Proceedings of the 28th IEEE
EMBS Annual International Conference
New York City, USA, Aug 30-Sept 3, 2006

Can a kinetic optimization criterion predict both arm trajectory and final arm posture?

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Abstract— **The following two characteristics have been well demonstrated concerning the features of a point-to-point human arm movement on a plane. (1) The path is a roughly straight line, but is slightly curved. (2) The velocity profile is bell shaped with a single peak. Several models have been proposed to explain these features. Four criteria for trajectory planning based on optimal principles have been proposed. They are the minimum hand jerk criterion, the minimum angle jerk criterion, the minimum torque change criterion, and the minimum commanded torque change criterion. Trajectories generated by the minimum commanded-torque change model have been shown to correspond well with measured trajectories in a horizontal and sagittal work space. However, previous works had been restricted to trajectories on a two-dimensional plane. Trajectories and final arm postures had not been examined in three-dimensional space. In this paper, we quantitatively discuss predictions based on these criteria for human arm trajectories and arm postures in three-dimensional space. Finally, we report that both measured hand trajectories and arm postures were closest to trajectories and postures predicted by the minimum commanded torque change criterion.**

I. Introduction

The following two characteristics have been found as features of a point-to-point human arm movement on a two-dimensional plane such as a horizontal plane or a sagittal plane [1]. (1) The path is a roughly straight line, but is slightly curved. (2) The velocity profile is bell shaped with a single peak. Several models based on optimization principles have been proposed to explain these characteristics. They include the minimum hand jerk criterion [2], the minimum angle jerk criterion [8], the minimum torque change criterion [9], and the minimum commanded torque change criterion [6]. In addition, the plausibility of these criteria as computational trajectory planning principles of the human brain have been discussed.

Nakano et al. [6] and Wada et al. [10] quantitatively examined how these criteria can predict measured human trajectories accurately, and then reported that minimum commanded torque change trajectories are the trajectories on two-dimensional planes closest to human trajectories among all of the trajectories predicted by the above criteria. However, these works were restricted to movements on two-dimensional planes. They had not examined trajectories and arm postures in three-dimensional space.

In this paper, we extend the equations of arm motion from two-joint, two-degree-of-freedom, and two-dimensional plane movements, to two-joint, four-degree-of-freedom, and three-dimensional space movements, as shown in Fig. 1. Next, we measure many point-to-point movements in three-dimensional space and compare the results with minimum hand jerk trajectories, minimum angle jerk trajectories, minimum torque change trajectories, and minimum commanded torque change trajectories. Then, we discuss our findings quantitatively and statistically. Moreover, we perform quantitative examinations between measured arm postures and arm postures predicted by the optimization criteria. Finally, we report that minimum commanded torque change trajectories are closest to trajectories measured in three-dimensional space as well as on a two-dimensional plane and the minimum commanded torque change criterion can explain measured arm postures well. We point out the possibility that both hand trajectories and arm postures can be predicted by the optimization criterion.

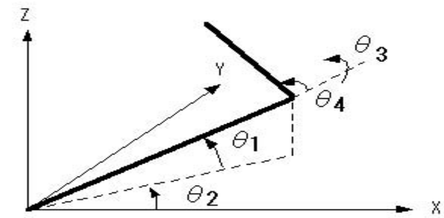

Fig. 1. Three-dimensional workspace

II. Equations of motion in three-dimensional space

The work space is determined by two joints and four degrees of freedom, that is, the shoulder joint takes three degrees of freedom and the elbow joint takes one degree of freedom. The following equations are the equations of motion, which depend on the dynamics parameters such as the arm length, mass, distance from the position of the joint to the position of the center of gravity, and inertia moment surroundings of links. Here, τ_p and θ_p are the torque of joint p $(p = 1, 2, 3, 4)$ and the joint angle, respectively. The first term, second term, third term, and fourth term show the inertial force, centrifugal force/Coriolis' force, viscous force, and gravitational force, respectively. B_{pj} denotes the viscosity coefficient expressing the influence of the angular velocity of joint j $(j = 1, 2, 3, 4)$ on the torque of joint p.

