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Identification of Time-varying Intrinsic and Reflex Joint Stiffness

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***Abstract*—We have developed a time-varying, parallel-cascade system identification algorithm to separate joint stiffness into intrinsic and reflex components at each point in time throughout rapid movements. The components are identified using an iterative algorithm in which intrinsic and reflex dynamics are identified using separate time-varying (TV) techniques based on ensemble methods. An ensemble of input-output records having the same TV behavior is acquired and used to identify the system dynamics as impulse response functions at time increments corresponding to the sampling interval. Simulation studies showed that the time-varying, parallel-cascade algorithm performed well under realistic conditions with 99.9% VAF between simulated and predicted torque. To evaluate the performance of the algorithm under realistic conditions we applied it to an ensemble of experimental data acquired under stationary conditions. Results demonstrated that the TV estimates converged to those of the established time-invariant algorithm and allowed us to determine how variance of the TV estimates varied with the number of realizations in the ensemble.**

I. Introduction

DYNAMIC joint stiffness quantifies the relationship between the position of the joint and the torque acting about it. It is an important mechanism used by our body to maintain posture and control movement. Joint stiffness can be separated into two pathways: intrinsic and reflex. Intrinsic stiffness encompasses the mechanical properties of the joint, active muscle and passive, visco-elastic tissues. Reflex stiffness arises from muscle activation in response to the stretch reflex. As yet, there is no consensus on how the CNS modulates stiffness or the role of intrinsic and reflex mechanisms in the control of posture and movement. It is known that intrinsic and reflex stiffness change during movement; however, these changes are difficult to characterize because they occur rapidly and therefore require time-varying (TV) identification techniques. Our lab has developed a time-varying, parallel-cascade identification algorithm capable of separating intrinsic and reflex mechanisms during rapid movements. This paper examines the algorithm's performance under simulated and experimental conditions. We compare the experimental results to those found using an established time-invariant (TI) algorithm [1].

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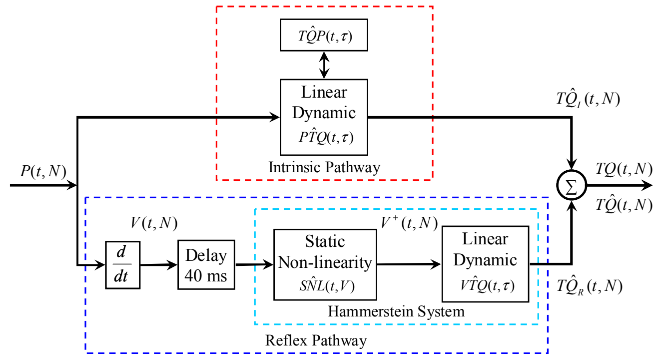

Fig. 1. Block diagram showing the time-varying, parallel-cascade model structure used to separate intrinsic and reflex contributions to total ankle torque. Position and torque are a function of time (*t*) and realization (*N*). IRFs are a function of time (*t*) and lag (τ).

II. Identification Algorithm

Fig. 1 illustrates the parallel-cascade model that describes the intrinsic and reflex contributions to joint stiffness [1]. The intrinsic pathway relates position of the joint to torque via a TV linear, dynamic element. The reflex pathway relates velocity of the joint to torque via a differentiator, delay and TV Hammerstein system. The Hammerstein system is composed of a static non-linear element, followed by a linear, dynamic element. The contribution from each pathway is assumed to add linearly to give the total torque produced by the joint.

To identify this TV model, we use an iterative algorithm in which the two pathways are identified using separate TV techniques based on ensemble methods. This requires the acquisition of an ensemble of input-output records having the same time-varying behavior. The algorithm proceeds as follows (refer to Fig. 1):

