

An Improved QT Interval Measurement based on Singular Value Decomposition

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Abstract— This paper proposes an improved method of automatic ECG QT interval measurement based on the singular value decomposition (SVD) of multiple lead ECG signals. SVD separates multiple lead ECG record into orthogonal signals. Major orthogonal signals associated with high singular values are selected first for subsequent analysis. Instantaneous norm of the major three orthogonal signals are used for estimating Q wave initiating time t_Q . Two dimensional trajectory of the major orthogonal signals are utilized for T wave end time t_{TE} estimation. The T wave trajectory stagnates at t_{TE} . For the accurate t_{TE} estimation, this stagnation of the trajectory is proposed to be detected by its change in tangential angle. The proposed method was applied to 17 ECG data from normal subjects, patients of long QT syndrome (LQTS) and left ventricular hypertrophy (LVH) to demonstrate its effectiveness. Good consistent agreement, mean relative error of 5.01%, between estimated QT intervals and those of manual measurement by an experienced cardiologist was achieved.

Keywords—ECG signal analysis, Singular value decomposition, QT interval, Automatic measurement, T-loop

I. INTRODUCTION

Systematic accurate QT interval measurement is important for automatic ECG evaluation. Methods previously documented in the literature [1]-[4] simulate the manual measurement by experienced cardiologists. A new method based on singular value decomposition (SVD) has been proposed[5] to overcome the instability caused by conventional methods such as lead selection or measurement noise. The method works on transformed orthogonal signals to improve the stability of the QT interval estimation. However, it still simulates the manual measurement on instantaneous norm of the orthogonal signals at the final stage to determine the T wave end time t_{TE} . Acar *et al.* proposed a unique method based on SVD to determine the T wave end time for appropriate segmentation of ECG waveform in their celebrated paper[6] on the precise characterization of T-wave morphology. The paper utilizes the fact that T-loop, two dimensional trajectory of major orthogonal signals, stagnates at the T wave end time t_{TE} . The goal of the method is to make automatic ECG segmentation rather than measuring T wave end time. Hence their method

estimates naturally the upper bound of t_{TE} rather than the exact t_{TE} . We shall demonstrate in this paper that the accurate and stable t_{TE} estimation is achieved by evaluating the change in tangential angle of the T-loop. To validate the proposed method, 8-lead (I, II, V1-V6) ECG signals from normal subjects, left ventricular hypertrophy (LVH) and long QT syndrome were analyzed. Comparison between estimated QT intervals and manual measurement by an experienced cardiologist was made.

II. METHODS

Let us denote the original ECG time series from the m^{th} lead as $x^{(m)}[n]$, $m=1,\dots,M$; $n=1,\dots,N$. Here, M and N respectively denote the number of ECG leads and sample data. By the singular value decomposition (SVD), original ECG data are decomposed into M orthogonal signals and expanded as:

$$x^{(m)}[n] = \sum_{r=1}^M a_{mr} s^{(r)}[n], \quad n = 1,\dots, N; m = 1,\dots, M \quad (1)$$

Here, $s^{(r)}[n]$ is the r^{th} singular vector (orthonormal function associated with the r^{th} singular value σ_r) multiplied by σ_r . Singular values are arranged in descending order of its value, *i.e.* $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_M$. To be more specific, $s^{(r)}[n]$ is the r^{th} row of the matrix S in the following SVD formula.

$$X = A\Sigma V = AS \quad (2)$$

Here, X is the matrix whose m^{th} row is original ECG signal $x^{(m)}[n]$, $n=1,\dots,N$, from the m^{th} lead. A is the coefficient matrix for the original signal restoration from decomposed orthogonal signals. Σ is a diagonal matrix whose main diagonal elements are singular values. V is the matrix whose r^{th} row is the r^{th} orthonormal singular vector. It is pointed out in the literature[6] that original ECG time series are approximated well by first two or three orthogonal functions. Relative approximation accuracy α by the signal reconstruction by the first r_0 ($\leq M$) orthogonal functions,

$$\tilde{x}^{(m)}[n] = \sum_{r=1}^{r_0} a_{mr} s^{(r)}[n], \quad n = 1, \dots, N; m = 1, \dots, M \quad (3)$$

is evaluated by

$$\alpha = \sqrt{\sum_{r=1}^{r_0} \sigma_r^2 / \sum_{r=1}^M \sigma_r^2} \quad (4).$$