$$\tau_p = \sum_{j=1}^{4}(M_{pj}\ddot{\theta}_j) + \sum_{j=1}^{4}\sum_{k=1}^{4}(H_{pjk}\dot{\theta}_j\dot{\theta}_k) + \sum_{j=1}^{4}(B_{pj}\dot{\theta}_j) + G_p$$

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TABLE I

APPROXIMATE LOCATIONS OF TARGETS RELATIVE TO THE SHOULDER. IN FRONT OF THE BODY, THE MINUS DIRECTION OF THE AZIMUTH IS THE LEFT DIRECTION FROM THE RIGHT SHOULDER POSITION. THE ELEVATION IS MINUS TOWARDS THE LOWER DIRECTION FROM THE RIGHT SHOULDER POSITION.

Target	Distance $[m]$	Azimuth $[°]$	Elevation $[°]$
1	0.32	-46	+2
2	0.28	- 8	+2
3	0.47	-24	+22
4	0.34	+15	+26
5	0.31	+27	-37

III. MEASUREMENT EXPERIMENT OF THREE-DIMENSIONAL MOVEMENTS

In the measurement experiment, the subjects performed point-to-point movements according to indicated starting and final points. They were not informed about the purpose of the experiment. The instructions for the starting point and the final point in three-dimensional space were markers suspended from the ceiling (Fig. 2). Each subject sat in a chair and his body, shoulder, and wrist were fixed. No arm posture instruction was given. The subject moved his hand to the position of the starting point naturally. Then, the subject moved his hand towards the final point. The position of the shoulder, the elbow, and the hand were measured by a three-dimensional position measurement device (OPTOTRAK3020). The sampling frequency was 200Hz. In the experiment, the subjects performed point-to-point movements according to the indicated movement time levels, which were instructed by "Fast", "Normal", and "Slow".

Experiment

The subjects were three right-handed men in their twenties. The target positions were selected by referring to the experiments in Nishikawa et al. [7]. The five points shown in Table I were determined. We did not indicate the arm posture at the starting position. Then, the subjects started the movements after assuming their own natural postures. Each subject performed thirty movements for the following ten combinations of starting and final points for three speed levels (fast, normal, and slow).

$$1 \longrightarrow 2,\ 1 \longrightarrow 3,\ 1 \longrightarrow 4,\ 1 \longrightarrow 5,\ 2 \longrightarrow 3,$$
$$2 \longrightarrow 4,\ 2 \longrightarrow 5,\ 3 \longrightarrow 4,\ 3 \longrightarrow 5,\ 4 \longrightarrow 5$$

We gave the instruction for the fast speed as faster than the normal speed and for the slow speed as slower than the normal speed.

The average motion durations of the ten measured trajectories for each subject in the order of fast, normal, and slow were as follows: $0.47 \pm 0.13[sec]$, $0.56 \pm 0.16[sec]$, $0.73 \pm 0.20[sec]$ for Subject 1, $0.56 \pm 0.08[sec]$, $0.80 \pm 0.09[sec]$, $1.05 \pm 0.19[sec]$ for Subject 2, and $0.74 \pm 0.18[sec]$, $0.97 \pm 0.24[sec]$, $1.17 \pm 0.28[sec]$ for Subject 3.

In all of the tests, the position data was filtered by a third-order Butterworth filter with an upper cutoff frequency of 15Hz. Because it was difficult for each subject to make his hand stationary at the starting point and at the final point and because the velocity profile oscillated, we determined the actual beginning and end positions of each movement using a velocity profile with a 5% threshold of the peak velocity. That is, the starting point was determined by searching for a point below 5% of the peak velocity from the peak velocity time towards the starting point, and the final point was determined by searching for a point below 5% of the peak velocity from the peak velocity time towards the final point.

