Proceedings of the 28th IEEE
EMBS Annual International Conference
New York City, USA, Aug 30-Sept 3, 2006

Finite Element Modelling of Breast Biomechanics: Directly Calculating the Reference State

V. Rajagopal, J. Chung, P.M.F. Nielsen and M.P. Nash
Bioengineering Institute, University of Auckland, New Zealand

Abstract— **Patient-specific models of the biomechanics of the breast based on finite deformation theory is potentially a valuable tool to assist clinicians in assimilating and assessing information obtained from different views of the breast, under different loading conditions and using different imaging modalities. It is anticipated that a computational model of the large deformation mechanics of the breast will also improve the accuracy of non-rigid registration techniques by restricting the deformations imposed by the algorithm to be those which are physcially plausible. Accurate registration will assist clinicians in tracking suspicious regions of tissue across multiple views of the breast, which are typically taken by applying different loads on the breast during imaging. For instance, a model that can predict deformations during mammography would help to track a region of tissue between a cranio-caudal (CC) view and a medio-lateral oblique (MLO) view. Due to the nonlinear deformations imposed on the breast during different imaging techniques, the finite element reference geometry from which deformations are predicted is important. Gravity loads act on the breast during all imaging modalities. In this paper, we describe a novel modification to solving the finite element implementation of finite deformation theory, which can predict the reference state of the breast from a deformed configuration that has been derived from images of a patient placed in a single known orientation with respect to the direction of gravity.**

I. INTRODUCTION

According to the World Health Organization, more than 1.2 million people would have been diagnosed with breast cancer during 2005 worldwide. The chance of developing invasive breast cancer during a woman's lifetime is approximately 1 in 7 (13.4%) [1].

The breast is a highly deformable soft tissue organ that assumes different shapes under the various loading conditions associated with different imaging modalities. For example, for x-ray mammography the woman stands and the breast is compressed between two plates, whilst for MRI the woman lies prone and the breast is allowed to droop under gravity. Increasingly, the information from these imaging modalities (and others) is being combined to provide a more complete picture of the health of the woman's breast. Due to the nonlinearity of the large deformations that take place between the different imaging modalities, non-rigid image registration techniques and image fusion techniques are being developed to synthesize the information. These techniques, such as maximization of mutual information, have previously used image warping to match intensity based criteria [2]. However, research has shown that incorporation of physical constraints can improve the accuracy of registration algorithms [3]. Finite element models of soft tissue mechanics are useful in computer aided surgeries and image fusion technologies because they introduce physics-based constraints on non-rigid registration algorithms.

Yu-Neifert [4] was one of the first to model breast tissue movement to determine the applicablilty of holographic interferometry in breast cancer detection. Yu-Neifert created a three-dimensional finite element model of the breast to predict deformations under gravity loading conditions. Azar *et al* [5] developed a detailed finite element model of the breast to track internal tissues during MR image-guided needle biopsy procedures. Tanner *et al* [6] provided more quantitative information by presenting a method to assess the accuracy of biomechanical models in predicting deformation under compression.

All images of the breast are obtained under gravity loading conditions (mammographic compression adds compressive loads as well). Therefore, any finite element geometry created from a set of breast images will represent a deformed state of the breast. As the prediction of large deformations is a nonlinear problem, it is important to perform forward simulations from an accurate estimate of the unloaded geometry (or reference state). To date, no model published in the literature has addressed this aspect of breast modelling. We have previously considered this issue by proposing an optimisation algorithm, which identifies a reference state from a series of loaded deformed configurations derived from images of a patient placed in different orientations with respect to the direction of gravity [7]. We have also previously proposed a method which requires the reformulation of the finite elasticity equations in which the integrals are defined over the deformed configuration, rather than the typical, standard way of defining them over the reference configuration [8]. In this paper, we describe a third method, which requires a small modification to the solution technique for forward deformation predictions, without the need for reformulating the integrals over the deformed configuration. This technique is not only applicable to breast mechanics, but to the wider field of finite deformation elasticity, as one could obtain the reference state from a deformed state, given the material properties, and loading and boundary conditions that imposed the deformation. Here, we illustrate the applicability of this technique to breast biomechanics.