We confirmed that $r_0=3$ is sufficient in describing original 8-lead ECG time series with 95% accuracy. To estimate Q wave initiating time t_Q we use major three orthogonal functions. First, instantaneous norm $p[n; r_0]$ is obtained as,

$$p[n; r_0] = \sqrt{\sum_{r=1}^{r_0} (s^{(r)}[n])^2} \quad (5).$$

Here we set $r_0=3$. The spline derivative $q[n; r_0]$ of the instantaneous norm $p[n; r_0]$ is obtained as an indicator of t_Q estimation. $q[n; r_0]$ is traced back from the R peak instance t_{RP} of the norm $p[n; r_0]$. t_Q is estimated as the time from which consecutive three signs of $q[n; r_0]$ remain negative. To determine the T wave end time t_{TE} , relevant T wave portion is extracted. $p[n; r_0]$ values are traced from t_{RP} and 48 (ms) advanced time from the time which gives 70% reduced amplitude from $p[n; r_0]$ peak at t_{RP} is set as T wave initiating time t_T . As you will see later, the selection of t_T is not sensitive to t_{TE} estimation accuracy. Now, original ECG data from t_T to the recording end are extracted and SVD decomposition is newly made to this T wave segment. Two major orthogonal signals ($r_0=2$) are typically sufficient to approximate the T wave portion of ECG waveform. We denote orthogonal signals obtained by SVD of the T wave as $s_T^{(r)}[n], r=1,2$. The instantaneous norm $p_T[n; r_0]$ of that region is defined in the same way as $p[n; r_0]$. Two dimensional plots of these signals are then made. An example of the 2D plots is shown in Fig. 1.

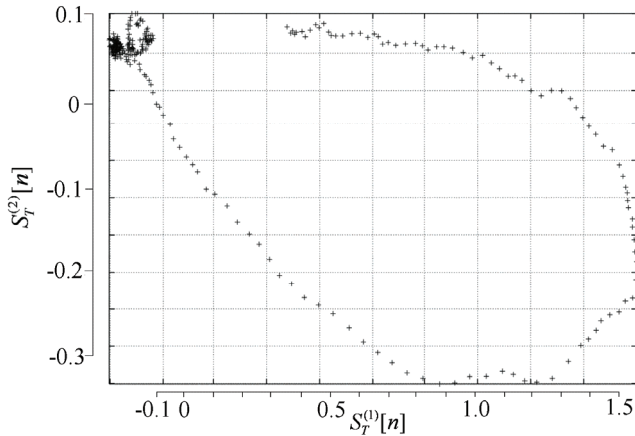


Fig. 1 2D plots of two major orthogonal signals

Acar et al. [6] proposed unique method to determine T wave terminating time. They divided the 2D plane by 10X10 rectangle blocks and counted the number of data points $D[k]$ fall in each block. The block number is denoted by k . The number of data points rapidly increases as time approaches to the T wave terminating time since the data values of both signals there stay at around the same small values close to zero. See upper left corner of Fig. 1. The average m_D and standard deviation s_D of $D[k]$ are estimated excluding empty blocks. Critical value is set at $m_D + \alpha \cdot s_D$. Here, α is a constant typically set at 3. The time corresponding to the first data point which falls in the first block including the number of data points exceeds the critical value is regarded as T wave end time t_{TE} . Since the method intends to make the appropriate data segmentation rather than estimating precise T wave end time t_{TE} , the estimate tends to become upper bound, apparently larger than the genuine t_{TE} . To narrow down the estimate of t_{TE} to the genuine value, we would like to propose to utilize the change in tangential angle of the 2D trajectory of orthogonal signals. We define the tangential angle $\theta[n]$ of the trajectory as,

$$\theta[n] = \tan^{-1} \frac{s_T^{(2)}[n] - s_T^{(2)}[n-1]}{s_T^{(1)}[n] - s_T^{(1)}[n-1]} \quad (6).$$

The change in the angle $\Delta\theta[n](= \theta[n] - \theta[n-1])$ is a good indicator to show the termination of T wave as illustrated by Fig. 2.