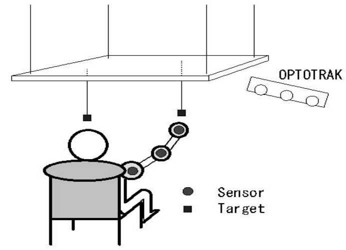

Fig. 2. Measurement setup for three-dimensional movements

IV. TRAJECTORY AND ARM POSTURE PLANNING BASED ON AN OPTIMIZATION PRINCIPLE

In the following analysis, we investigated differences between measured human movements and movements predicted by the planning criteria when varying the speed of motion. Finally, we considered whether the arm posture at the final position can be predicted by the optimization criterion for trajectory planning.

A. Quantitative examination of trajectory prediction

In this section, we look at quantitative examinations from the data of experiment.

1) Dynamics parameters : The dynamics parameter values for the three subjects were as follows: L_1: $0.29 - 0.32[m]$, L_2: $0.28 - 0.31[m]$, M_1: $1.50 - 1.60[kg]$, M_2: $0.92 - 1.03[kg]$, S_1: $0.11 - 0.12[m]$, S_2: $0.14 - 0.15[m]$, I_1: $0.0098 - 0.0120[kg \cdot m^2]$, I_2: $0.0088 - 0.0124[kg \cdot m^2]$, $I_{1\theta_3}$: $0.0020 - 0.0024[kg \cdot m^2]$. I_i, M_i, L_i, and S_i show the inertia moment surroundings of link i $(i = 1, 2)$, mass, arm length, and the distance from the position of the joint to the position of the center of gravity. $I_{1\theta_3}$ expresses the inertia moment of the inner rotation and lateral rotation for the upper arm. Viscosity B_{ij} is basically calculated by the same method as Nakano et al. [6]. The viscosity values for the subjects were as follows: diagonal element B_{11}: $0.99-1.05[N \cdot m \cdot sec/rad]$, B_{22}: $0.64-0.66[N \cdot m \cdot sec/rad]$, B_{33}: $0.66 - 0.69[N \cdot m \cdot sec/rad]$, B_{44}: $0.80 - 0.88[N \cdot m \cdot sec/rad]$. The non-diagonal elements were $B_{14}(= B_{41})$, $B_{24}(= B_{42})$,$B_{34}(= B_{43})$: $0.18 - 0.19[N \cdot m \cdot sec/rad]$, $B_{12}(= B_{21})$, $B_{13}(= B_{31})$,$B_{23}(= B_{32})$: $0[N \cdot m \cdot sec/rad]$.

2) Quantitative examination : To compare the measured trajectories with the trajectories predicted by the optimization criteria quantitatively, the square error between the predicted trajectory and the measured trajectory was calculated for (A) position, (B) joint angle, (C) velocity, and (D) joint torque.

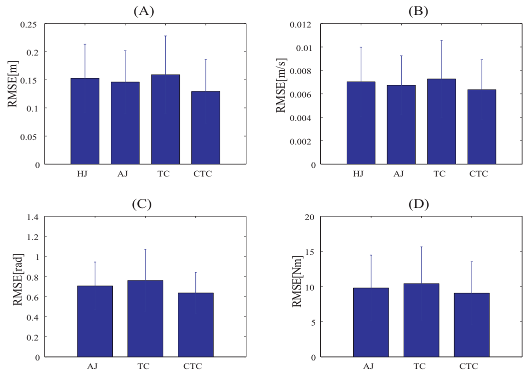

Fig. 3. Mean squared errors of measured trajectories and simulated trajectories (subject 201) (normal speed). HJ: minimum hand jerk criterion, AJ: minimum angle jerk criterion, TC: minimum torque change criterion, CTC: minimum commanded torque change criterion

Figure 3 shows the average of the square error of (A) the position, (B) joint angle, (C) velocity, and (D) joint torque for 10 normal speed trajectories of subject 201.

Next, a t-test was performed for the errors of (A) - (D) for the minimum hand jerk trajectory, minimum angle jerk trajectory, minimum torque change trajectory, and minimum commanded torque change trajectory containing the gravity term at every speed (Table II). Table II shows t values with respect to errors between the minimum commanded torque change trajectory and the other criteria as well as a significant smaller criterion. The table shows results of the t test for 30 trajectories for each speed of all subjects and for 90 trajectories involving all of the subjects and speed.

