Proceedings of the 28th IEEE
EMBS Annual International Conference
New York City, USA, Aug 30-Sept 3, 2006

ThC03.2

Further Study of the Asymmetry for Multifractal Spectra of Heartbeat Time Series

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Abstract— **We study the asymmetry of multifractal spectra of diurnal heartbeat time series from healthy young subjects, healthy elderly subjects and patients with congestive heart failure (CHF). Aging and CHF causes loss of multifractality. We report here some ways of analyzing the asymmetry of these spectra and we show how the joint analysis of the degree of multifractality and the parameters that characterizes the asymmetry can differentiate between the cardiac interbeat time series of young and elderly persons and it can also separate healthy subjects and CHF patients.**

I. Introduction

In recent years, scaling and fractal properties of many real time series have attracted the attention of researchers from different disciplines [1, 2]. The monofractal and multifractal properties of irregular and nonstationary signals can be described with global and local invariant quantities. Multifractal structures have been found in a growing number of physical problems such as spatial distribution of the dissipation field of fully developed turbulence [3], voltage drops across a random resistor network [4] or diffusion limited aggregation [5]. Recently, the multifractal analysis has begun to be used for the study of physiological time series [6, 7, 8]. Physiological signals are generated by complex self-regulating systems that process inputs with a broad range of characteristics. These time series are extremely inhomogeneous and fluctuate in an irregular and complex way. In reference [7], Ivanov et al. established the relevance of the multifractal formalism for the description of heartbeat time series. They found that multifractal analysis reveals important differences between the heartbeat time series of healthy subjects and those of subjects that have heart affections. They suggest that is natural to adopt multifractal concepts for the description of heartbeats since they are a result of the interaction of many physiological components operating in different scales [9]. The hypothesis of heartbeat monofractality is inadequate because monofractal signals are homogeneous in the sense that they have the same scaling properties throughout time, characterized locally by a single scaling exponent or by a single fractal dimension. On the other hand, multifractal signals can be decomposed into many subsets characterized by different local fractal dimensions, which quantify the local singularities and thus relate to the local scaling of the time series, so they are intrinsically more complex and inhomogeneous that monofractals [9].

We have applied the multifractal formalism to the analysis of interbeat time series from healthy young subjects and elderly healthy subjects. We show that the multifractal analysis could be a useful tool to evaluate the aging effect on the heartbeat dynamics. We also reproduce the results of Ivanov et al. [7] with loss of multifractality in patients with congestive heart failure (CHF) respect to healthy subjects. Our multifractal analysis is based on the Hölder exponent. Our results are qualitatively equivalent to those obtained by Ivanov et al. [7] in terms of the Hurst exponent. We also present here the behavior analysis of fractal and generalized dimensions against the q-th moment of the measure. Based on these results, we interpret the loss of multifractality of the heart interbeat time series as a loss of complexity in the heartbeat dynamics with aging and with disease (CHF). In this work we present a study of the shape of the singularity spectra in terms of their asymmetry. The first part of our asymmetry analysis consists of a graphical analysis, we plot the right part of the multifractality degree and the left part, if the spectra were symmetrical the left and the right part would be equal, but usually they are different, if the right part is greater than the left part we have a right skewed spectrum, if the left part is greater than the right part we have a left skewed spectrum. The second part of our analysis is based on the suggestion of Shimizu et al. [10], they suggested to fit the multifractal spectrum to a quadratic function and they used asymmetry parameters as a measure of complexity in an study of human posture; this idea was also used by Telesca et al. [11] in an investigation of scaling properties in temporal patterns of seismic sequences. Finally, we propose to fit the multifractal spectrum with a fourth-degree function of the same type proposed by Shimizu et al. The fitting is good, but there is a problem because of the difficulty of characterizing through a single parameter the spectrum asymmetry. However, the results are interesting, because we can separate the spectra of persons with different health conditions, and the spectra of young persons from the spectra of elderly persons. The analysis of asymmetry is important because the width of the multifractal spectra has been used together with other fractal or statistical properties of the time series to distinguish the differences between spectra of time series of healthy persons from the

Manuscript received April 3, 2006. This work was supported in part by the SEPI-IPN-MEXICO.

