

# Real-Time Estimation of Intrinsic and Reflex Stiffness

Daniel Ludvig, *Student Member, IEEE*, and Robert E. Kearney, *Fellow, IEEE*

**Abstract**—Joint stiffness is defined as the dynamic relationship between the position of the joint and torque acting about it. Joint stiffness is composed of two components: intrinsic and reflex stiffness. Measuring the two stiffness components cannot be done simply because the two components appear and change together. A number of approaches have been used to estimate the components, but all those approaches are inherently off-line. We have developed a novel algorithm that separates and estimates the two components in real-time. Intrinsic stiffness was estimated by finding the cross-correlations between the position, its derivatives and the torque. Reflex stiffness was estimated by finding the IRF between the half-wave rectified velocity and the estimated reflex torque. A novel position perturbation, consisting of pseudo random series of pulses of different lengths, was used to eliminate covariance of intrinsic and reflex stiffness estimates. Using simulated data, the real-time estimates were shown to be estimated accurately. The real-time estimation algorithm was validated by comparing the real-time estimates with estimates generated by the parallel-cascade identification, an established off-line intrinsic and reflex stiffness identification algorithm, using simulated and experimental data. The estimates produced by the two algorithms were in agreement for both simulated and experimental data.

## I. INTRODUCTION

Joint stiffness defines the dynamic relationship between the position of the joint and the torque acting about it [1]; hence it plays a role in the control of posture in the face of perturbations. It is also vital in control of movement, as it is the torque produced by the muscles that controls the final joint position. Joint stiffness is composed of two components: intrinsic stiffness, which is generated by the viscoelastic properties of the joint, muscles and connective tissue, and reflex stiffness, which is generated by active muscle contraction in response to the stretch of a muscle.

Despite the fact that much is known about the two stiffness pathways, measuring each component separately is challenging because intrinsic and reflex stiffness appear and change together. A number of groups have developed analytical tools to separate intrinsic and reflex stiffness. Kearney et al. [2] developed a parallel-cascade identification algorithm which uses the reflex delay to separate intrinsic

and reflex components. Zhang and Rymer [3] used mathematical modeling to estimate intrinsic and reflex components of joint stiffness. However both these approaches are inherently off-line, and cannot be used to estimate intrinsic and reflex stiffness in real-time.

The main reason for estimating intrinsic and reflex stiffness in real-time is so that it may be used by subjects as feedback. This type of feedback could be given to a subject in order to test whether intrinsic and reflex stiffness can be varied independently. This feedback could also be useful in training spastic patients to reduce their exaggerated stretch reflex.

Therefore, an algorithm is needed to separate and estimate reflex and intrinsic stiffness in a computationally efficient manner so that the calculation can be performed in real-time. Any such algorithm must:

1. Estimate reflex and intrinsic stiffness independently.
2. Rely on short segments of data to provide the subjects with feedback of current stiffness estimates.
3. Be computationally efficient, so that the calculation can be performed in real-time.

We have achieved this using the zero lag cross-correlation between position, its derivatives, and torque to estimate the intrinsic stiffness. The impulse response function (IRF) between a half-wave rectified velocity signal and torque is used to estimate the reflex stiffness.

## II. REAL-TIME ESTIMATION ALGORITHM

The real-time estimation algorithm (Figure 1) can be separated into three components: separation of reflex and intrinsic torque, estimation of intrinsic stiffness and estimation of reflex stiffness.

### A. Separation of Intrinsic and Reflex Stiffness

We wish to estimate intrinsic stiffness by calculating the cross-correlation between position and torque. However, this estimate would be biased if the position and the reflex torque are correlated. Thus we designed a perturbation sequence which was uncorrelated to the reflex torque. The perturbation sequence consisted of a random mix of 500 ms “pulse” and “step” segments (Figure 2A). “Pulses” consisted of a 40 ms pulse, while “steps” were 460 ms pulses. The average of the position-reflex torque product for this input perturbation is zero as shown by the dotted line in Figure 2C. Furthermore, since velocity and acceleration are non-zero only during stretch, and 40 and 460 ms following the stretch, and reflex torque is non-zero between 40-400 ms following the stretch there is no correlation between them.