1) The input auto-correlation and input-output cross-correlation matrix are estimated at each sample time using data across the ensemble. A nonparametric estimate of intrinsic stiffness is estimated as an impulse response function (IRF) by solving the matrix equation relating the auto-correlation and cross-correlation. Previous methods used least-squares minimization to solve for the IRF [2]; this was improved for noise resistance and colored inputs by replacing the matrix equation with a pseudoinverse [3]. The length of the IRF is fixed at a value less than the reflex delay to prevent reflex contamination in the intrinsic estimate. The process is repeated at each sample time to produce a series of intrinsic stiffness estimates, $P\hat{T}Q(t,\tau)$.
2) $P\hat{T}Q(t,\tau)$ is convolved with the position ensemble to

predict intrinsic torque, $T\hat{Q}_I(t,N)$. The intrinsic residual torque is computed as

$$T\hat{Q}_{IR}(t,N) = TQ(t,N) - T\hat{Q}_I(t,N)$$

3) The static non-linear, $S\hat{N}L(t,V)$, and linear dynamic, $V\hat{T}Q(t,\tau)$, elements of the reflex pathway are estimated using an ensemble method, treating velocity as the input and intrinsic residual torque as the output [4]. A correlation approach is used first to obtain initial, nonparametric estimates of the linear subsystem. Then an iterative optimization algorithm is used to produce final estimates of the system parameters. This process is repeated at each sample time to produce a series of nonparametric, Hammerstein system estimates.
4) The non-linear Hammerstein system is used with the differentiated, input ensemble to generate $T\hat{Q}_R(t,N)$, an estimate of the reflex torque.
5) The total predicted torque is computed as

$$T\hat{Q}(t,N) = T\hat{Q}_I(t,N) + T\hat{Q}_R(t,N)$$

6) To evaluate the quality of the identification, the percent variance accounted for (%VAF) is computed between observed and predicted total torque at each time instant, *i*. In general, the %VAF is computed as

$$\%VAF = 100\left(1 - \frac{\operatorname{var}(X_i - \hat{X}_i)}{\operatorname{var}(X_i)}\right) \qquad (1)$$

where X is the true value, and $\hat{X}$ is the predicted value.
7) The procedure is repeated using reflex residual torque to compute $P\hat{T}Q(t,\tau)$, in effect increasing the signal to noise ratio. This is done until successive iterations fail to improve the %VAF.

III. Simulation Study

A. Methods

We evaluated the identification method using data from simulating a simple model of ankle stiffness. A diagram of the simulation scheme is shown in Fig. 2. Second-order parametric models were used to approximate intrinsic and reflex dynamics. Intrinsic stiffness was represented with inertial, I, viscous, B, and elastic, K, elements. Reflex stiffness was represented with gain, G, damping, ζ, and natural frequency, ω_n elements. The static non-linearity was approximated as a third-order polynomial.

Intrinsic and reflex parameters were varied with time, resulting in TV behavior of the simulated systems. First K, then G, and finally B underwent a ramp change to half their initial value. The simulated coefficients of the third order static non-linearity remained constant in this investigation.

Five hundred, 2 second pseudo-random binary sequences (PRBS) were used as the input ensemble and convolved with the simulated systems to generate an ensemble of output torque. Fig. 3 shows the position input and torque output for several realizations.

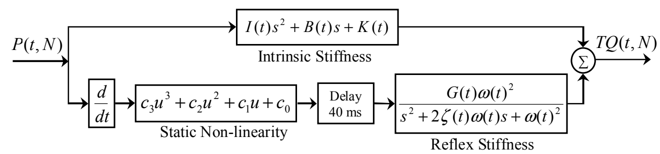

Fig. 2. Block diagram showing the simulation scheme used to evaluate the TV parallel-cascade identification algorithm. Intrinsic elasticity (K), viscosity (B), and reflex gain (G) varied with time, while the coefficients of the static non-linearity (c_0, c_1, c_2, c_3) remained constant.

B. Results

Time-varying, parallel-cascade system identification was performed on the input-output data at a sampling rate of 100Hz. The identification algorithm yielded excellent results. Predicted torques are overlaid on simulated torques in Fig. 3 and match with near perfect agreement. VAF between simulated and predicted torque was computed at each point in time and the mean value obtained. The mean VAF for intrinsic, reflex and total torque were all greater than 99%. The accuracy and precision of the algorithm was also evaluated by comparing the simulated and estimated systems. At each point in time, the VAF between the simulated and estimated system was computed; VAFs ranged from 97-99.9%.