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spectra of time series of elderly persons. We can also separate spectra of time series of healthy persons from the spectra of time series of ill persons (CHF patients).

II. MULTIFRACTAL ANALYSIS

Multifractal signals require many local exponents to fully characterize their scaling properties [8]. The multifractal distributions are characterized by the function $f(\alpha)$ (fractal dimension) against α (Hölder exponent) that is called the multifractal spectrum [12]. Chhabra and Jensen [13, 14] developed a simple and precise method for the direct calculation of the singularity spectrum $f(\alpha)$. We can consider a normalized time series as a singular measure P. We calculate the $f(\alpha)$ curve covering the measure with boxes of length L and computing the probabilities $P_i(L)$ in each box. First a 1-parameter manifold of normalized measures $\mu_i(q)$ is defined:

$$\mu_i(q,L) = \frac{[P_i(L)]^q}{\sum_j [P_j(L)]^q} \tag{1}$$

with q the qth moment of the measure. For values of $q>1$ the strongly singular structures are enhanced, for values of $q<1$ the less singular areas are more emphasized, and for $q=1$ the original measure $\mu(1)$ is replicated. The fractal dimension can be obtained from

$$f(q) = \lim_{L\to 0} \frac{\sum_i \mu_i(q,L)\log[\mu_i(q,L)]}{\log L} \tag{2}$$

And the mean strength of the singularity is obtained from

$$\alpha(q) = \lim_{L\to 0} \frac{\sum_i \mu_i(q,L)\log[P_i(L)]}{\log L} \tag{3}$$

The equations (2) and (3) provide a relationship between a fractal dimension f and the average singularity strength α as implicit functions of the parameter. According with this interpretation a multifractal can be visualized as an interwoven ensemble of independent monofractals of dimension $f(\alpha_i)$. On each of these fractals i the observable P scales with the Hölder exponent α [11, 15]. In order to make quantitative statements about the multifractal spectra of different time series, it is common to calculate the spectrum width or degree of multifractality of the time series, $\Delta\alpha = \alpha_{max} - \alpha_{min}$, it measures the length of the range of fractal exponents in the time series; therefore, if $\Delta\alpha$ is high, the signal is richer in structure. Shimizu et al. [10] and Telesca et al. [11] suggested other parameters to describe the multifractal spectrum, they proposed to fit, by a least square method, the spectrum to a quadratic function around the position of their maximum α_0: $f(\alpha)=a+b(\alpha - \alpha_0)+c(\alpha - \alpha_0)^2$. The parameter b, which is the coefficient of the linear term, measures the asymmetry of the curve, which is zero for symmetric shapes, positive or negative for left-skewed or right-skewed shapes respectively. b informs about the dominance of low or high fractal exponents respect to the other, a right skewed spectrum denotes relatively strongly weighted high fractal exponents, corresponding to fine structures, and low ones (more smooth looking) for left-skewed spectra. Shimizu et al. [10] and Telesca et al. [11] used the fitting to the quadratic function to evaluate the parameters b and $\Delta\alpha$, but we have found that it is more adequate to evaluate $\Delta\alpha$ by using cubic splines, however, cubic splines is not adequate to evaluate asymmetry.

In the first part of our asymmetry analysis we evaluate $\alpha_{max} - \alpha_0$ (right part) and $\alpha_0 - \alpha_{min}$ (left part) these quantities must be equal if the spectrum is symmetric, but usually they are different, if the right part is greater than the left part we have a right skewed spectrum, if the left part is greater than the right part we have a left skewed spectrum.