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D. Ludvig is with the Department of Biomedical Engineering, McGill University, Montreal, QC H3A 2B4 Canada (phone: 514-398-6736, fax: 514-398-7461, email: daniel.ludvig@mail.mcgill.ca)

R. E. Kearney is with the Department of Biomedical Engineering, McGill University, Montreal, QC H3A 2B4 Canada (email: robert.kearney@mcgill.ca)

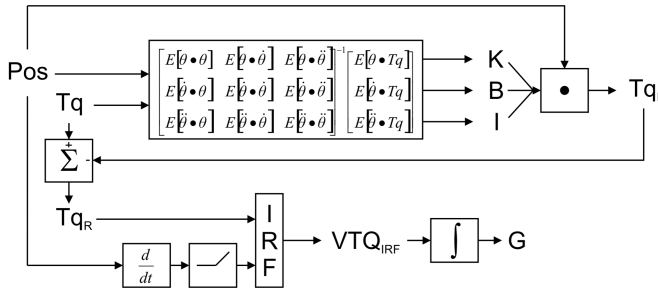


Figure 1. Schematic diagram of real-time estimation algorithm. Position, velocity, acceleration (denoted by the vector POS) and torque (TQ) are used to estimate elastic (K), viscous (B) and (I) inertial stiffness. The dot product between POS and K, B, and I produces an estimate for the intrinsic torque (TQ<sub>i</sub>). Reflex torque (TQ<sub>R</sub>) is estimated by subtracting TQ<sub>i</sub> from the TQ. Reflex stiffness (VTQ<sub>IRF</sub>) is then estimated by finding the impulse response function (IRF) between TQ<sub>R</sub> and the half-wave rectified velocity. Finally, the reflex gain (G) is calculated by integrating VTQ<sub>IRF</sub>. This entire calculation is carried out at every sample time.

### B. Estimation of Intrinsic Stiffness

Intrinsic stiffness consists of three components: elastic, viscous and inertial. Since all three components are unknown, determining the cross-correlation between position and torque cannot by itself estimate the three components. To determine all three components, the cross-correlations between velocity and torque and acceleration and torque were calculated. Combining these three cross-correlations, we can determine the intrinsic components by solving the following equation

$$\begin{bmatrix} \hat{K} \\ \hat{B} \\ \hat{I} \end{bmatrix} = \begin{bmatrix} E[\theta \cdot \theta] & E[\theta \cdot \dot{\theta}] & E[\theta \cdot \ddot{\theta}] \\ E[\dot{\theta} \cdot \theta] & E[\dot{\theta} \cdot \dot{\theta}] & E[\dot{\theta} \cdot \ddot{\theta}] \\ E[\ddot{\theta} \cdot \theta] & E[\ddot{\theta} \cdot \dot{\theta}] & E[\ddot{\theta} \cdot \ddot{\theta}] \end{bmatrix}^{-1} \begin{bmatrix} E[\theta \cdot Tq] \\ E[\dot{\theta} \cdot Tq] \\ E[\ddot{\theta} \cdot Tq] \end{bmatrix} \quad (1)$$

The averaging in each matrix entry is performed using a 2<sup>nd</sup> order Bessel low-pass filter. Different cut-off frequencies were tested to see which produced the best estimates.

### C. Estimation of Reflex Stiffness

Reflex stiffness has been modeled as a uni-directional rate sensitive element followed by a low-pass system and a delay. While the delay varies from subject to subject, it is about 40 ms [2, 4]. Therefore, to estimate reflex stiffness we can calculate the impulse response function (IRF) between half-wave rectified velocity and reflex torque. Reflex torque is estimated by subtracting the intrinsic stiffness estimated in the previous step from the total torque. The impulse response function is calculated at every sample time using least-squares linear regression, using lags of 10 ms starting at 50 ms and ending at 400 ms for 1 second worth of data. The reflex gain is then calculated by integrating the reflex stiffness IRF.

