

# Can Human Isochrony be Explained by a Computational Theory?

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**Abstract**— Previous studies on human motor control reported a phenomenon called isochrony, which is the compensatory increase of movement speed with increasing movement distance. On the other hand, in complex via-points trajectory formation, a possible computational model that can estimate via-point time has been proposed. This model is optimized on the condition that the Duration average of the Commanded Torque Change (DCTC) between each via-point is equal. In this paper, we consider the possibility that human isochrony can be explained by the computational theory and investigate the human drawing movement of a set of figure eight and double elliptical patterns. Results show that isochrony was observed in both duration and DCTC and that relative variance with an increasing movement perimeter ratio of DCTC was significantly less than duration. The possibility is suggested that human isochrony is a phenomenon observed as a result of movement time planning to equalize DCTC.

## I. INTRODUCTION

Previous studies on human motor control reported a phenomenon called isochrony [1], which is the compensatory increase of movement speed with increasing movement distance. For example, this phenomenon is observed in such handwriting movement situations as when “e” and “ℓ” are continuously drawn. Isochrony is a general principle of movement control, not only for such arm movements as handwriting and drawing but also for head rotation and speech production [2] [3]. Human isochrony has been extensively researched in relation to the two-thirds power law [4] and stroke-based segmentation hypotheses. As part of such research, even though Viviani & Flash [6] attempted to explain isochrony by a minimum jerk hypothesis [5], which is a possible computational model for human point-to-point movement, isochrony has still never been explained computationally.

On the other hand, in complex via-points trajectory formation, Wada & Kawato [7] proposed a possible computational trajectory generation model that can estimate the time to pass through via-points. This model differs from traditional via-points movement generation models by suggesting that via-point time is not required as a constrained condition. This model is optimized on the condition that the Duration average of the Commanded Torque Change (DCTC) between each via-point is equal, suggesting that CNS plans via-point time according to the computational theory. We suppose that the above human isochrony can be explained by the computational theory. In this paper, we designed two tracing

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tasks, figure eight and double elliptical patterns, which were researched in previous isochrony studies. Measured movements were performed on seven subjects to investigate the possibility.

## II. A VIA-POINT TIME OPTIMIZATION MODEL

In trajectory formation passing through via-points, a via-point time optimization model [7] decides via-point time  $t_i$  to minimize the commanded torque change, as shown in the following equation:

$$C_\tau = \int_0^{t_f} \sum_{k=1}^K \left( \frac{d\tau^k}{dt} \right)^2 dt \\ = \sum_i C_i(t_i) = \sum_i \int_{t_{i-1}}^{t_i} \sum_{k=1}^K \left( \frac{d\tau_i^k}{dt} \right)^2 dt, \quad (1)$$

where  $t_f$  is the entire motion duration,  $i$  are the indices of via-points ( $i = 1, 2, \dots, n$ ),  $t_i$  is the time passing through via-point  $i$ ,  $\tau^k$  is the commanded torque of joint  $k$ , and  $K$  is the number of joints.

This model estimates via-point time as follows.

Step 1 A trajectory is generated by Forward Inverse Relaxation Model (FIRM) [8] according to a set of initial via-point times.

Step 2 The movement time between each via-point is updated to reduce the performance index of the minimum commanded torque change using the steepest descent method. Then the entire movement time is basically lengthened.

Step 3 To satisfy the given entire motion duration, the via-point time obtained in Step 2 is corrected.

In Step 2, the via-point time  $t_i$  must be extended by the steepest descent method. The movement time between each via-point is updated with the following equation:

$$\Delta t_i = \varepsilon \frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} \sum_{k=1}^K \left( \frac{d\tau_i^k}{dt} \right)^2 dt, \quad (2)$$

where  $\tau^*$  represents the commanded torque by which  $C_i$  is minimized and  $\varepsilon$  is an appropriate positive coefficient.

The via-point time is corrected according to the following equation:

$$t_i \leftarrow \frac{t_i + \Delta t_i}{t_f + \Delta t_f} t_f, \quad (3)$$

where  $\Delta t_f = \sum \Delta t_i$ . Thus, the given entire motion duration is satisfied.