II. MODELLING THEORY

We model the deformation of breast tissues using finite deformation theory ([9], [10], [11]), which can accurately capture soft tissue deformation. Here, we outline the important aspects of finite deformation theory.

1-4244-0033-3/06/$20.00 ©2006 IEEE.

A. Finite Deformation Theory

1) Kinematics: The problem is to find the coordinates ($\mathbf{x}$) of the deformed body, (v), given the coordinates, ($\mathbf{X}$), of the undeformed body (V). The deformation gradient tensor $\mathbf{F}$ provides the relationship to map between the undeformed and the deformed states, and is defined as

$$\mathbf{F} = \left\{ \frac{\partial x_i}{\partial X_M} \right\} \tag{1}$$

The Lagrangian Green strain tensor $\mathbf{E}$ is calculated using:

$$\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T\mathbf{F} - \mathbf{I}\right) \tag{2}$$

The aim is to find a solution vector $\mathbf{x}$ representing the degrees of freedom defining the deformed state, such that the principles of conservation of mass, linear momentum, and angular momentum are satisfied.

2) Principle Laws of Continuum Mechanics: The modelling framework has been expressed with respect to the reference configuration and thus the equations below are written in terms of the undeformed state.

When a body is in equilibrium, all of the forces (body and traction) must balance. This is achieved by satisfying the principle of conservation of linear momentum:

$$\frac{\partial}{\partial X_M}\left(T^{MN}\frac{\partial x_j}{\partial X_N}\right) + \rho_0 b^j = \rho_0 f^j \tag{3}$$

where T^{MN} are components of the second Piola Kirchhoff stress tensor (force per unit area of the undeformed body), b^j are the body forces (such as gravity), and f^j are components of the surface tractions acting on the body.

The stress equilibrium equations (3) can be expressed in an alternative form using the principle of virtual work as follows ([12]):

$$\int_{S_2} \mathbf{s}\cdot\delta\mathbf{v}dS + \int_V \rho\left(b^j - f^j\right)dV - \int_V \frac{1}{J}T^{MN}\frac{\partial x_j}{\partial X_M}\frac{\partial \delta v_j}{\partial X_N}dV = 0 \tag{4}$$

where δv_j are the virtual displacements expressed in terms of the reference coordinate system, S_2 is the free boundary surface on which virtual displacements are applied, and J is the determinant of the deformation gradient tensor.

These nonlinear governing equations have been formulated in such a way that they can be solved using an iterative finite element modelling technique. A description of the finite element modelling method can be obtained from [12]. The integrals in equations (4) are evaluated over the undeformed volume. These equations can be reformulated to integrate over the deformed configuration. The reader is pointed to [8] for an outline of this formulation.

B. CALCULATING THE DEFORMED STATE

Given the reference coordinates $\mathbf{X}$, the unknowns in the set of nonlinear differential equations are the deformed coordinates and the hydrostatic pressure variables $\mathbf{x}$. A linearised system of equations can be obtained by reformulating equation (4) to be a set of residuals

$$\mathbf{R}(\mathbf{x}) = \mathbf{0} \tag{5}$$

that must be minimised with respect to the unknowns. A search direction for the minimisation may be obtained using the Newton Raphson technique [13].

The method is derived from the Taylor series expansion of a function value at a point, $\mathbf{r}$, from the current point, $\mathbf{x}$. Let us consider the system of n nonlinear residuals $R_i(\mathbf{x})$ with the current position in the parameter space being ($\mathbf{x}_o$) and the increment from the current position being $\mathbf{r}$. The Taylor series expansion of each of the equations is

$$R_i(\mathbf{x_o}) + \frac{\partial R_i}{\partial x_1}(\mathbf{x_o})r_1 + \frac{\partial R_i}{\partial x_2}(\mathbf{x_o})r_2 + \cdots + \frac{\partial R_i}{\partial x_n}(\mathbf{x_o})r_n + O(\mathbf{r}^2) = 0 \tag{6}$$

