Proceedings of the 28th IEEE
EMBS Annual International Conference
New York City, USA, Aug 30-Sept 3, 2006

ON THE COMPLIMENTARITY OF SENSE AND GRAPPA IN PARALLEL MR IMAGING

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ABSTRACT

Two image reconstruction methods currently dominate parallel MR imaging: SENSE and GRAPPA. While both seek to reconstruct images from subsampled multi-channel MRI data, there exist fundamental differences between the two. In particular, SENSE reconstructs an image of the excited spin-density directly whereas GRAPPA reconstructs estimates of the fully sampled raw coil data and then combines them to obtain an image. In this work we show that these differences can be exploited such that each method can compliment the other. In the case of SENSE, which requires an estimate of the coil sensitivity map before reconstruction, one can use GRAPPA to improve the coil sensitivity estimates. Alternatively, using coil sensitivity estimates and the SENSE reconstruction equations, one can improve the GRAPPA reconstruction parameter estimation. Together, these approaches can provide higher image quality than either method alone.

1. INTRODUCTION

Parallel MRI (PMRI) has proven to be an effective technique to reduce data acquisition latency in MR imaging. PMRI employs multiple receiver coils to introduce spatial-domain encoding of the FOV that compliments the Fourier encoding traditionally employed in MRI. Two image reconstruction methods currently dominate the parallel MR imaging field. One is SENSE [1] which seeks to reconstruct *an image* from subsampled data acquired through multiple receiver coils. The other is GRAPPA [2], which seeks to estimate the *missing data in each coil*. The final image is then reconstructed through combination of the reconstructed coil data, e.g. via a root-sum-of-squares combination of the spatial domain representation. While both seek to reconstruct a image of the excited spin density within the MR scanner, there exist fundamental differences between the two approaches. These differences can be exploited such that each method can compliment the other.

SENSE requires an estimate of the coil sensitivity map before reconstruction. The sensitivity estimates can be acquired in a separate acquisition, i.e. a pre-scan, or data from within the accelerated scan can be used to produce self-referenced estimates. There exists an inherent trade-off in self-referenced approaches. Specifically, one must acquire a smaller low-frequency set of data at the Nyquist sampling frequency from which to produce low-resolution estimates of the coil sensitivity. At a given subsampling factor, this additional data is acquired in place of higher spatial frequencies that may be desired. Typically, one aims to acquire as little low-frequency data as possible. Below, we demonstrate how GRAPPA can be used to provide high-quality coil sensitivity estimates for SENSE and related methods from very little low-resolution data.

Alternatively, the SENSE reconstruction equations lend themselves naturally to various regularization approaches [3]. The use of regularization in GRAPPA has been less well explored. Here, we demonstrate how one can use an estimate of the coil sensitivities and the SENSE reconstruction equations to improve the GRAPPA reconstruction parameter estimation through regularization. This leads to less variance in the GRAPPA parameters, leading to potentially higher quality images.

2. THEORY

The aim of parallel MR imaging methods is to reconstruct an image of the excited spin distribution from sub-sampled k-space data acquired in multiple coils. The signal acquired in each coil, $l = 1, 2, \cdots, L$, can be modeled by the signal equation:

$$s_l(\mathbf{k}) = \int_V W_l(\mathbf{r})\rho(\mathbf{r})e^{j2\pi\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}. \quad (1)$$

Here, $\rho(\mathbf{r})$ is the excited spin density function throughout the volume V, $\mathbf{r}$ is a vector describing the spatial position within the FOV, $W_l(\mathbf{r})$ is the spatial sensitivity of coil l at spatial point $\mathbf{r}$, and $\mathbf{k}$ is a reciprocal spatial (wave number) vector determined by the gradients employed between RF excitation and data acquisition. Note that the data is sampled in the $\mathbf{k}$ coordinate frame, termed *k-space*, and reducing the number of acquired k-space lines proportionally reduces the image acquisition time. The solution of a discretized version of this linear system gives an estimated image of the spin distribution computed from the sampled data.

