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Estimation of elastic and viscous properties of the left ventricle based on annulus plane harmonic behavior

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Abstract— **Assessment of left ventricular (LV) function with an emphasis on contractility has been a challenge in cardiac mechanics during the recent decades. The LV function is usually described by the LV pressure-volume (P-V) diagram. The standard P-V diagrams are easy to interpret but difficult to obtain and require invasive instrumentation for measuring the corresponding volume and pressure data. In the present study, we introduce a technique that can estimate the viscoelastic properties of the LV based on harmonic behavior of the ventricular chamber and it can be applied non-invasively as well. The estimation technique is based on modeling the actual long axis displacement of the mitral annulus plane toward the cardiac base as a linear damped oscillator with time-varying coefficients. The time-varying parameters of the model were estimated by a standard Recursive Linear Least Squares (RLLS) technique. LV stiffness at end-systole and end diastole was in the range of 61.86-136.00 dyne/g.cm and 1.25-21.02 dyne/g.cm, respectively. The only input used in this model was the long axis displacement of the annulus plane, which can also be obtained non-invasively using tissue Doppler or MR imaging.**

I. Introduction

ASSESSMENT of left ventricular function with an emphasis on contractility has been a major challenge in cardiac mechanics during the recent decades. To date, extensive work has been done to develop models for describing left ventricular (LV) dynamics. However, an applicable model that can differentiate between different pathophysiological states based on mechanical properties of the heart is still needed. With regards to describing ventricular function, Suga and Sagawa [1] introduced a diagram for instantaneous ventricular pressure-volume (P-V) relationship. Based on their diagram, they described the ratio of instantaneous pressure to instantaneous volume ($P(t)/(V(t)-V_d)$) as the time varying stiffness for ventricular chamber. The standard P-V diagrams are easy to interpret and give a rough estimate of the mechanical work done by the LV. However, the pressure and the corresponding volume of the LV need to be measured invasively using sophisticated techniques, such as intravascular micromanometers and conductance catheters, which restrict the clinical applications of this model. Furthermore, this model ignores the viscoelasticity of the heart by not considering the viscous damping. It has also been shown that although dp/dv provides a useful portrait of simultaneous LV pressure and volume events, it does not represent actual LV physical properties.

In the present study, we introduce a noninvasive technique to assess the LV contractile behavior during the entire cardiac cycle. The estimation technique is based on modeling the long axis displacement of the mitral annulus plane toward the cardiac base as a linear damped oscillator with time-varying coefficients. We derive longitudinal rather than global indices for stiffness and damping of the LV. Elastic deformations resulting from the changes in the mechanical properties of myocardium are represented as a spring model with variable coefficients. The viscous components of the model include a time-varying viscous damper, representing relaxation and the frictional energy loss.

II. Method

A. Mathematical Model

Longitudinal displacement of the mitral annulus plane toward the apex during a cardiac cycle can be considered

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analogous to motion of a damped harmonic oscillator with time-varying coefficients. The time dependency of the coefficients is an advantage of this model, which allows the model to describe systole, diastole and the transitional isovolumic phases of the cardiac cycle. This description of model is also consistent with the fact that the LV acts as two distinct pumps in a cardiac cycle; acting as a suction pump during diastole and as a positive-displacement pump during systole in which the pressure of the chamber depends on the wall displacement and the blood volume. The abbreviations and the acronyms used in the paper are summarized in table 1.

Due to the natural time-dependency of the LV mass (blood and tissue), identification of LV mass, separated from the rest of the heart, as a function of time was impractical. Therefore, the equation of motion for a linear harmonic oscillator with time varying coefficients was divided by the instantaneous mass of the system and rewritten as:

$$\ddot{y} + h(t)\dot{y} + K(t)y = 0 \quad (1)$$

where (y) is the zero-mean displacement of the system and (x) is the longitudinal base to apex displacement:

$$y = (x - \bar{x}) \quad (2)$$

The bar indicates mean value, the dot denotes differentiation with respect to time and "*h*" and *"K"* are the damping and elastic (stiffness) coefficients per unit mass, respectively. Equation (1) can also be reorganized to a constant-coefficient harmonic oscillator with time varying forcing term, as follows:

$$\ddot{y} + \bar{h}\dot{y} + \bar{K}y = (\bar{h} - h(t))\dot{y} + (\bar{K} - K(t))y \quad (3)$$

In (3), "$\bar{h}$" and "$\bar{K}$" are the averaged values of damping (h) and stiffness (K) coefficients during a cardiac cycle, respectively.