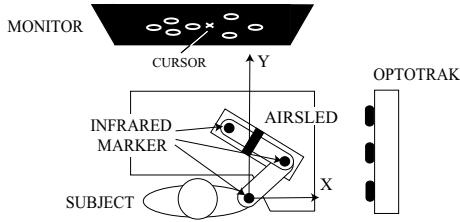


Fig. 1. Experimental setup

In the above algorithm, if (4), which is the DCTC between each via-point, is equal, cost function (1) converges, and each via-point time ( $t_1, t_2, \dots, t_n$ ) is optimized [7].

$$C_i^{via} = \frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} \sum_{k=1}^K \left( \frac{d\tau_i^k}{dt} \right)^2 dt \quad (4)$$

The above computational theory can be applied to such isochronous movements as a drawing motion in which large and small loops are continuously traced (two segment movement). Assuming that each loop corresponds to each via-point interval in the computational theory, the possibility is suggested that trajectory is generated to uphold the following equation:

$$C_L^{via} = C_S^{via}. \quad (5)$$

where  $C_L^{via}$  is DCTC in large loop and  $C_S^{via}$  is DCTC in small loop.

We investigate the possibility in the double joint version of (4) using human cyclic tracing movements.

### III. METHOD

#### A. Procedure and Tasks

In each experimental tasks, subjects were seven right-handed males, 21-23 years old. Experimental setup is shown in Fig. ???. Subjects sat in chairs adjusted to lift their arms to shoulder level, and their wrists were supported by a brace mounted on an air-sled to reduce the influence of friction. Shoulder, elbow, and hand positions were measured by OPTOTRAK3020, which detected infrared markers at a sampling frequency of 500 (Hz). Subjects were given

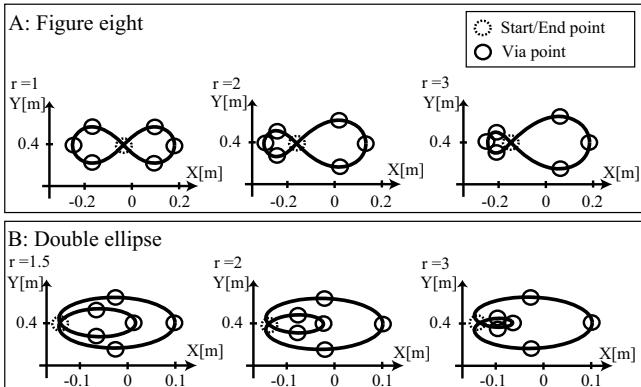


Fig. 2. Start/End and via points in experimental tasks

online feedback of hand position as indicated by cursors on monitors.

In this experiment, continuous cyclic tracing tasks of figure eight (Fig. 2 A) and a double elliptical pattern (Fig. 2 B) that consisted of three kinds of large and small loops were designed. When subjects set their arms to Start/End (S/E) position, they received a beep sound as a cue. After starting S/E point, subjects moved their arms clockwise from large to small loop. The five cycle movements (1 cycle: large  $\rightarrow$  small) were measured in each trial. In each pattern, subjects performed 10 trials after many practices. In practice sessions, subjects were given S/E points, via-points, and guide trajectory, and in real sessions, they were not given guide trajectory to reduce the influence of visual feedback.

#### B. Data Analysis

Position data were filtered by a third order Butter-worth filter with a cutoff frequency of 15 (Hz). The actual starting and ending positions of each movement were determined using tangential velocity with 5% of maximum threshold. The commanded torque ( $\tau_1, \tau_2$ ) was computed from the following inverse dynamics model:

$$\begin{aligned} \tau_1 &= (I_1 + I_2 + 2M_2L_1S_2 \cos(\theta_2) + M_2(L_1)^2)\ddot{\theta}_1 \\ &\quad + (I_2 + M_2L_1S_2 \cos(\theta_2))\ddot{\theta}_2 \\ &\quad - (M_2L_1S_2(2\dot{\theta}_1 + \dot{\theta}_2))\dot{\theta}_2 \sin(\theta_2) \\ &\quad + B_{11}\dot{\theta}_1 + B_{12}\dot{\theta}_2 \\ \tau_2 &= (I_2 + M_2I_1S_2 \cos(\theta_2))\ddot{\theta}_1 + I_2\ddot{\theta}_2 \\ &\quad + M_2L_1S_2(\dot{\theta}_1)^2 \sin(\theta_2) + B_{21}\dot{\theta}_1 + B_{22}\dot{\theta}_2. \end{aligned}$$