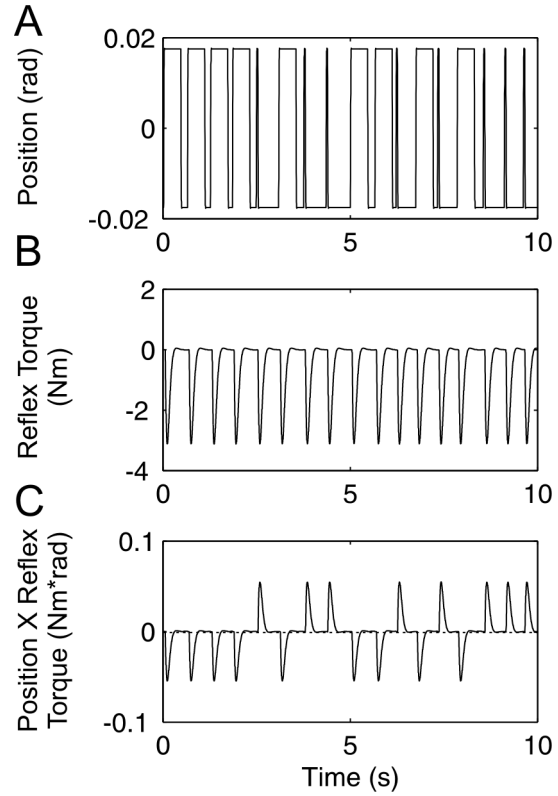


Figure 2. A) “Pulse-step” position perturbation with B) simulated reflex torque and C) position-reflex torque product. The mean of the product, shown by the dotted line in C, is equal to zero.

### D. Implementation

The real-time estimation of intrinsic and reflex stiffness was implemented using Simulink and xPC Target (Mathworks Inc.) The estimation algorithm ran at a fixed rate of 1 kHz. The following steps outline the estimation procedure.

1. The torque was high-pass filtered at .033 Hz.
2. Equation 1 was solved using an LDL solver in Simulink and xPC Target.
3. Intrinsic torque was estimated by multiplying the estimates of  $K$ ,  $B$ , and  $I$  with position, velocity and acceleration respectively.
4. Reflex torque was calculated by subtracting the estimated intrinsic torque—following high-pass filtering—from the high-pass filtered torque.
5. The estimated reflex torque and half-wave rectified velocity signals were filtered with an 8<sup>th</sup> order, type 1, low-pass Chebyshev filter, with a cutoff of 40 Hz and passband ripple of .05 dB, and then downsampled to 100 Hz.
6. A 36 lag IRF was found between the estimated reflex and the delayed half-wave rectified velocity signals, starting with a delay of 50 ms. A QR solver was used to calculate the IRF using the least-squares approach.
7. Reflex stiffness gain was calculated by integrating the reflex stiffness IRF.

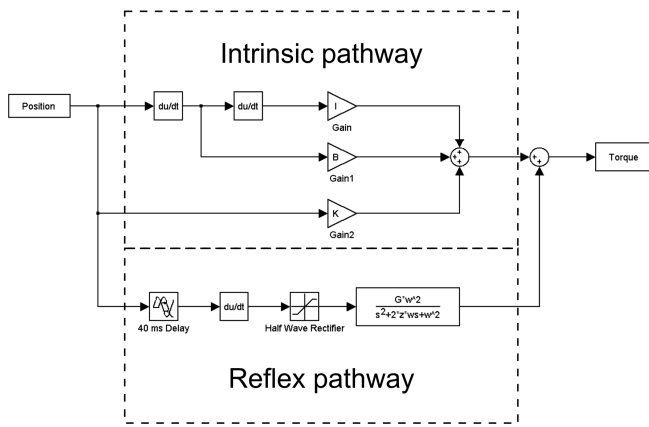


Figure 3. Intrinsic and reflex components of total joint stiffness. Intrinsic pathway is composed of three components: inertial, viscous and elastic; reflex pathway consists of a differentiator followed by a Hammerstein system where the non-linearity is half-wave rectifier and the linearity is a 2<sup>nd</sup> order low-pass system with a delay.