Ignoring nonlinear terms, this expansion can be reformulated with $R_i(\mathbf{x_o})$ on the right hand side, resulting in the matrix form :

$$\mathbf{J}(\mathbf{x_o})\mathbf{r} = -\mathbf{R}(\mathbf{x_o}) \tag{7}$$

$\mathbf{J}$ is the Jacobian (or global stiffness) matrix with the set of derivatives of the each of the n residuals with respect to the n solution degrees of freedom. The unknowns are the increments $\mathbf{r}$ to the solution degrees of freedom for the subsequent solution iteration. The entries in the Jacobian matrix can be calculated numerically using a finite difference approach (or analytically). For a one sided finite difference approximation, a derivative is numerically calculated using

$$\frac{\partial R_i(\mathbf{x})}{\partial x_j} = \frac{R_i(\mathbf{x} + \Delta\cdot\mathbf{e}_j) - R_i(\mathbf{x})}{\Delta} \tag{8}$$

Finite difference derivatives for each of the residual equations are calculated by perturbing each of the solution degrees of freedom by an amount Δ using equation (8) to form the global stiffness matrix in equation (7). $\mathbf{e}_j$ is a vector of dimension n with a value of 1 at the j^th index and 0 everywhere else. The initial estimate of the deformed coordinates is typically taken as the undeformed coordinates. The increments $\mathbf{r}$ are then found using direct solver techniques such as LU decomposition or iterative solvers such as the Generalised Minimum Residual (GMRES) method. The above steps are repeated until all the residuals are minimised.

C. CALCULATING THE UNDEFORMED STATE

Idenitification of the reference state has been addressed in the wider field of finite element modelling. Previous studies reformulated the finite elasticity equilibrium equations in terms of the deformed configuration such that the undeformed state degrees of freedom were considered to be the dependent variables in the equations [14], [15]. This section will show that the undeformed state can be accurately determined without the complexities of this reformulation of the standard finite elasticity equations.

Consider again the finite elasticity equations (4). For the forward problem, $\mathbf{X}$ are known and $\mathbf{x}$ are unknown.

In order to find the values of the unknowns, the system of residuals are differentiated with respect to each of the unknown solution degrees of freedom.

On the other hand, for the reverse problem, $\mathbf{x}$ can be considered as the knowns, and $\mathbf{X}$ the unknowns. Hence, the Jacobian matrix could be constructed with the first derivatives of the same finite elasticity residuals with respect to the undeformed coordinates

$$\frac{\partial R_i(\mathbf{X})}{\partial X_j} = \frac{R_i(\mathbf{X}+\Delta\cdot\mathbf{e}_j)-R_i(\mathbf{X})}{\Delta} \tag{9}$$

An initial estimate of the undeformed coordinates are the deformed coordinates. The boundary conditions on the undeformed state are still applied as if the forward problem was being solved. The entire problem is set up as if a forward problem was being solved with the only difference being that the undeformed coordinates are perturbed to satisfy the usual equilibrium equations.

The primary advantage of this method is the ease in setting up the procedure, given an implementation of the conventional formulation. The advantage of formulating the equilibrium equations with respect to the reference configuration is that the integrals are all expressed over domains that do not change as the body deforms and thus are not affected by variation or linearization steps [12].

III. Validation Studies Using Silicon Gel Phantoms

We have previously validated the proposed method [16] using the analytic solution to the deformation of a thick-walled cylinder under uniaxial extension and pressure inflation [17]. The analytic solution provided unique deformed internal and external radii for a specific pressure inflation and axial extension. Therefore, it was possible to validate the proposed method by predicting the undeformed state from the deformed state to the pressure inflation and axial extension of the cylinder. In this paper, we present the applicability of the method to breast biomechanics based on experiments on a homogeneous breast phantom.

A. Methods

We systematically validate assumptions made during the modelling by measuring the accuracy of our model in predicting deformations of a silicon gel phantom (Fig. 1). For this paper, the gel was oriented at several angles to the direction of gravity and the deformed surface was scanned to provide a dense sampling of data points. We have conducted independent experiments to accurately characterise the mechanical behaviour of the silicon gel [18]. Our mechanical tests showed that the silicon gel could be modelled as a neo-Hookean material with the strain energy function $W = c_1(I_1-3)$. The material parameter, c_1 was estimated for this validation exercise using a univariate nonlinear optimisation technique coupled to a finite element model of the silicon gel phantom under gravity loading. One deformed configuration data set was used to estimate the material parameter, and the other deformed configurations were then used to verify the accuracy of the material parameter in modelling the gel behaviour.

