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Decomposition of a Parallel Cascade Model of Ankle Stiffness Using Subspace Methods

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***Abstract*—Joint stiffness, defined as the relation between the angular position of a joint and the torque acting about it, can be used to describe the dynamical behavior of the human ankle during posture and movement. Joint stiffness can be separated into intrinsic stiffness and reflex stiffness, which are modeled as a linear system and a Hammerstein system, respectively. A two-pathway parallel cascade model, with the intrinsic stiffness on one pathway and the reflex stiffness on the other, can be used to describe the joint stiffness. In this paper, we present a new method to separate the torque from each pathway from the total torque measurement. A subspace based system identification method is used to estimate the dynamics of each pathway directly from measured data without iteration. Simulation studies demonstrate that the method produces accurate results without the need of iteration.**

***Keywords*—decomposition, parallel cascade systems, ankle dynamics, subspace method**

I. INTRODUCTION

To study the mechanical behavior of the mechanisms acting about the ankle, the concept of joint dynamic stiffness is used. Joint stiffness is defined as the dynamic relation between the angular position of a joint and the torques or forces acting about it. Joint stiffness can be separated into two components: an intrinsic component due to the mechanical properties of the joint, passive tissue, and active muscle fibers; and a reflex component due to muscle activation in response to the activation of stretch receptors in the muscle. Kearney and Hunter [1] found that a parallel cascade model could be used to describe the joint dynamic stiffness, as shown in Figure 1.

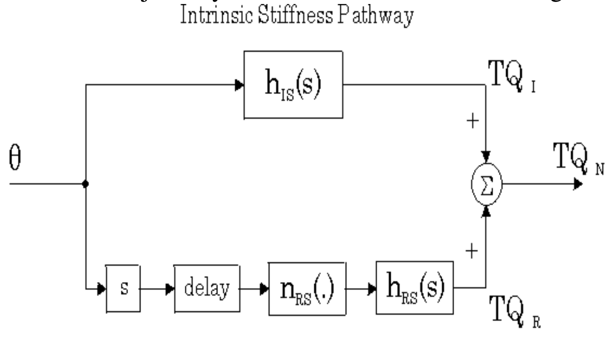

Figure 1 Parallel-cascade structure of ankle dynamics. θ, TQ_I, TQ_R and TQ_N denote position, intrinsic torque, reflex torque and net torque. $h_{IS}(s)$ and $h_{RS}(s)$ represent Impulse Response Functions for intrinsic and reflex stiffness. $n_{RS}(s)$ denotes a static nonlinearity.

TQ_I and TQ_R cannot be measured directly from the experiment, so a direct estimation of $h_{IS}(s)$ and $h_{RS}(s)$ is not feasible. An iterative algorithm [2] has been developed to efficiently separate and estimate the intrinsic stiffness and reflex stiffness respectively from the measured position θ and the measured net torque TQ_N.

In this paper, we present a new non-iterative method to separate the torque from the intrinsic stiffness and the torque from the reflex stiffness. MOESP (MIMO output error state space), a family of subspace based identification methods, is used to estimate a nonlinear state-space model from the measured position θ and the measured net torque TQ_N. The past output is used as instrumental variable to eliminate the noise from the measurement. Afterwards, the torque from each pathway is calculated by simulating the estimated state space model with corresponding inputs. Compared with the iterative algorithm, the new method requires no priori knowledge of any stiffness pathway, is faster in computation and more robust in SNR (signal to noise ratio), and does not require iterations.

II. IDENTIFICATION

Subspace methods have proven to be an efficient alternative to classic identification technologies. These methods estimate state space models directly from the input-output data. Multiple inputs and multiple outputs (MIMO) system are treated in the same way as single input and single output systems (SISO) systems [3]. Different instrumental variable structures, past input or past output, can be implemented to eliminate the affects from the noise source.

A. State Space Model for Intrinsic Stiffness age

Intrinsic stiffness can be modeled by a dynamic linear system between the position θ and the torque TQ_I. Thus a SISO state space model can be used to describe intrinsic stiffness.

$$\begin{aligned} X^l{}_{k+1} &= A_l X^l{}_k + B_l u_k \\ y_k^l &= C_l X^l{}_k + D_l u_k \end{aligned} \quad (1)$$

where u_k, y_k and X_k^l denote the input position, torque from the intrinsic stiffness, and the internal states. A_l, B_l, C_l, D_l are the system matrices for intrinsic stiffness.