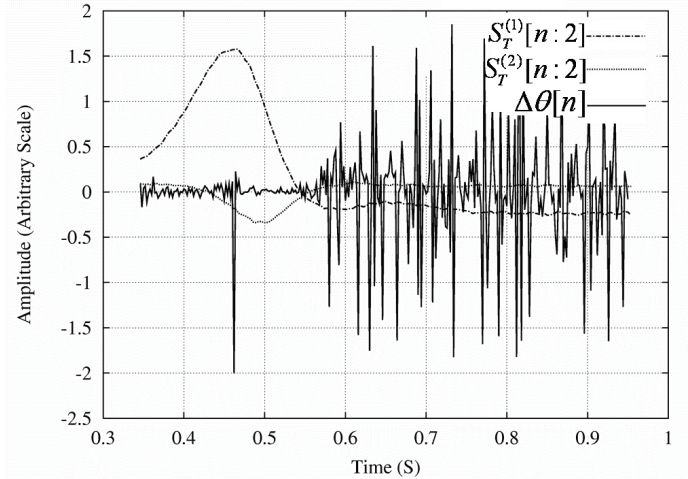


Fig. 2 $\Delta\theta[n]$ as an indicator of estimating T wave terminating time t_{TE} .

Solid lines in the figure showing $\Delta\theta[n]$ is superimposed on $s_T^{(1)}[n]$ and $s_T^{(2)}[n]$. It is clearly seen that $\Delta\theta[n]$ starts to show large instable fluctuations at the end of T -wave. To quantify

the estimation, we first identify the stable region of $\Delta\theta[n]$ as the time range where the instantaneous norm $p_T[n;2]$ takes values between 20% and 80% of the peak T wave values. Then mean $m_{\Delta\theta}$ and standard deviation $s_{\Delta\theta}$ are estimated. T wave end time t_{TE} is estimated as the time when $\Delta\theta[n]$ exceeds the threshold value $m_{\Delta\theta} + \beta \cdot s_{\Delta\theta}$. Here β is a constant typically set at 2.

III. RESULTS

Eight lead (I, II, V1-V6) ECG signals from nine normal subjects, six patients of long QT syndrome ($LQTS$) and of two left ventricular hypertrophy (LVH) were analyzed to validate the proposed method. Data were sampled at 500 Hz. Fig. 3 shows a typical example of the instantaneous norm $p[n; 3]$. Estimated t_Q (marked by the open circle) and t_{TES} by Acar's method (open rectangle) and proposed method (filled triangle) are superimposed on the figure.

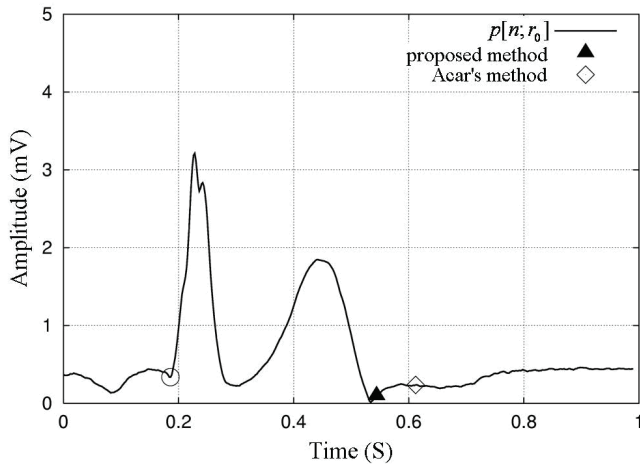


Fig. 3 A typical norm signal and estimated Q-wave initiating time t_Q and T-wave end time t_{TES} by two methods discussed in this paper.