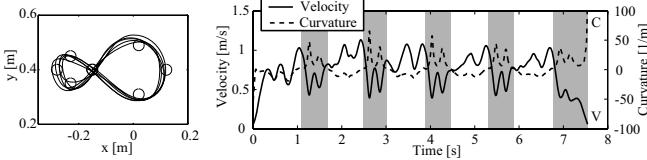
Here,  $\theta_i$ ,  $\dot{\theta}_i$ , and  $\ddot{\theta}_i$  represent the position, velocity, and acceleration of each joint, respectively.  $M_i, L_i, S_i, I_i, B_{ij}$  represent the mass, length, distance from the mass center to the joint, the rotary inertia of link  $i$  around the joint, and the coefficients of viscosity, respectively.  $B_{ij}$  shows that the joint angle velocity of link  $j$  influences the actuated torque of link  $i$ . Joints 1 and 2 correspond to the shoulder and elbow, respectively. The parameters of dynamics were estimated by the methods of Gomi & Osu [9] and Nakano et al. [10]. The ranges of the parameters of dynamics of all subjects are shown in Table I.

Duration ratios  $r_d = t_L/t_S$  and DCTC ratios  $r_C = C_L^{via}/C_S^{via}$  were statistically analyzed. Additionally, we analyzed isochrony coefficient  $I = \frac{r - r_*}{r - 1}$  [6]. Here,  $r$  is the

TABLE I  
RANGE OF PARAMETERS OF DYNAMICS FOR ALL SUBJECTS

Parameter	Link 1	Link 2
$L_i$ (m)	0.21-0.29	0.30-0.34
$M_i$ (kg)	1.03-1.43	0.98-1.09
$S_i$ (m)	0.078-0.108	0.148-0.165
$I_i$ (kg m <sup>2</sup> )	0.0095-0.0257	0.0319-0.0441
$B_i$ (kg m <sup>2</sup> /s)	0.69-0.88	0.80-0.88
$B_{ij}$ (kg m <sup>2</sup> /s)	0.18-0.20	0.18-0.20

A: Figure eight



B: Double ellipse

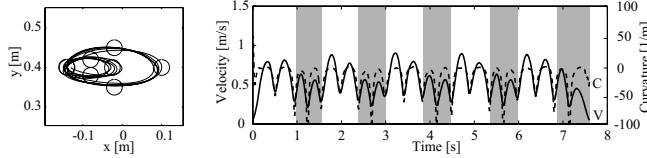


Fig. 3. Trajectory, velocity and curvature profile performed by subject TA (A) and YK (B) for  $r = 2$ . In velocity and curvature plot, white areas indicate the time in large segments, gray areas indicate the time in the small segments.

actual movement perimeter ratio, and  $r_*$  is  $r_d$  or  $r_C$ . The isochrony coefficient ranges between 0 ( $r_* \geq r$ : isochrony's complete absence) and 1 ( $r_* = 1$ : perfect isochrony). In the case of  $r_* < 1$ ,  $r_*$  is substituted  $1/r_*$  so that  $I$  does not exceed 1.

In each trial even though 5 cycle movements were measured, the stable 2-4 cycle movements, which are rejected first and final cycle movements, were accepted for analyze. Hence 3 cycles  $\times$  10 trials = 30 cycles per subject were analyzed.

#### IV. RESULTS

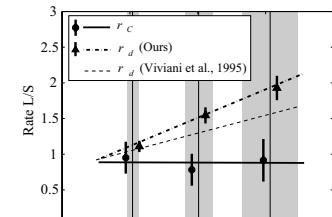
##### A. Typical Cases of Measured Features

Fig. 3 shows the trajectories, tangential velocity, and curvature profiles performed by subject TA (figure eight task) and subject YK (double ellipse task) as an example of experimental results. In each task, the velocity in large segments (white areas) tends to be qualitatively higher than in small segments (gray areas). This means that movement speed are increased to compensate for increasing movement perimeter; an isochronous tendency can be observed qualitatively.