TABLE I
ABBREVIATIONS AND ACRONYMS

x	longitudinal base to apex displacement
y	zero-mean displacement
$\bar{K}$	mean of stiffness coefficient
K_{ED}	end diastolic stiffness coefficient
K_{ES}	end systolic stiffness coefficient
$\bar{h}$	mean of damping coefficient
μ_K	mean of the mean of stiffness coefficient
μ_h	mean of the mean of damping coefficient
i	sample index
t	time

The function on the right-hand side of (3) is the intrinsic forcing function, which can represent the contractile elements of LV [2-4]. The intrinsic forcing function is described as:

$$f(t) = (\bar{h} - h(t))\dot{y} + (\bar{K} - K(t))y \quad (4)$$

Considering that we measured the longitudinal displacement "*x(t)*", the parameters in this model, as well as the forcing function, would be estimated using a standard identification technique [5-6] which will be described after the data acquisition procedure.

B. *Animal Preparation*

In order to assess the model behavior, an animal study with limited cases was performed. Animal data were collected at the Harrison department of surgical research, University of Pennsylvania School of Medicine. To measure the left ventricular axial displacement and intraventricular pressure, ten healthy sheep between 35 and 45 kg underwent left thoracotomy after induction of anesthesia. Sonomicrometry transducers (*1.0 mm; Sonometrics Corp., London, Ontario, Canada*) were implanted at the apex and base of the left ventricle to characterize ventricular motion.

C. *Equation of motion and parameter estimations*

The relative displacements obtained from sonomicrometry transducers were substituted in equation (1). The problem of tracking the parameters was tackled by re-sorting to a class of recursive linear least squares algorithms [5]. To estimate the model parameters, equation (1) was re-written as:

$$\ddot{y} = -h\dot{y} - Ky = \boldsymbol{\theta} \cdot \varphi^T \quad (5)$$

where $\boldsymbol{\theta} = \{h, K\}^T$, and $\varphi = \{-\dot{y}, -y\}^T$. The coefficients $\boldsymbol{\theta}$ were estimated by a standard Recursive Linear Least Squares (RLLS) technique [5] coupled with an estimator for continuous time derivatives described in [6].

III. RESULTS

The forced form of the oscillator (3) provides information about the global (averaged) state of elasticity and damping of the system, in addition to the instantaneous intrinsic longitudinal force generated over a cardiac cycle. Values of mean stiffness and damping coefficients for each case used in the forced form of the equation (3) are provided in table 2. To determine whether the coefficients were from the same distribution and possessed the same mean, we applied Student's t-test for each sample of coefficients.

TABLE II
MAGNITUDE OF COEFFICIENTS

	$\overline{K}$ (dyne/g.cm)		$\overline{h}$ (dyne.s/g.cm)		K_{ES} (dyne/g.cm)	K_{ED} (dyne/g.cm)
	magnitude	*P-value*	*magnitude*	*P-value*	*magnitude*	*magnitude*
Sheep1	49.11	0.16	0.05	0.90	85.81	2 .00
Sheep2	51.32	0.50	0.02	0.90	83.83	5.84
Sheep3	62.39	0.39	0.09	0.68	61.86	1.25
Sheep4	47.16	0.12	0.01	0.64	89.64	16.62
Sheep5	44.40	0.05	0.04	0.79	68.74	4.80
Sheep6	56.86	0.83	0.04	0.79	86.41	5.43
Sheep7	63.62	0.15	0.07	0.70	66.86	3.42
Sheep8	83.40	0.05	0.12	0.68	134.50	7.07
Sheep9	83.48	0.05	0.12	0.68	136.00	7.07
Sheep10	47.84	0.06	0.02	0.85	86.79	21.02

For each coefficient of stiffness and damping, we calculated the mean of the means of all 10 cases, "μ_K" and "μ_h", respectively. Then the null hypothesis was tested as each sample of coefficients had the same mean as "μ_K" and "μ_h" (table 2). The estimated "μ_K" and "μ_h" for 10 healthy cases were 58.63±12.8 dyne/g.cm and zero dyne.s/g.cm, respectively. The p-value for each sample is shown in table 2.

The forcing term of the equation (3) was also computed based on (4), using the estimated parameters, zero-mean longitudinal displacement (y) and zero-mean longitudinal velocity ($\dot{y}$). Evolution of the forcing term within a cardiac cycle is shown in figure 1.