Q -wave initiating time t_Q is accurately estimated by the simple algorithm described in the methods section. It is clearly seen that t_{TE} by Acar's method indicates a larger upper bound of the T -wave end time suited for its automatic segmentation. While t_{TE} by the proposed method is close to the time on which the exact T -wave end should be. To validate this claim, measured QT intervals ($t_{TE} - t_Q$) are compared with those manually measured by an experienced cardiologist. Manual measurements were done on printed ECG traces of lead II, V2 and V5. Measured QT intervals were averaged to obtain reference QT interval values. Figure 4 shows the result of this comparison. Measured t_{TES} by Acar's method are shown by asterisks while t_{TES} by the proposed method are shown by filled circles. As implied by an example shown by Fig. 3, t_{TES} by Acar's method lie on

above manual measurements indicating that they are positively biased estimates of QT intervals. It is seen that QT intervals by proposed method are unbiased and closer to manual measurements. Mean value of relative estimation errors was 5.01%. Figures 5-7 compares manually measured and estimated QT intervals from single leads (Fig. 5 from II; Fig. 6 from V2; Fig. 7 from V5.) Relative mean estimate errors were 6.13%(II), 7.87%(V2) and 4.52%(V5). Estimated QT intervals from single leads also showed good agreement with manually measured values although comparatively large dispersions were observed in case of the measurement by the lead V2.

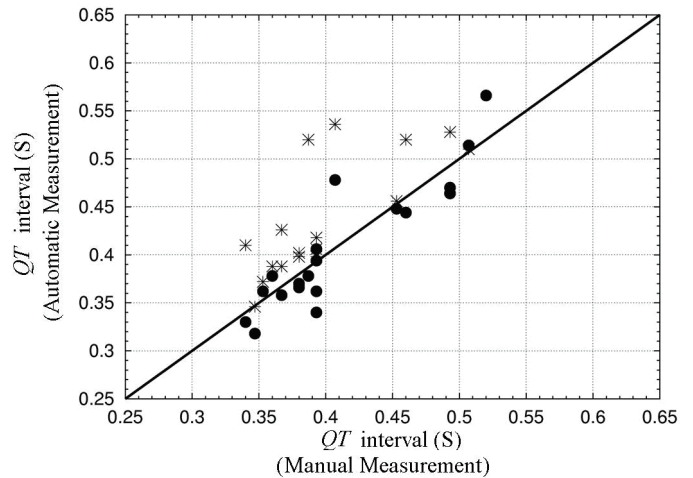


Fig. 4. Comparison between estimated and manually measured average QT intervals from lead II, V2 and V5.

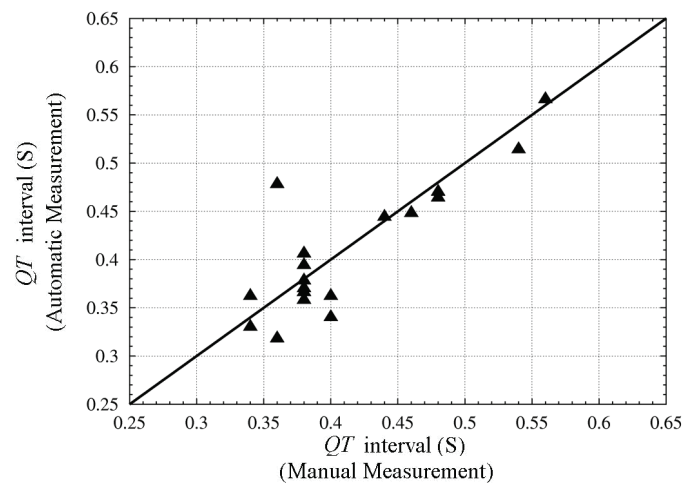


Fig.5. Comparison between estimated and manually measured QT intervals from the single lead II.