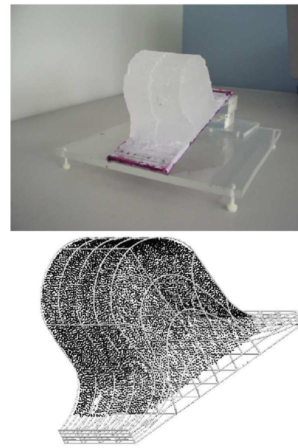

Fig. 1. Top: Gel phantom used to validate our homogeneous model predictions. Bottom: Predicted deformation matching scanned surface data with RMS error 0.9 mm. Gel positioned at an angle of 30 degrees to the horizontal.

A geometric model of one deformed configuration (positioned at 30 degrees to the horizontal direction) was then created using a surface fitting procedure developed by Nielsen [19] to fit a generic model to the deformed surface data set. This fitted model was then used to calculate the undeformed state using the proposed method. The predicted undeformed state was then compared to the known undeformed state, obtained from the known dimensions of the mould used to make the gel.

B. Results

The material parameter, c_1 in the neo-Hookean strain energy function, was found to be 1.38 kPa. This parameter was used to predict the deformations of the five deformed states using a forward model with RMS errors in the range of 0.7 mm and 0.9 mm (Fig 1). A model of the gel phantom was fitted to the deformed data set in Fig 1 with an RMS error of 0.4 mm. The proposed method predicted the undeformed state using the fitted mesh with an RMS error of 0.9 mm.

Based on the surface profile comparisons, it is clear that the proposed method accurately predicts the reference configuration when given the deformed configuration, material properties and loading conditions. However, it must be noted that this method will predict slightly different reference configurations for different deformed configurations. This is primarily due to the errors that are inherently present in all experimental measurements. To illustrate this issue, figure 2 consists of surface data sets of two predicted reference configurations using two deformed configurations of the silicon gel phantom. It is clear from the figure that the two data sets do not represent the same reference configuration. This discrepancy in the reference configurations is potentially more pronounced for more complex systems such as the breast, due to the inhomogeneity and complex boundary conditions applied to obtain different images of the breast.

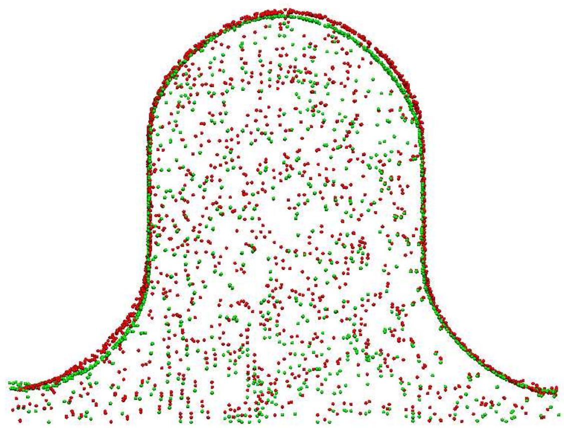

Fig. 2. Two predicted reference state surface data sets. The green data set was obtained from the predicted reference state for deformed state in Fig 1. The red data set was obtained from the predicted reference state for the gel lying flat on a table

Therefore, it is important to compare the performance of the optimisation algorithm presented in [7] and the direct calculation method proposed in this paper to choose the best method of determining the reference state.

IV. CONCLUSIONS

We have developed a novel but straightforward technique to identify the reference state of a body given a deformed configuration, material properties, and loading conditions. The method requires a simple modification to the standard implementation of finite elasticity using finite elements, and is applicable to a wide variety of problems that require the determination of the reference state. The applicability of the method was demonstrated on a silicon gel breast phantom, which showed that the method could predict the reference configuration with an RMS error of 0.9 mm. Future studies will further validate the performance of this method for modelling breast biomechanics.