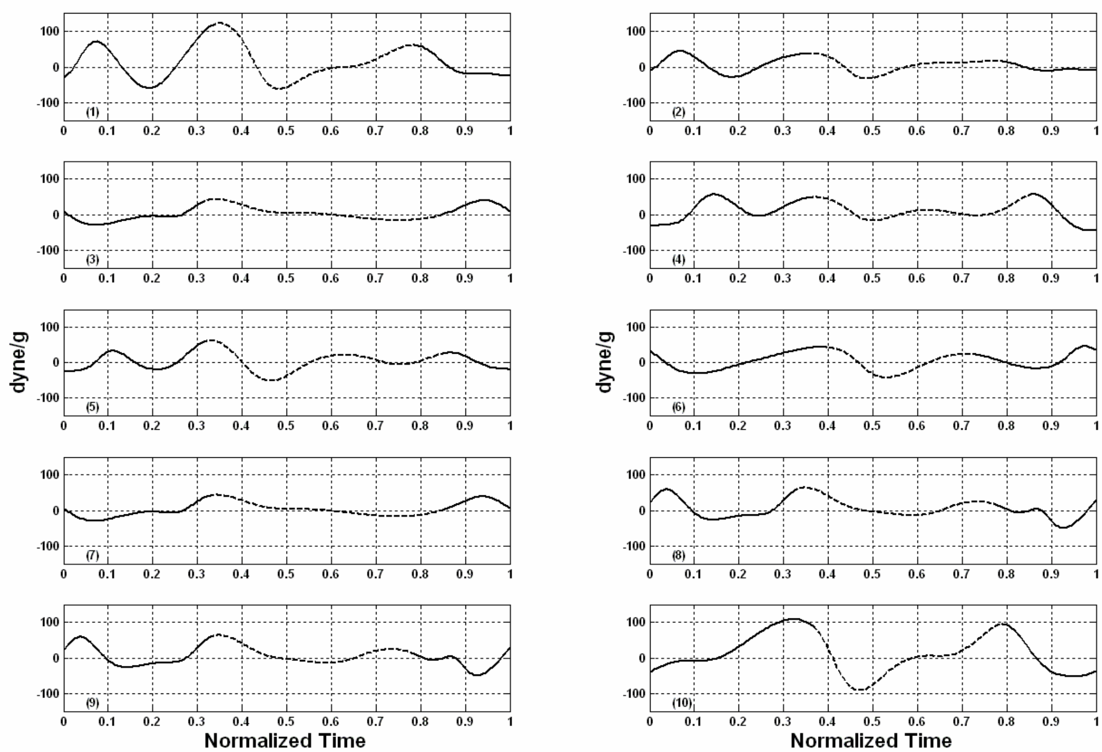

Fig. 1. Time evolution of forcing function for all the 10 cases.

I. Conclusion

We developed a technique that employs a dynamic model for longitudinal displacement of the annulus plane toward the apex. Using this technique enables us to estimate longitudinal elastic and damping coefficients for the left ventricle based only on the mitral annulus displacement. Although the estimated values of coefficients were considered one-dimensional parameters, their time-varying trends were consistent with the previous studies that considered *dP/dV* as an index of stiffness [1,8]. The only input used in this model is the real-time long axis displacement of the annulus plane, which can also be obtained non-invasively using tissue Doppler. The fact that the technique can be used as a non-invasive way to evaluate the LV function is the key advantage of this model with respect to other existing techniques. In all the cases, the stiffness of the model has a minimum at the end-diastole, increasing during systole. It reaches a maximal peak at the end-systole. This observation was consistent with the P-V diagram of Suga and Sagawa [1] and the other models that described LV stiffness based on P-V relationship [7-11].

Another interesting observation resulting from the forced form of the harmonic oscillator was the similarity between the magnitude of K and h in all the 10 cases ($p>0.05$). However, in only two cases (sheep 4 and sheep 10) the magnitude of K_{ED} was greater than the rest of the cases. This can be a result of inaccuracy in defining the onset of end-diastole due to ECG irregularities during surgery. Based on the present model, the mean damping coefficient can be considered zero in healthy hearts, denoting that the viscous damping is minimal in the normal LV [12]. Further study is in progress to observe the variations of coefficients in cases where the physical state of the LV has been changed due to the acquired or congenital heart diseases.

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