B. State Space Model for Reflex Stiffness

Reflex stiffness has a Hammerstein structure, consisting of a series connection of a static non-linearity followed by a linear dynamic system. An extended subspace method for Hammerstein systems has been developed to identify the nonlinear function and the system matrices of the linear part. [4]. Assuming the static nonlinearity $n_{RS}(s)$ can be approximated by a basis function $g(\cdot)$, we have:

$$\begin{aligned} z_k &= g(u_k, \tau) \\ &= \sum_{i=1}^{r} \tau_i g_i(u_k) \end{aligned} \tag{2}$$

where $g_i(u_k)$ is the kernel of the basis function and τ is the parameter of the basis function. u_k and z_k denote the input to the nonlinearity and the output from the nonlinearity.

Furthermore, a state space model can be used to describe the linear part of the reflex stiffness:

$$\begin{aligned} X^l{}_{k+1} &= A_l X^l{}_k + B_l z_k \\ y_k^l &= C_l X^l{}_k + D_l z_k \end{aligned} \tag{3}$$

From equation (3), the output from the nonlinearity is the product of parameter and the kernel of the basis function, Defining now $\tilde{B} = [B\tau_1, \quad \cdots \quad B\tau_r]$, $\tilde{D} = [D\tau_1, \quad \cdots \quad D\tau_r]$ and $U_k = [g_1(u_k), \quad \cdots \quad g_r(u_k)]^T$, the Hammerstein system can be rewritten as

$$\begin{aligned} X^h{}_{k+1} &= A_h X^h{}_k + \tilde{B} U_k \\ y_k^h &= C_h X^h{}_k + \tilde{D} U_k \end{aligned} \tag{4}$$

where y_k^h and X_k^h denote the torque from the reflex stiffness and the internal states. A_l, C_l, represent the system matrices for intrinsic stiffness.

Thus using the chosen basis function, the SISO Hammerstein system can be described by a MISO linear state space model with multiple constructed inputs.

C. Estimate the state space model for joint stiffness

It is still not yet clear how to estimate the state space models for intrinsic stiffness and the reflex stiffness respectively since y_k^l and y_k^h are not available from the experiment. Considering that the measurement torque TQ_N is the sum of the torque from the intrinsic stiffness and the torque from the reflex stiffness, let $Y = y_k^l + y_k^h$, equation (2) and the equation (3) can be combined as:

$$\begin{aligned} \begin{bmatrix} X^l{}_{k+1} \\ X^h{}_{k+1} \end{bmatrix} &= \begin{bmatrix} A_l & 0 \\ 0 & A_h \end{bmatrix} \begin{bmatrix} X^l \\ X^h \end{bmatrix} + \begin{bmatrix} B_l & \tilde{B} \end{bmatrix} \begin{bmatrix} u_k \\ U_k \end{bmatrix} \\ Y &= \begin{bmatrix} C_l & C_h \end{bmatrix} \begin{bmatrix} X^l \\ X^h \end{bmatrix} + \begin{bmatrix} D_l & \tilde{D} \end{bmatrix} \begin{bmatrix} u_k \\ U_k \end{bmatrix} \end{aligned} \tag{5}$$

Specifically for the above state space model, the input $\begin{bmatrix} u_k \\ U_k \end{bmatrix}$ and the output Y contain u_k , Y direct from the experiment and U_k which is construct from the measured u_k . Thus PO-MOESP can be used to estimate the system matrices $\hat{A}, \hat{B}, \hat{C}, \hat{D}$ for state space model (5) from input $\begin{bmatrix} u_k \\ U_k \end{bmatrix}$ to output Y .

$\hat{A}, \hat{B}, \hat{C}, \hat{D}$ do not estimate the intrinsic stiffness and the reflex stiffness directly. However, by simulating the estimated system with appropriate inputs, it is possible to estimate the torque from intrinsic stiffness and the torque from reflex stiffness. Specifically, the output from the simulation with input signal $\begin{bmatrix} u_k \\ 0 \end{bmatrix}$ provides an estimate of the intrinsic torque TQ_I . Similarly, simulating the estimated model with $\begin{bmatrix} 0 \\ U_k \end{bmatrix}$ provides an estimate of the output from the reflex stiffness.

