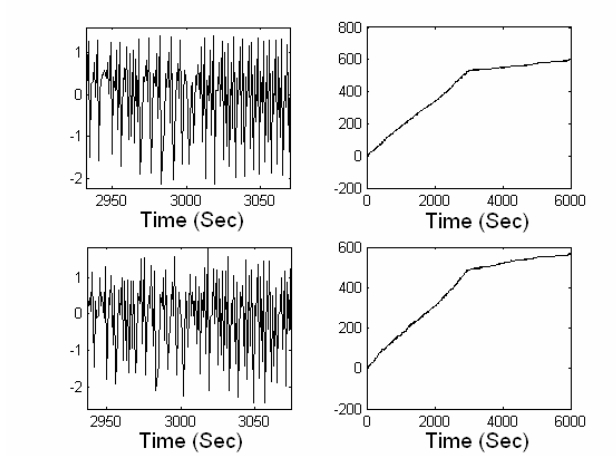

Fig. 5. The top left panel shows the change in dynamics from $y(n)=3.168y(n-1)+0.3y(n-2)- y(n-1)^2$ to $y(n)=2.93y(n-1)+0.3y(n-2)- y(n-1)^2$ at time point 4000. The top right panel shows the modified time series indicating switching dynamics at time point 4000. The bottom left and right panels correspond to the top panels but with this Henon time series corrupted by 10 dB Gaussian white noise.

The modified time series of Eq. (4) and the corresponding IS are displayed in the top left and right panels of Fig. 5, respectively. The modified time series corrupted by 10 dB GWN and the corresponding IS are displayed in the bottom left and right panels of Fig. 5, respectively. The change in the slope values of the IS clearly indicates two different dynamics and this change occurs exactly at the time index of 4000. With 10 dB GWN noise corruption, it is difficult to observe the change in dynamics at time point 4000, but the IS method is robust enough to identify the switching dynamics in the time series. The importance of this result is that the IS is essentially a time-varying detector of determinism. For example, the IS is able to identify a portion of the signal that may have deterministic component. Current methods for detecting determinism in time series are not designed for time-varying signals, and will not provide accurate results.

IV. CONCLUSION

In this study, we have presented a new, simple, and computationally efficient technique known as the incremental slope which can detect the presence of nonlinear determinism even when the variance of the noise is equal to or greater than the variance of the signal. An additional capability of the incremental slope is its ability to detect changes in dynamics. These salient features and the simple implementation of the method should facilitate screening of deterministic systems that may arise from diverse fields.

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