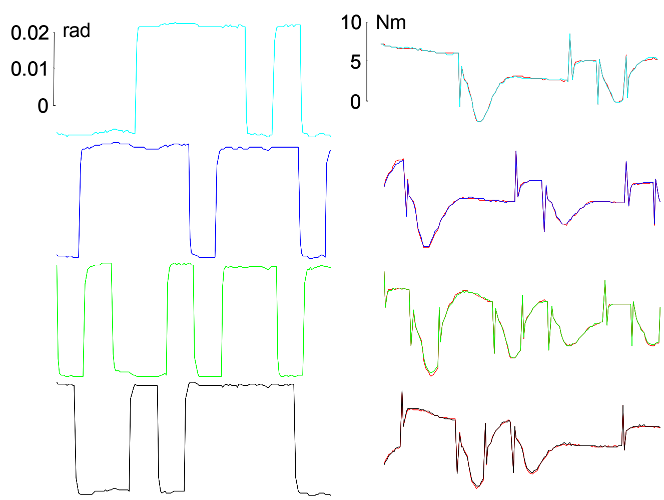

Fig. 3. Several realizations of input position (left) and output torque (right) used for system identification in the simulation study. Predicted torques are overlaid on simulated torques to illustrate excellent agreement between the signals.

C. Parametric Fits

The parametric equations used to simulate intrinsic and reflex dynamics were also fit to the IRF estimates. The time-varying properties of each system were extracted from these fits. Intrinsic dynamics were parameterized by first inverting intrinsic stiffness, $P\hat{T}Q$, to obtain compliance, $T\hat{Q}P$, then applying a Levenberg-Marquardt non-linear, least-square fit algorithm to find the second-order, lowpass system relating torque to position. Intrinsic compliance is used rather than stiffness, because the time-varying properties of interest (viscosity and elasticity) are more evident. Reflex stiffness was parameterized using the second-order, lowpass system relating rectified velocity to torque. However, prior to parameterizing the reflex stiffness, the gain was redistributed to the static non-linearity so that the area under each IRF was unity, while maintaining the same overall gain for the reflex pathway. Therefore, we expect that the parameter *G*, will be unity for the normalized system. Fig. 4 shows the recovered time-varying parameters for the estimated systems. Recall that *K*, *B,* and *G* underwent a ramp change to half their initial value. It is clear from these results that the identification algorithm correctly tracked the TV properties of the simulated systems with no *a priori* information of the TV behavior.

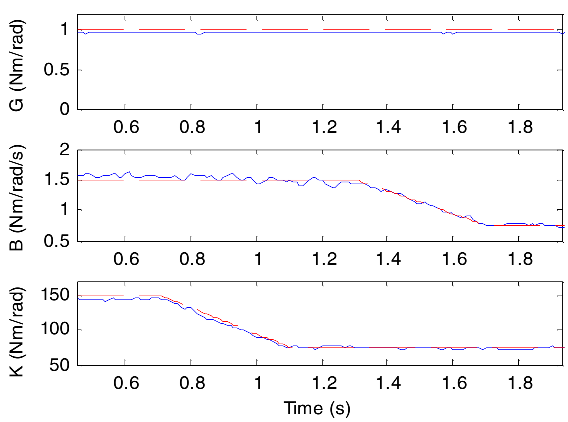

Fig. 4. Intrinsic and reflex second-order parameter estimates. Estimated parameters are shown in solid and simulated in dashed. Only those parameters that varied with time are shown: reflex gain (*G*), intrinsic viscosity (*B*), and intrinsic elasticity (*K*).

D. *Additive Noise*

To simulate noisy conditions, Gaussian white noise was added to the output signals at various signal-to-noise ratios (SNR). The input signals and noisy output signals were used to identify TV intrinsic and reflex systems. Fig. 5 shows the quality of identification as a function of SNR. The identification algorithm returned good system estimates above 10dB SNR, demonstrating that the time-varying, parallel-cascade method is capable of producing very good results even in the presence of significant output noise.

IV. Experimental Study

The algorithm was tested with real data to further validate its ability to predict intrinsic and reflex dynamics during experimental conditions. For example, background noise due to variable levels of voluntary activation will adversely affect the quality of identification. To provide a benchmark for the experimental results, we employed a time-invariant paradigm where the dynamic properties of the joint were stationary. This enabled us to compare the TV identification results to that of our established TI algorithm.