For each speed level, there were cases without a statistically significant difference between the square error of the minimum commanded torque change trajectory and the minimum hand jerk trajectory, minimum torque change trajectory, and minimum angle jerk trajectory. However, there was no case in which the error for the minimum commanded torque change trajectory was significantly larger. Moreover, for all of the subjects and all of the speed levels, the error for the minimum commanded torque change trajectory was significantly smaller.

B. *Examination of human arm posture prediction*

In this section, we discuss the possibility of arm posture prediction based on optimization criteria using the data of experiment. The arm dynamics parameters for the prediction calculation are the same as those in the above section. The criteria we consider are the minimum commanded torque change trajectory, the minimum torque change trajectory, and the minimum angle jerk trajectory. In the calculations, the movement time, the hand positions at the starting point and final point, and the elbow position at the starting point were used as the common boundary conditions. By varying the elbow position at the final point, the optimal trajectory was calculated. That is, we looked at the variation in the performance index of each criterion with respect to the elbow position at the final point. Each elbow position at the final point was determined by rotating the measured elbow position as in Fig. 4. The rotation angle of the measured elbow position is denoted as 0 degree. The rotation angle range is $-60^\circ \sim +60^\circ$, and nine rotation angles (-60°, -40°, -20°, -10°, 0°, $+10^\circ$, $+20^\circ$, $+40^\circ$, $+60^\circ$) are selected to generate the minimum angle jerk trajectory, the minimum torque change trajectory, and the minimum commanded torque change trajectory. Then, we estimated the elbow position at the final point that took the minimum performance index value. The differences between the measured elbow positions and the elbow positions minimized the individual performance indices were compared quantitatively.

TABLE II

RESULTS OF A t-TEST FOR THE MEAN SQUARED ERRORS OF MEASURED TRAJECTORIES AND SIMULATED TRAJECTORIES. (FAST, NORMAL, SLOW SPEEDS) t: T VALUE, $*$: SIGNIFICANCE < 0.05, $**$: SIGNIFICANCE < 0.01. A MODEL WITH SIGNIFICANTLY LESS MEAN SQUARED ERRORS IS SHOWN ON THE RIGHT OF THE LEVEL SIGNIFICANCE.

(a) Fast (three subjects)

		t-value	Degree of freedom	
HJ - CTC	(A)	2.08	29	
	(B)	3.03*	29	CTC
AJ - CTC	(A)	2.61*	29	CTC
	(B)	3.51**	29	CTC
	(C)	3.24**	29	CTC
	(D)	2.76*	29	CTC
TC - CTC	(A)	3.49**	29	CTC
	(B)	3.90**	29	CTC
	(C)	4.02**	29	CTC
	(D)	3.25**	29	CTC

(b) Normal (three subjects)

		t-value	Degree of freedom	
HJ - CTC	(A)	3.42**	29	CTC
	(B)	2.50*	29	CTC
AJ - CTC	(A)	3.40**	29	CTC
	(B)	2.72*	29	CTC
	(C)	2.76*	29	CTC
	(D)	2.42*	29	CTC
TC - CTC	(A)	3.61**	29	CTC
	(B)	3.19**	29	CTC
	(C)	4.18**	29	CTC
	(D)	3.38**	29	CTC

(c) Slow (three subjects)

		t-value	Degree of freedom	
HJ - CTC	(A)	3.09**	29	CTC
	(B)	1.86	29	
AJ - CTC	(A)	2.92*	29	CTC
	(B)	2.67*	29	CTC
	(C)	4.15**	29	CTC
	(D)	1.91	29	
TC - CTC	(A)	3.45**	29	CTC
	(B)	3.17**	29	CTC
	(C)	4.87**	29	CTC
	(D)	2.98*	29	CTC

(d) All

		t-value	Degree of freedom	
HJ - CTC	(A)	4.85**	89	CTC
	(B)	4.32**	89	CTC
AJ - CTC	(A)	4.93**	89	CTC
	(B)	5.15**	89	CTC
	(C)	5.86**	89	CTC
	(D)	4.15**	89	CTC
TC - CTC	(A)	5.91**	89	CTC
	(B)	5.78**	89	CTC
	(C)	7.51**	89	CTC
	(D)	5.61**	89	CTC