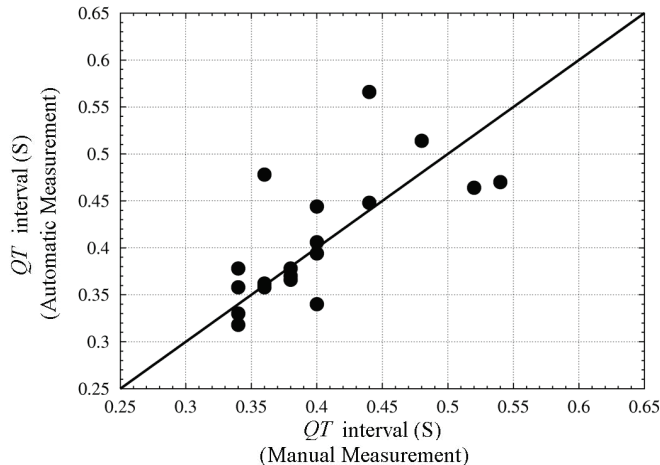


Fig.6. Comparison between estimated and manually measured QT intervals from the single lead V2.

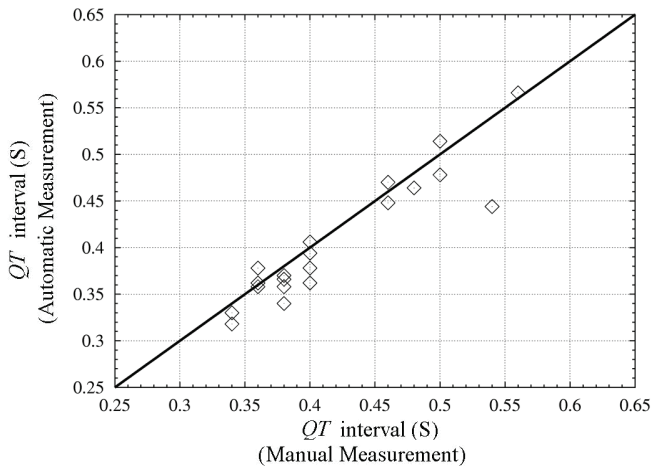


Fig.7. Comparison between estimated and manually measured QT intervals from the single lead V5.

IV. DISCUSSION

Acar's method gives biased estimate of t_{TE} as shown by Fig. 4. This is because the method doesn't intend the accurate measurement of t_{TE} but for automatic segmentation of T -wave as a step toward precise characterization of T wave morphology. For that purpose the estimate rather should be positively biased to include whole T -wave portion of the ECG signals. This paper showed that the basic idea of the method is not only be suited for T -wave segmentation but be suited for accurate estimation of t_{TE} by replacing the method of finding the starting point of data stagnation in two dimensional trajectory of orthogonal signals. One of advantages to introduce SVD for estimating t_{TE} is the measurement noise insensitivity. Selecting a few major orthogonal functions for the estimation eliminates the effect of irregular noise components. Another advantage is the

insensitivity to the lead waveform variations. ECG records from individual leads usually show large waveform variations. Orthogonal signals obtained by SVD however show rather consistent waveforms resulting in efficient systematic and stable estimation of t_{TE} . Manually measured QT intervals vary depending on the lead selection as shown in Figs. 5-7. Hence, lead selection for the manual QT interval measurement is important. We have seen good agreement between estimated QT intervals and averaged manual measurement from leads II, V2 and V5. More data analysis may be necessary to confirm and to interpret why the best agreement was achieved between estimated QT interval and manual measurement from the single lead V5.

V. CONCLUSION

A method for accurate and efficient QT -interval estimation has been introduced. The instantaneous norms $S[n; Re]$ of major orthogonal signals derived from the singular value decomposition of original ECG signals are shown to be a promising tool for accurate and stable estimation of QT intervals. The SVD method is potentially useful in many ways. Appropriate selection and grouping of ECG signals from different leads may be utilized for the estimation of QT dispersion. $q[n; r_0]$ and major orthogonal signal themselves $s^{(r)}[n]$'s may also be useful for more general ECG morphology analysis for clinical applications since the set of orthogonal signals obtained from SVD preserve wave form information of all set of original ECG record.

ACKNOWLEDGMENT

This work has been supported by the ministry of education, culture sports science and technology (MEXT) of Japan as one of national Open Research Center programs.

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