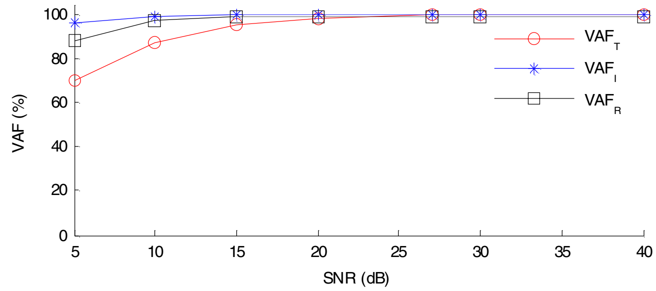

Fig. 5. Mean VAF between simulated and predicted intrinsic, reflex and total torque as a function of SNR.

A. *Apparatus*

Subjects laid supine with their ankle attached to an electro-hydraulic actuator by means of a custom made fiberglass boot. Their leg was immobilized and ankle movement restricted to plantarflexion and dorsiflexion. Perturbations were applied to the subject's ankle and measured with a potentiometer (BI Technologies 6273). Torque produced in response to the perturbations was measured with a torque transducer (Lebow 2110-5K). Position and torque were sampled at 1 kHz using a NI-4472 data acquisition card.

B. *Experimental Paradigm & Post-processing*

Three normal male subjects were examined. Subjects were required to provide a constant level of voluntary activation by maintaining a tonic 5 Nm contraction of the gastrocnemius-soleus. Subjects were assisted by a visual feedback displaying lowpass filtered torque.

Small amplitude PRBS perturbations of 0.03rad peak-to-peak were applied to the subject's ankle about the optimal operating point for reflex activity. System dynamics were assumed to remain constant at perturbations of this magnitude [5]. The perturbations were applied continuously for 5 minute periods, followed by a 1 minute rest period. This was repeated 3-5 times resulting in a total of 15-30 minutes of position and torque data. The data was then decimated to 100Hz and analyzed in two ways: First, we used a stationary, time-invariant (TI) algorithm. Second, we segmented the data into a series of 3 second trials and treated as if it were an ensemble of responses to a time-varying (TV) condition. The data ensemble for subject TG, CB, and BE consisted of 1011, 1084, and 1050 realizations respectively.

C. *Identified Systems*

A system estimate of intrinsic stiffness, intrinsic compliance, reflex stiffness, and static non-linearity was produced at each 10ms time increment. Fig. 6 shows for Subject TG that the IRFs and SNL are the same at all times. This was expected since the experiment operated under time-invariant conditions, which means there was no larger movement invoking TV system dynamics. Therefore, the system estimates confirm stationary behavior.

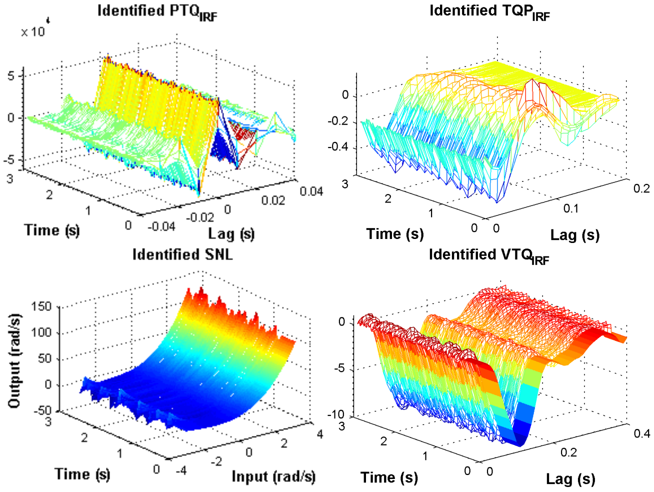

Fig. 6. Identified intrinsic stiffness, $\hat{PTQ}$, intrinsic compliance, $\hat{TQP}$, reflex static non-linearity, $\hat{SNL}$ and reflex stiffness, $\hat{VTQ}$.