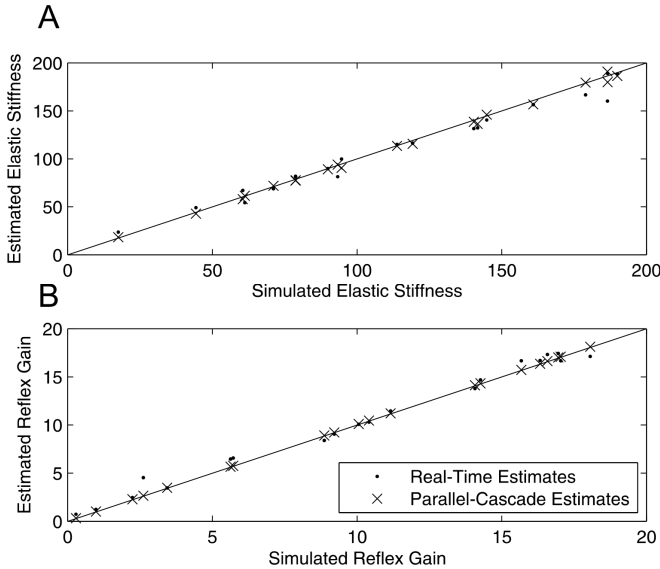


Figure 4. Real-time and parallel-cascade estimates of elastic stiffness and reflex gain. Both methods of estimation performed very well in identifying A) elastic stiffness and B) reflex gain, when both  $K$  and  $G$  were varied simultaneously.

### III. SIMULATION STUDIES

#### A. Accuracy

Intrinsic and reflex torques were simulated using a “pulse-step” position perturbation, using the model shown in Figure 3 with a 1 ms sample time. Three different simulation sets were performed. In the first set, elastic stiffness ( $K$ ) and reflex gain ( $G$ ), the most variable parameters under normal physiological conditions [4], were varied randomly and independently between 0-200 and 0-20 respectively while the other parameters remained constant. The real-time algorithm produced estimates of  $K$  and  $G$  which were close to the simulated values (% variance accounted for of 98.6% for  $K$  and 99.3% for  $G$ ). Figure 4 shows the real-time and parallel-cascade estimates for each simulation. Notice that

the real-time estimation algorithm performed as well as the parallel-cascade algorithm in estimating both parameters.

#### B. Variability

The second set of simulations evaluated the variance of the parameter estimates generated by the real-time estimation algorithm. Reflex and intrinsic torque were simulated once with  $K = 100$  and  $G = 10$ . Elastic and reflex stiffness were estimated using two different filters to perform the averaging. The standard deviation of the estimates was lower when a .033 Hz filter was used (7.9% of the mean for  $K$  and 5.2% for  $G$ ) than when a .1 Hz filter was used (11.9% for  $K$  and 7.2% for  $G$ ). However this increased averaging does have a drawback, which is shown in the next simulation.

#### C. Response Time

The third set of simulations examined the response time of the estimation algorithm. Reflex and intrinsic torque were simulated with  $K$  and  $G$  undergoing step changes from 50-150 and 5-15 at 100 and 150 seconds respectively. The risetime—the time taken for the parameter estimates to go from 10% to 90% of its change—was calculated for both changing parameters. Figure 5 shows that the response of the estimates was more sluggish when the .033 Hz filter was used (14.2 s for  $K$  and 11.9 s for  $G$ ) than when the .1 Hz filter was used (4.6 s for both  $K$  and  $G$ ).

### IV. EXPERIMENTAL STUDIES

To further validate the real-time algorithm, the algorithm was tested with experimental data. This was done by recording the real-time estimates generated by the algorithm along with the corresponding position and torque signals. The position and torque data were then used to identify reflex and intrinsic stiffness using the parallel-cascade identification algorithm.

The experimental paradigm was a simple torque matching task. Subjects were provided with a display of their average torque on an LCD monitor hung overhead and were asked to maintain it at a fixed level. The torque target began at 8 Nm, and was decreased by 4 Nm every 90 seconds, until the completion of a 90 second interval at -8 Nm, at which point the target returned to zero (see Figure 6A). All experiments were done with the low-pass filters at .033 Hz because subjects were most comfortable controlling joint stiffness with the increased averaging.

Figure 6A shows the mean torque with the real-time and parallel-cascade estimates for elastic stiffness (Figure 6B) and reflex gain (Figure 6C) recorded during an experimental trial. As mentioned previously, the risetime for the elastic stiffness was 14.2 seconds with the .033 Hz filter, so the real-time estimates for at least the first 15 s of each interval should be ignored. For both elastic stiffness and reflex gain, the mean of the real-time estimate for each interval closely matched the parallel-cascade estimates for that interval.