##### B. Statistical Results

Fig. 4 shows the analysis results using all subject data of mean  $\pm$  standard deviation (S.D.) for duration ratio  $r_d = t_L/t_S$ , DCTC ratio  $r_C = C_L^{via}/C_S^{via}$ , and their isochrony coefficients ( $I_{r_d}$  and  $I_{r_C}$ ) in each perimeter ratio. In Figs. 4 A and B, the upper side shows the results for each ratio, and the lower side shows the results for each isochrony coefficient, which is indefinite when the perimeter ratio is  $r = 1$ . In the ratio plot, regression lines representing the variance of  $r_d$  or  $r_C$  with increasing perimeter ratio  $r$  are shown in Fig. 4. To compare duration ratio  $r_d$  measured in our experiments with previous experiments, the regression line calculated from data in Viviani et al. [6] is also shown. Even though the slopes of the regression lines of  $r_d$  measured in our experiments are slightly greater than those of Viviani et al., both regression lines show qualitatively identical levels

A: Figure eight



B: Double ellipse

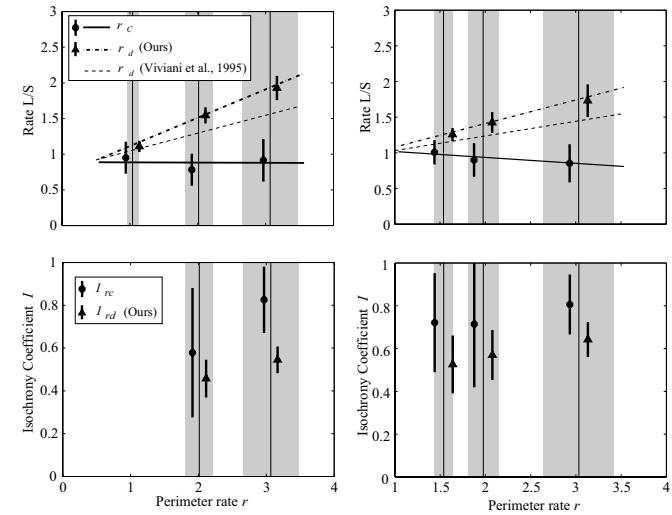


Fig. 4. Rates and Isochrony Coefficients for  $t_L/t_S$  and  $C_L^{via}/C_S^{via}$ . In each perimeter ratio, gray areas indicate S.D. for actual perimeter rate, the vertical lines indicate S.D. for rates or isochrony coefficients.

TABLE II  
RESULTS COMPARING THE SLOPE OF REGRESSION LINE WITH  
 $r_C = C_L^{via}/C_S^{via}$  AND  $r_d = t_L/t_S$

Subject	Figure eight task		
	Slope $r_C$	Slope $r_d$	F value
KK	-0.024	0.415	407.03***
TT	-0.021	0.407	441.08***
YK	-0.079	0.410	595.62***
MT	-0.133	0.488	503.87***
TA1	0.068	0.325	47.92***
YS	0.144	0.355	37.96***
SS	-0.042	0.422	154.30***
All	-0.003	0.398	1001.70***
Double ellipse task			
	Slope $r_C$	Slope $r_d$	F value
	-0.176	0.359	541.84***
TA2	0.091	0.240	11.57**
TM	-0.169	0.349	167.78***
TT	0.145	0.243	8.00**
KK	-0.044	0.265	94.42***
MT	-0.226	0.418	330.9***
AI	-0.187	0.379	366.63***
YK	-0.083	0.330	710.17***

\* $p < .05$ , \*\* $p < .01$ , \*\*\* $p < .0001$ ,

$df = 176$  (All:  $df = 1256$ )

of isochronous tendency. We performed statistical tests for the slope differences of the regression lines of  $r_d$  between the data from each subject in our experiments and in Viviani et al.; they had no significant differences except for subject TT in the double ellipse task ( $p < .05$ ).