III. SIMULATIONS

To test and validate the algorithm, simulated data was generated using Matlab's Simulink. Intrinsic stiffness was modeled as a linear, second-order high-pass system with transfer function:

$$\frac{TQ_I(s)}{\theta(s)} = I(\lambda)s^2 + B(\lambda)s + K(\lambda) \tag{6}$$

where θ is joint angle, TQ_I is torque from the intrinsic stiffness, *I* is inertia, *B* is viscous parameter, and *K* is elastic parameter. Reflex stiffness was described by a half-wave rectifier followed by a second order low pass filter as

$$\frac{TQ_R(s)}{V_R(s)} = \frac{G_R \omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)} \tag{7}$$

where TQ_R is reflex torque, V_R is half-wave rectified joint angular velocity, G_R is reflex gain, ω_n is 2nd order natural frequency and ξ is damping parameter. Table 1 shows the set of parameters for this simulation.

A Gaussian white noise was used as the position input. It was low-pass filtered by a 2nd order low pass Butterworth filter with the cutoff frequency as 30 Hz to simulate the actuator dynamics in the real experiment. The input to the reflex stiffness was the differentiating position. There was a 60 ms delay between the position signal and the velocity. The simulation lasted for 50 seconds. In addition, a Gaussian white noise was used to test the robustness of the algorithm. The SNR was 11.4 dB. Figure 2 shows the first two seconds of a simulated trial.

Table 1. Intrinsic and reflex parameter values of simulated ankle dynamics

h_{IS}^{-1} *Parameters*		
I (*Nm/rad/s*^{2})	B (*Nm/rad/s*)	K (*Nm/rad*)
0.015	0.8	150
H_{RS}^{-1} *Parameters*		
ω_n (*rad/s*)	ξ	G_R (*Nm/rad/s*)
40	1	2

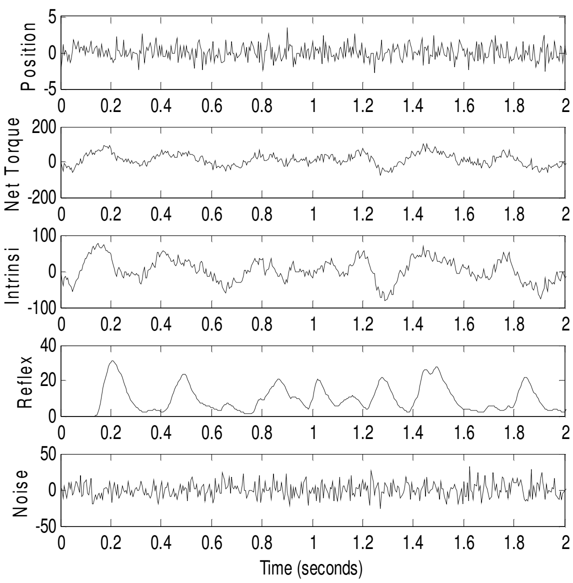

Fig. 2 Simulation data

IV. IDENTIFICATION AND VALIDATION

Torque from each stiffness was estimated as described above. To describe the nonlinearity in the reflex stiffness, it is necessary to select a basis function before the identification starts. A number of options are available including, Chebyshev polynomials, wavelets, sigmoids, radial basis functions etc. We select a 5th order Chebyshev polynomial [5] to avoid unconditional matrix caused by the high order components in regular polynomials. The constructed input matrix becomes:

$$U_k = \left[T_1(x) \quad T_2(x) \quad \cdots \quad T_{n_N}(x) \right]' \qquad (8)$$

where the Chebyshev polynomials are given by:

$$\begin{aligned} T_1(x) &= 1 \\ T_2(x) &= u_k \\ T_n(x) &= 2 \cdot u_k \cdot T_{n-1}(x) - T_{n-2}(x) \end{aligned} \qquad (9)$$

Then a state space model, $\hat{A}, \hat{B}, \hat{C}, \hat{D}$, can be estimated from the input $\begin{bmatrix} u_k \\ U_k \end{bmatrix}$ to the output Y using the SMI toolbox [6]. Simulating the response of estimated, $\hat{A}, \hat{B}, \hat{C}, \hat{D}$, to the input $\begin{bmatrix} u_k \\ 0 \end{bmatrix}$ gives an estimate of intrinsic torque, while the output from the same simulation with the input as $\begin{bmatrix} 0 \\ U_k \end{bmatrix}$ provides an estimate of reflex torque. VAF is used to measure the accuracy of the estimation. .

$$\%VAF = \left(1 - \frac{\mathrm{var}(y - y_e)}{\mathrm{var}(y)} \right) \times 100 \qquad (10)$$

Figure 3 shows the comparison between the simulated net torque and the estimated net torque. The estimated system is able to provide a fit with VAF = 99.98. The estimation of the torque from the intrinsic stiffness is as good as that of the net torque with VAF = 99.9. The accuracy for reflex stiffness is shown in Figure 5. The estimated torque from the reflex stiffness fits the simulated torque with VAF = 98.4.