In the second part of our analysis we follow the suggestion of Shimizu et al. [10], we begin by doing an approximation of the multifractal spectrum by one quadratic function (a quadratic polynomial):

$$P_2(\alpha) = a_1 + b_1\alpha + c_1\alpha^2 \tag{4}$$

If we plot α vs. $f(\alpha)$ this polynomial passes trough the points $(\alpha_{min}, 0)$, $(\alpha_{max}, 0)$ and as the support is of dimension 1, it must also passes by the point $(\alpha_0, 1)$. With these conditions we find the following values of the coefficients a_1, b_1 and c_1 in terms of the multifractal spectrum parameters:

$$a_1 = \frac{\alpha_{\max}\alpha_{\min}}{(\alpha_{\min}-\alpha_0)(\alpha_{\max}-\alpha_0)} \tag{5}$$

$$b_1 = \frac{\alpha_{\max}+\alpha_{\min}}{(\alpha_{\min}-\alpha_0)(\alpha_{\max}-\alpha_0)} \tag{6}$$

$$a_1 = \frac{1}{(\alpha_{\min}-\alpha_0)(\alpha_{\max}-\alpha_0)} \tag{7}$$

so the multifractal spectrum can be approximated by

$$f(\alpha) \approx P_2(\alpha) = \frac{1}{(\alpha_{\min}-\alpha_0)(\alpha_{\max}-\alpha_0)} \bullet \left[\alpha_{\max}\alpha_{\min} - (\alpha_{\max}\alpha_{\min})\alpha + \alpha^2\right] \tag{8}$$

This approximation is a parabola that opens downwards. To study the symmetry we make a translation to the origin, so we have the function

$$f(\alpha) = a + b(\alpha-\alpha_0) + c(\alpha-\alpha_0)^2 \tag{9}$$

And the coefficients a, b and c can be written in terms of a_1, b_1 and c_1. This means that $a = a_1 + b_1\alpha_0 + c_1\alpha_0^2$, $b = b_1 + 2c_1\alpha_0$ and $c = c_1$. The most interesting coefficient is b, it is

$$b = \frac{(\alpha_0-\alpha_{\min})-(\alpha_{\max}-\alpha_0)}{(\alpha_{\min}-\alpha_0)(\alpha_{\max}-\alpha_0)} \tag{10}$$

and we have three cases, if $\alpha_0 - \alpha_{min} = \alpha_{max} - \alpha_0$ then $b = 0$ and we have the symmetric case. If $\alpha_0 - \alpha_{min} < \alpha_{max} - \alpha_0$ then $b < 0$ and the spectrum is right skewed. Finally, if $\alpha_0 - \alpha_{min} > \alpha_{max} - \alpha_0$ then $b > 0$ and the spectrum is left skewed. The fitting of the spectrum to a quadratic function is not good in some cases, so we propose to use instead of the quadratic function a fourth-degree function of the same kind,

$f(\alpha)=A+B(\alpha - \alpha_0)+C(\alpha - \alpha_0)^2 +D((\alpha - \alpha_0)^3 + E(\alpha - \alpha_0)^4$, the problem is that the asymmetry in this case depends on two coefficients: the *B* coefficient of the linear term and the *D* coefficient of the cubic term. However, we have obtained interesting results also for this case.

III. RESULTS

We analyzed cardiac interbeat time series arising from two different databases taken from the reference [16], the first database is the "Fantasia database" that has data from two groups, the first is a group of twenty healthy young subjects (21-34 years old) and the second has twenty healthy elderly subjects (68-85 years old) [17]. The data consist on the registration of the interbeat interval during two hours of the subjects in rest conditions while watching the movie *Fantasia* (Disney, 1940). The second database has interbeat sequences from two another groups: 16 healthy subjects (mean age: 32.6 years) and 11 patients (mean age 54.4 years) with congestive heart failure (CHF), for our study we consider diurnal periods of ECG records (approximately 8 h). We calculate the multifractal spectra by means of the Chhabra and Jensen algorithm from $q=-30$ to $q=30$, the curve was smoothed using cubic splines and then we extrapolate to find α_{min}, α_{max} and interpolate to find α_0.