In the ratio results in each task, the variation of DCTC ratio  $r_C$  with increasing perimeter ratio  $r$  tends to be less than the variation of duration ratio  $r_d$ . Also in results for isochrony coefficient,  $I_{r_C}$  tends to be higher than  $I_{r_d}$ . To quantitatively evaluate this tendency, we performed statistical tests on the slope differences of the regression lines between duration ratio  $r_d$  and DCTC ratio  $r_C$  in each subject (see

TABLE III  
RESULTS COMPARING MEAN OF ISOCHRONY COEFFICIENT WITH  $r_C$  AND  $r_d$

Subject	Figure eight task		
	$r = 1$ $I_{r_C}, I_{r_d}, t$ value	$r = 2$ $I_{r_C}, I_{r_d}, t$ value	$r = 3$ $I_{r_C}, I_{r_d}, t$ value
KK	—	0.74, 0.47, 7.16***	0.93, 0.56, 38.74***
TT	—	0.38, 0.41, -0.56	0.78, 0.52, 11.14***
YK	—	0.70, 0.45, 6.94***	0.86, 0.54, 22.97***
MT	—	0.19, 0.36, -3.63	0.62, 0.47, 3.46**
TA1	—	0.66, 0.54, 2.61*	0.89, 0.60, 18.50***
YS	—	0.76, 0.55, 6.09***	0.83, 0.61, 8.74***
SS	—	0.62, 0.42, 3.55**	0.86, 0.51, 16.47***
All	—	0.58, 0.46, 5.57***	0.83, 0.55, 24.35***
Double ellipse task			
$r = 1.5$			
Subject	$I_{r_C}, I_{r_d}, t$ value	$I_{r_C}, I_{r_d}, t$ value	$I_{r_C}, I_{r_d}, t$ value
	TA2	0.79, 0.55, 6.67***	0.81, 0.52, 8.65***
TM	0.62, 0.47, 2.73**	0.79, 0.66, 3.07**	0.89, 0.71, 8.89***
TT	0.66, 0.60, 1.01	0.72, 0.58, 2.62*	0.74, 0.63, 3.57**
KK	0.71, 0.50, 5.17***	0.82, 0.61, 6.92***	0.86, 0.70, 8.29***
MT	0.73, 0.46, 6.18***	0.78, 0.55, 4.67***	0.90, 0.66, 18.10***
AI	0.72, 0.55, 3.33**	0.19, 0.42, -6.31	0.62, 0.56, 2.72**
YK	0.81, 0.56, 7.11***	0.89, 0.65, 11.58***	0.86, 0.63, 10.60***
All	0.72, 0.53, 10.55***	0.71, 0.57, 6.60***	0.81, 0.64, 14.65***

\* $p < .05$ , \*\* $p < .01$ , \*\*\* $p < .0001$ , df = 58 (All: df = 418)

Table II). Additionally, we statistically tested the difference of the mean value of isochrony coefficient with  $I_{r_d}$  and  $I_{r_C}$  in each subject (see Table III). In Table II, the differences between each slope of the regression line are significant for all subjects; in Table III,  $I_{r_C}$  is significantly higher than  $I_{r_d}$  for most subjects. Thus, these mean the tendency that the varying with an increasing movement perimeter ratio of the DCTC ratio is significantly less than the duration ratio.

The results of the analysis of duration and DCTC ratios suggest a tendency that human isochrony is a phenomenon where DCTC equals rather than duration. That is, our experiments suggest that isochrony is observed as a secondary effect caused by movement time planning in CNS to equalize DCTC.

## V. CONCLUSION

In this paper, we measured two kinds of cyclic tracing movements performed by seven subjects to investigate the possibility that computational theory, which argues that trajectory is generated to equalize the Duration average of the Commanded Torque Change (DCTC) between via-points, can explain human isochrony. Results show a tendency where isochrony is the phenomenon that DCTC equals rather than duration, and the possibility is suggested that isochrony is observed as a secondary effect caused by movement time planning to equalize DCTC.

The double joint commanded torque evaluated in this report was approximately estimated by dynamic equations. In future studies, the above possibility must be investigated at higher levels such as the electromyogram (EMG) level. However, such research has to wait for appropriate models that can estimate dynamic torque from EMG with high accuracy. Additionally, important future work includes an investigation whether the hardware model implemented by the computational theory generates the two-thirds power law

and compensatory increase of movement speed. In future studies, we are going to approach not only local movement time planning in each movement segmentation but also global movement time planning in the entire movement.

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