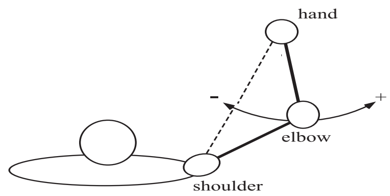

Fig. 4. Rotation of elbow at final location

TABLE III
AVERAGE AND STANDARD DEVIATION OF ANGLE OF ELBOW TO WHICH THE EVALUATION VALUE IS MINIMIZED

	Fast	Normal	Slow
AJ	26.46±38.64[deg]	23.06±40.37[deg]	24.81±35.61[deg]
TC	2.59±10.41[deg]	2.18±10.18[deg]	1.93±10.30[deg]
CTC	1.58±8.44[deg]	1.85±8.78[deg]	2.60±8.34[deg]

AJ: minimum angle jerk criterion, TC: minimum torque change criterion, CTC: minimum commanded torque change criterion

The mean and standard deviation of the rotation angle that minimizes the value of the performance index are shown in Table III for the three subjects at each movement speed. Results of a t-test are shown in Table IV. By comparing the minimum commanded torque change trajectory with the minimum angle jerk trajectory, we can see that the rotation angle that minimizes the minimum commanded torque change criterion is significantly closer to the rotation angle of 0 degree than the minimum angle jerk criterion for all of the movement speeds. However, there is no significant difference between the minimum commanded torque change trajectory and the minimum torque change trajectory. In the point-to-point movement, the arm posture at the final point predicted by the minimum commanded torque change criterion and the minimum torque change criterion corresponds better to the measured arm posture than the prediction by the minimum angle jerk criterion. Considering this together with the results of the previous section, it is conceivable that the minimum commanded torque change criterion is the most adequate criterion for trajectory and posture planning among all of the criteria in this paper based on optimization principles.

V. DISCUSSION

In this paper, we examined the possibility of trajectory and final arm posture planning based on optimization criteria in three-dimensional space using equations of two-link manipulators with four degrees of freedom. In addition, we looked at the influence of the motion duration. In regard to the trajectory planning, we generated point-to-point trajectories in three-dimensional space based on the minimum hand jerk criterion, minimum angle jerk criterion, minimum torque change criterion, and minimum commanded torque change criterion, and confirmed that measured trajectories were closest to the minimum commanded torque change trajectory. In point-to-point movements, the arm posture at the final point predicted by the minimum commanded torque change criterion and the minimum torque change criterion was found to be closer to the measured arm posture than the prediction by the minimum angle jerk criterion. Finally, we suggested that the minimum commanded torque change criterion is the most adequate criterion for trajectory and posture planning among all of the criteria in this paper based on optimization principles.

As future problems, we need to further consider the viscosity of the inner rotation and lateral rotation for the shoulder joint, which is not estimated in Gomi&Osu [3]. Several posture control systems [4][5] have been proposed. Computational approaches, e.g., a trajectory formation model (Forward-Inverse Relaxation Model) based on the minimum commanded torque change model, have been proposed by Wada and Kawato [11], but no model has been proposed that can calculate both the hand trajectory and arm posture based on the minimum commanded torque change criterion. It is an important issue to study a one-shot algorithm that can calculate both the hand trajectory and arm posture.

TABLE IV
RESULTS OF A t-TEST FOR ANGLE OF ELBOW TO WHICH THE EVALUATION VALUE IS MINIMIZED. t: T VALUE, $*$: SIGNIFICANCE < 0.05, $**$: SIGNIFICANCE < 0.01.

	AJ-CTC			TC-CTC		
	t-value	df		t-value	df	
Fast	3.51**	29	CTC	0.83	29	—
Normal	3.01*	29	CTC	0.44	29	—
Slow	3.39**	29	CTC	0.70	29	—
all (90trajectories)	5.77**	89	CTC	0.39	89	—

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