We show in Fig. 1, the behavior of $\alpha_{max} - \alpha_0$ and $\alpha_0 - \alpha_{min}$ for each one of the twenty healthy young persons and the twenty elderly persons. There is a difference between them but it is not easily appreciated, it is better if we define the ratio

$$r_s = \frac{\alpha_{\max} - \alpha_0}{\alpha_0 - \alpha_{\min}}, \tag{11}$$

if $r_s = 1$ we have the symmetric case, $r_s > 1$ (right skewed case) and $r_s < 1$ (left skewed case). We show in Fig. 2 the plot of this quantity versus $\Delta\alpha$, the separation between young and elderly persons is clearer than in Fig. 1.

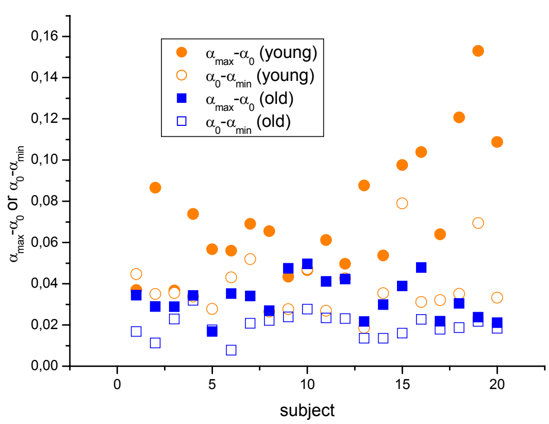

Fig. 1. α_{max} - α_0 and α_0 - α_{min} for each one of the twenty healthy young persons and the twenty elderly persons. Full symbols are separated and the open symbols are also separated. However the separation is not clear as in Fig. 2.

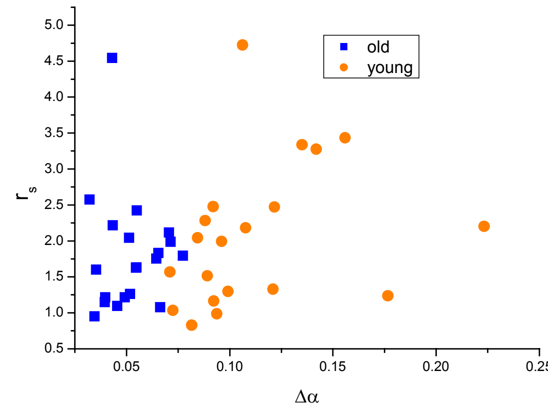

Fig. 2. In this figure we show the same situation than in figure 1, but we use the ratio r_s versus $\Delta\alpha$ instead of the differences α_{max} - α_0 and α_0 - α_{min} . The separation between young and elderly persons is clearer than in Fig. 1.

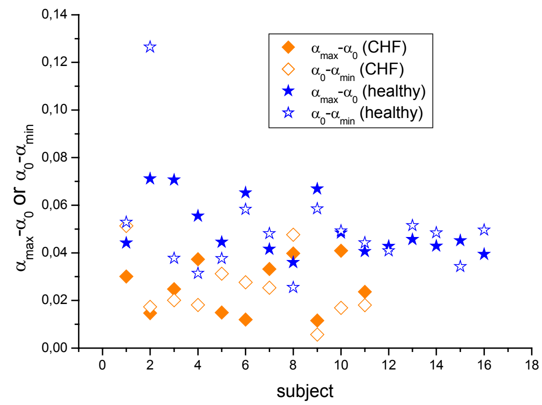

Fig. 3. α_{max} - α_0 and α_0 - α_{min} for each one of persons in the second database. As in Fig. 1 full symbols are separated and open symbols are also separated. Although there is overlapping the separation is clearer than in Fig. 1.

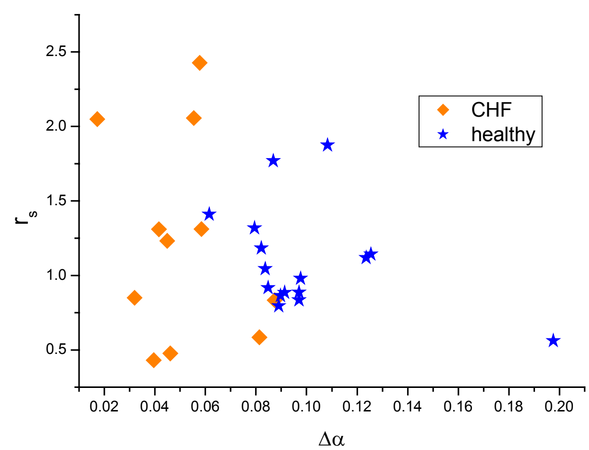

Fig. 4. The ratio r_s versus $\Delta\alpha$. The separation between the group of healthy persons and the CHF patients is clearer than in Fig. 3.