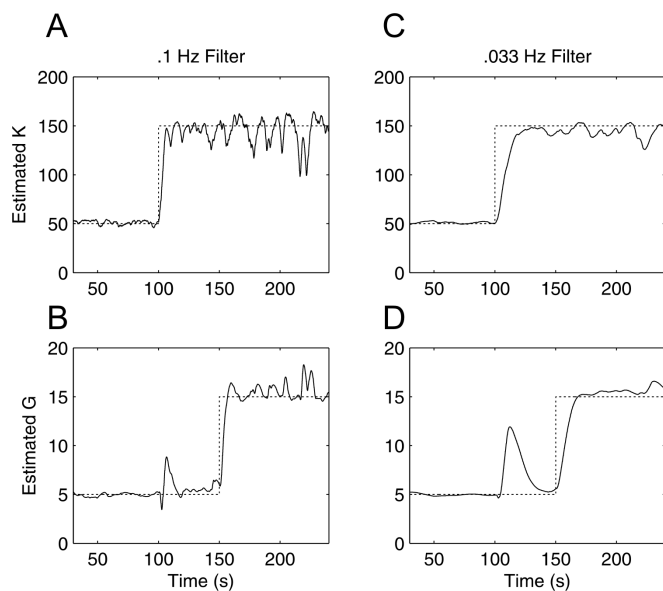


Figure 5. Response time of real-time estimates. Position and torque were simulated with step changes in elastic stiffness and reflex gain occurring at 100 and 150 s respectively. Using the .1 Hz filter (left) the response time is much quicker for both A) elastic stiffness and B) reflex gain than the response time for the C) elastic stiffness and D) reflex gain estimates produced by the .033 Hz filter (right).

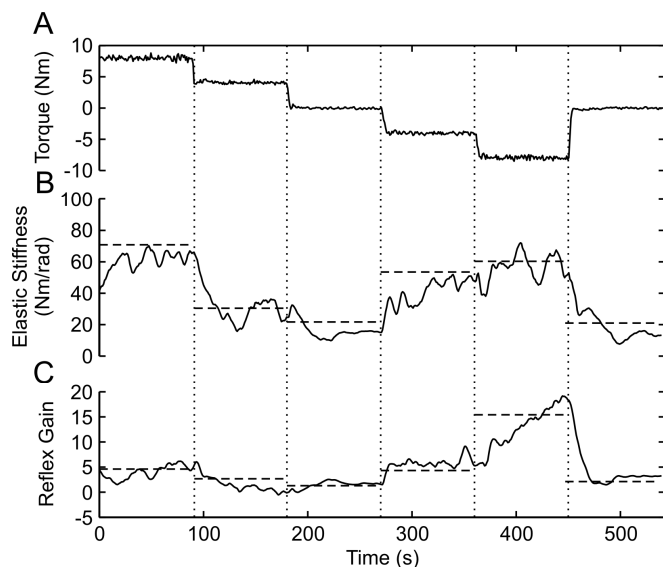


Figure 6. Real-time and parallel-cascade estimates of reflex gain and elastic stiffness in experimental data. Real-time estimates (solid line) were collected in an experimental trial where the subject varied her A) torque at 90 s intervals. Parallel-cascade estimates (dashed line) were generated following the experiment using recorded position and torque values. Parallel-cascade estimates for B) elastic stiffness and C) reflex gain closely matched the real-time estimates.

## V. DISCUSSION

We have shown that using the “pulse-step” displacement perturbations, intrinsic and reflex stiffness can be separated and estimated in real-time. Intrinsic stiffness was estimated by finding the cross-correlation between the position, its derivatives and the torque. Reflex stiffness was estimated by finding the IRF between a delayed half-wave rectified

velocity signal and the torque. Using simulated data, it was found that the real-time algorithm could accurately estimate the parameter values. Using two different filters to perform averaging, we found that there is a trade-off between precision and response time. Validating the real-time estimates with the parallel-cascade identification algorithm showed that both estimates were in agreement when both simulated and experimental data were used.

The main benefit of being able to accurately and independently estimate intrinsic and reflex stiffness is the ability to use the estimates as feedback for subjects. This was done by Ludvig et al. (Paper submitted), where they used a simplified version of this real-time estimation algorithm to estimate elastic stiffness and reflex gain in real-time and use it as feedback. They found that subjects could map out many different combinations of intrinsic and reflex stiffness. They also found that subject could vary the two components independently of each other, shining light on the issue of whether the two pathways are independent or invariably linked. Additionally this type of feedback could be used to help patients, with spinal cord injuries or other injuries which cause spasticity, reduce their exaggerated stretch reflex response, similar to what was done in [5-8]. It could also be used by clinician as a way of quantifying the degree of spasticity in spastic patients in real-time.

## VI. REFERENCES

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