REFERENCES

[1] website article by Imaginis-The Breast Cancer Resource, April 2005. [Online]. Available: http://imaginis.com/breasthealth/statistics.asp

[2] M. Wirth, J. Narhan, and D. Gray, "Nonrigid mammogram registration using mutual information," *SPIE Medical Imaging: Image Processing*, vol. 4684, pp. 562–573, San Diego 2002.

[3] C. Tanner, J. Schnabel, A. Degenhard, A. Castellano Smith, C. Hayes, M. Leach, D. Rose, D. Hill, and D. Hawkes, "Validation of volume-preserving non-rigid registration: Application to contrast-enhanced mr-mammography," in *Medical Image Computing and Computer-Assisted Intervention (MICCAI 2002). Lecture notes in Computer science*, vol. 2489-I, 2002, pp. 307–314.

[4] Q. Yu-Neifert, "A three-dimensional finite element model to predict the applicability of holographic interferometry to breast tumour detection," Ph.D. dissertation, Graduate Faculty of the University of Akron, 1995.

[5] F. Azar, D. Metaxas, and M. Schnall, "A finite element model of the breast for predicting mechanical deformations during biopsy procedures," *Mathematical Methods in Biomedical Image Analysis*, pp. 38–45, 2000.

[6] C. Tanner, A. Degenhard, J. Schnabel, A. Smith, C. Hayes, L. Sonoda, M. Leach, D. Hose, D. Hill, and D. Hawkes, "A method for the comparison of biomechanical breast models," in *Proceedings of the Workshop on Mathematical Methods in Biomedical Image Analysis*, 2001, pp. 11–18.

[7] V. Rajagopal, J. Chung, P. Nielsen, and M. Nash, "Finite element modelling of breast biomechanics: Finding a reference state," in *27th Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, vol. 27, September 2005.

[8] P. Pathmanathan, D. Gavaghan, J. Whiteley, M. Brady, M. Nash, P. Nielsen, and V. Rajagopal, "Predicting tumour location by simulating large deformations of the breast using a 3D finite elementmodel and nonlinear elasticity," in *Lecutre Notes in Computer Science*, vol. 3217, 2004, pp. 217–224.

[9] M. Nash and P. Hunter, "Computational mechanics of the heart," *Journal of Elasticity*, vol. 61, pp. 113–141, 2000.

[10] J. Oden, *Finite Elements of Nonlinear Continua*. McGraw-Hill, 1972.

[11] L. Malvern, *Introduction to the Mechanics of a Continuous Medium*. Prentice-Hall, 1969.

[12] O. Zienkiewicz and R. Taylor, *The Finite Element Method: Solid Mechanics*, 5th ed. Butterworth-Heinemann, 2000, vol. 2.

[13] W. Press, S. Teukolsky, W. Vellerling, and B. Flannery, *Numerical Recipes in C*, 2nd ed. Cambridge University Press, 1992.

[14] S. Govindjee and P. Mihalic, "Computational methods for inverse finite elastostatics," *Computer Methods in Applied Mechanics and Engineering*, vol. 136, pp. 47–57, 1996.

[15] S. Govindjee and P. Mihalic, "Computational methods for inverse deformations in quasi-incompressible finite elasticity," *International Journal for Numerical Methods in Engineering*, vol. 43, pp. 821–838, 1998.

[16] V. Rajagopal, J. Chung, D. Bullivant, P. Nielsen, and M. Nash, "Finite elasticity: Determining the reference state from a loaded configuration," *International Journal of Numerical Methods in Engineering*, 2006, submitted.

[17] R. Rivlin, "Large elastic deformations of isotropic materials vi. further results in the theory of torsion, shear and flexure," *Philosophical Transactions*, vol. A242, pp. 173–195, 1950.

[18] V. Rajagopal, P. Nielsen, and M. Nash, "Development of a three-dimensional finite element model of breast mechanics," in *26th Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, vol. 26, September 2004, pp. 5080–5083.

[19] P. Nielsen, "The anatomy of the heart: A finite element model," Ph.D. dissertation, University of Auckland, 1987.