The same situation is shown in Fig. 3 and Fig. 4 for the time series in the second database. There is a separation between the spectra of the healthy persons and the spectra of

CHF patients, there is a little overlapping, but it is reasonable because researchers usually do not use a single technique to distinguish between both time series.

The fitting of the multifractal spectrum to a quadratic polynomial allows obtaining experimentally the parameter *b* that defines the symmetry of the multifractal spectrum.

This parameter results in a good separation of the multifractal spectra, between healthy young and elderly persons (Fig. 5) and also between healthy and CHF patients (Fig. 6).

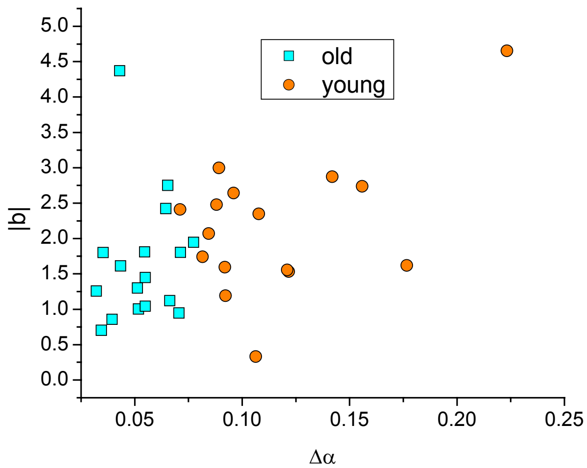

Fig. 5. The absolute value of the *b* parameter versus *Δα*. Again we observe the importance of the parameters that characterize the asymmetry, because the group of elderly persons is separated from the group of young persons.

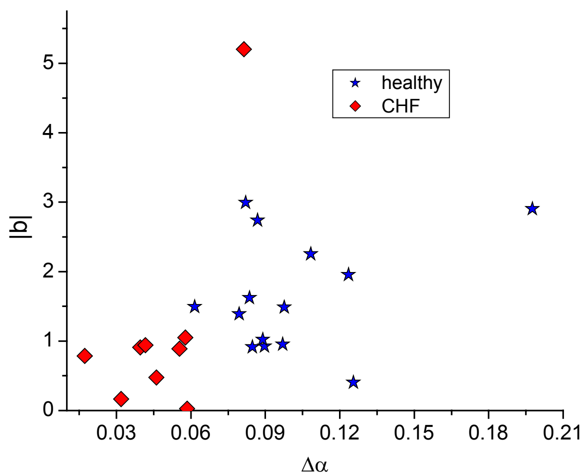

Fig. 6. The absolute value of the *b* parameter versus *Δα*. Again we observe the separation of the group of healthy persons from the group CHF patients.

Finally, we adjusted each spectrum to a fourth-degree function in order to obtain the parameters *B* and *D*. The results are similar. We do not show for brevity these results, but the best separation is obtained when we plot absolute value of the parameter D (the coefficient of the cubic term) as a function of *Δα*. We can see in this work, that additionally to the degree of multifractality the parameters that define the symmetry or the asymmetry of the multifractal spectra can be important to effectively distinguish between multifractal spectra of persons with different health conditions or with different ages.

IV. CONCLUSIONS

We have tried to show he importance of the parameters that can characterize the asymmetry of the heartbeat time series in separating the time series that belong to the group of elderly persons from the time series of the group of young persons. We have also applied these concepts to try to separate the time series of one group with C HF patients from another group of healthy persons. It seems to be that this separation is possible using the joint analysis of the degree of multifractality and the asymmetry parameters.

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