D. *Predicted Torque*

The VAF between observed and predicted total torque was computed and the mean value obtained. The relative contribution of the intrinsic and reflex pathway to the total torque was evaluated as the VAF between total torque and predicted intrinsic and reflex torque, respectively. Table I outlines the mean VAF between predicted and observed torque for all three subjects. We see from these results that the TV algorithm produced good predictions of torque with 19-30% of the output variance attributed to noise.

E. *TV versus TI System Identification*

The TV system estimates were averaged to produce a mean intrinsic stiffness, compliance, reflex stiffness and static non-linearity. The mean TV systems were compared with those produced by the TI analysis. The TI and mean TV system estimates for Subject TG are shown in Fig. 7; the estimates are in close agreement, with nearly 100% VAF for each system. Similar for Subjects CB and BE, the VAF ranged from 87-99%. This indicates that the TV method is capable of identifying reflex and intrinsic mechanisms consistent with our established TI method.

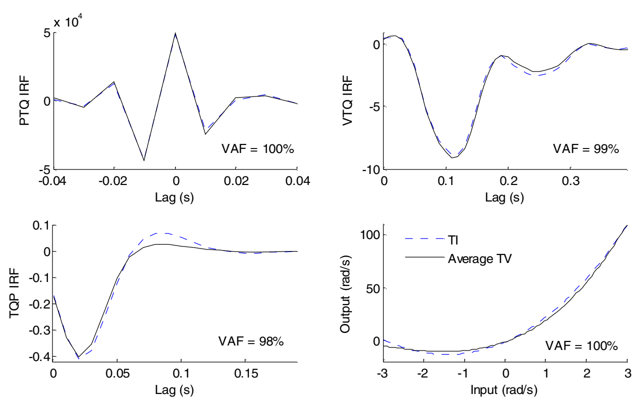

Fig. 7. TI and mean TV systems: intrinsic stiffness, *PTQ*, intrinsic compliance, *TQP*, reflex stiffness, *VTQ*, and static non-linearity, *SNL*.

TABLE I
MEAN VAF FOR PREDICTED AND OBSERVED TORQUE

Torque VAF (%)		Subject TG	CB	BE
TQ_I	mean ± std	27 ± 3	62 ± 3	36 ± 5
TQ_R	mean ± std	66 ± 2	29 ± 6	41 ± 4
TQ_{Total}	mean ± std	81 ± 1	75 ± 1	72 ± 2

F. *Identification versus Ensemble Size*

System identification was performed using various ensemble sizes to determine the amount of data required for future experiments. Data from Subject TG was used for this analysis. The VAF between predicted and observed total, intrinsic and reflex torque were examined as a function of ensemble size. It is clear from Fig. 8 that the TV algorithm converges to the TI results for ensemble sizes greater than 600 realizations. In true time-varying conditions, more than 1000 realizations will be used to ensure reliable results.

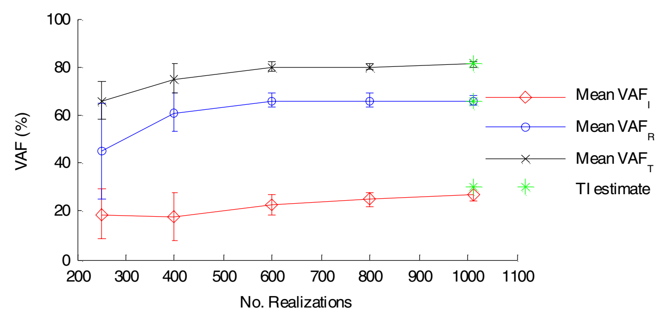

Fig. 8. Mean VAF for predicted and observed torque versus ensemble size. Mean TV values converge to TI values at 600+ realizations.

V. DISCUSSION & CONCLUSION

These studies demonstrate that the time-varying, parallel-cascade method is capable of tracking rapid changes in intrinsic and reflex stiffness and produces reliable nonparametric estimates of system dynamics. This is shown by high VAF in both simulated and experimental conditions. We have determined that ensembles sizes exceeding 600 realizations are required for reliable results. A time-varying experiment that uses a periodic movement such as a gait cycle, superimposed on a small amplitude PRBS perturbation, will be examined next by our lab.

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