V. DISCUSSION

The subspace method we have present successfully separates the intrinsic and reflex torque from the measured net torque. This is shown by the high VAFs between the simulated data and estimated data. However, the estimated discrete time model is not suitable to study the human ankle's dynamic behavior in continuous time. A direct conversion from the discrete time model to the continuous model will produce a biased result. Thus, decomposition of the torque from intrinsic stiffness and reflex stiffness can be viewed as the first step to study the dynamic behavior of the human ankle. With the estimates of the torque from each stiffness, continuous time parameters for intrinsic and reflex stiffness can be estimated using nonlinear parametric identification methods. By analyzing the parameters, a better understanding of how human ankle behaves during the movement will be obtained.

Subspace methods are good alternatives for biological system modeling with parallel cascade structures and nonlinear contributions. The decomposition of parallel cascade system provides the possibility to fit the continuous time parameters without converting from the discrete time parameters. Further analysis will implement more experimental data with different input signal properties, various sampling rates and SNR to testify the consistency and accuracy of the algorithm

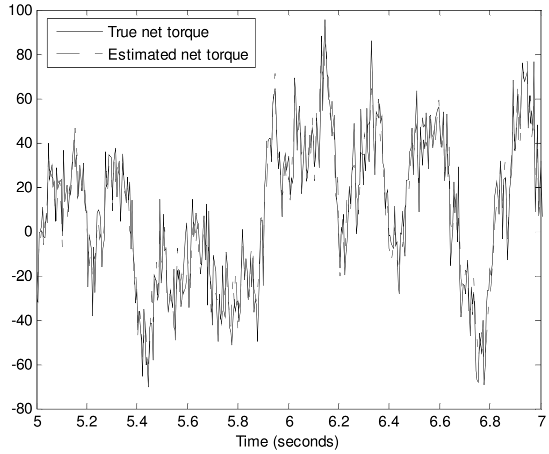

Fig. 3 Compare the Net Torque

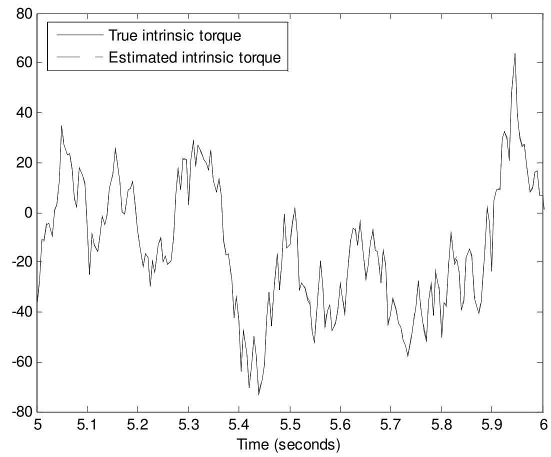

Fig. 4 Compare the Torque from intrinsic stiffness

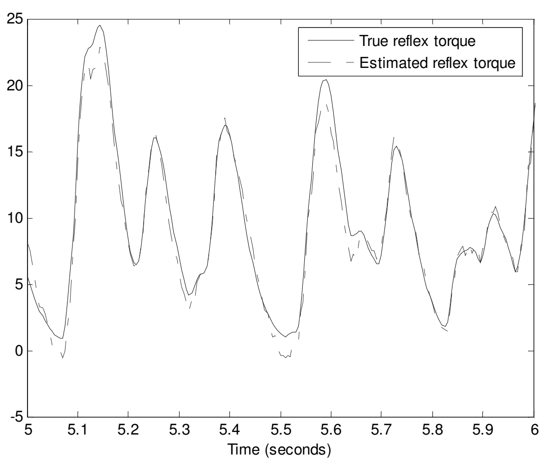

Fig. 5 Compare the Torque from reflex stiffness

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