In SENSE-like methods, Eq. (1) is typically discretized and expressed as a linear system of equations $s = P\rho$, where s is a vector holding the acquired data, P is defined by the spatial and Fourier encoding [3], and ρ is the image data to reconstruct. The solution of this system of equations produces the image. The equation can also be represented as

Support for this research provided in part by NIH U41 RR019703-01A2 (Jolesz, PI)

$s = (PF^H)F\rho = GF^H\kappa$, where F is a unitary Fourier transform operator, and κ is the k-space representation of ρ, that is, $F\rho = \kappa$. This representation leads nicely to the GRAPPA algorithm, as recently shown by Kholmovski and Parker [6]. Specifically, the data estimation process in GRAPPA can be modeled by the system of equations

$$\begin{aligned} s_l &= G\kappa \\ \begin{bmatrix} s_l^{(a)} \\ s_l^{(u)} \end{bmatrix} &= \begin{bmatrix} G_a \\ G_u \end{bmatrix} \kappa. \end{aligned} \tag{2}$$

Here, the vector s_l is composed of two parts: a known SENSE-like part, giving acquired phase encode lines: $s_l^{(a)} = G_a\kappa$ and the unknown GRAPPA-extension part, giving unacquired phase encode lines: $s_l^{(u)} = G_u\kappa$. Assuming a suitable pseudo-inverse for G_a exists, one can replace κ in the unacquired part with the acquired data terms to give

$$s_l^{(u)} = G_u(G_a^{\dagger} s_l^{(a)}). \tag{3}$$

This illustrates that the GRAPPA algorithm seeks to estimate the composite $G_uG_a^{\dagger}$ encoding matrix, to reconstruct unacquired coil data, $s_l^{(u)}$, from acquired coil data, $s_l^{(a)}$.

This distinction is significant: SENSE-like methods and coil-by-coil methods each seek to model separate interactions of the same acquisition process. Thus, it is possible to *combine* the two methods and employ each of their strengths.

2.1. Combining GRAPPA and SENSE

We have shown previously that SENSE performs better in estimating high-frequency k-space components, but only in the presence of high-quality coil sensitivity estimates [3]. Alternatively, GRAPPA performs better in estimating the low-frequency components of k-space [3], due to its emphasis in convolution estimation. Here, we present two methods that combine each of these strengths.

SENSE-like methods require high-quality estimates of the coil sensitivities before reconstructing the image. Here we show that GRAPPA can provide additional data to increase the number of k-space lines used in auto-calibration techniques for SENSE. Alternatively, using estimates of the coil sensitivity, one can compute an estimate of $G_uG_a^{\dagger}$ in a SENSE-like manner. One can then regularize the linear system of equations used to find the GRAPPA reconstruction parameters, using the SENSE-like estimate as a regularization *prior*. In both cases, from a given data set, these combined approaches can provide higher quality images than either method alone.

2.1.1. GRAPPA-guided-SENSE

The use of GRAPPA to enhance the coil sensitivity estimates used in SENSE-like methods is straight forward. First we tailor our data subsampling scheme to acquire a small number of lines separated in frequency by $(1\Delta k)$ in the low frequency region of k-space. As one moves farther from the center of k-space, a significant region of lines spaced $(2\Delta k)$ apart are acquired. Beyond this second region, higher acceleration factors can be employed, as shown in Fig 1.

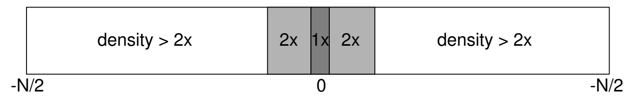

Fig. 1. Sampling density distribution

Note that this sampling strategy is similar to SHRUG, as described in [7]. The difference is that for SHRUG, multiple frames are interleaved to create the $(1\Delta k)$ sampling necessary for coil sensitivity estimation. In the current work, we use GRAPPA to perform a similar task, using lines in the $(1\Delta k)$ density as GRAPPA auto-calibration signals, and then using GRAPPA to fill in the missing lines within the $(2\Delta k)$ region to reduce the spacing to $(1\Delta k)$. This allows our GRAPPA-guided-SENSE approach to be employed in acquisition paradigms where temporal data is unavailable, while still reducing the number of auto-calibration lines needed by SENSE-like methods to construct high-quality images.

2.1.2. SENSE-guided-GRAPPA

The GRAPPA estimation parameters, n, are found by mapping a number of acquired k-space lines to a set of auto-calibration signal (ACS) lines. For simplicity, we consider here a 4x1 GRAPPA kernel, which produces the system of equations

$$s_{ACS}^{(l)}(j) = \sum_{m=1}^{L} \sum_{c=-2}^{2} s_m(c + k_x, k_y) n(m, c, l) \tag{4}$$

where m runs over the coils, c runs over the kernel size, and $j = k_x + N_x * (k_y - 1)$. This is effectively a least-squares problem, $s = Jn$, where the elements of J are acquired data points, $s_m(c + k_x, k_y)$, organized by order of summation.

As described in the Theory section above, GRAPPA seeks to estimate the composite encoding matrix $G_uG_a^{\dagger}$ for each coil. The acquired-portion corresponds to the encode matrix used in SENSE-like methods, e.g. $G_a = PF^H$. This is the G-SMASH [5] version of the SENSE formulation, and we have demonstrated previously that, when ordered first by phase-encode index and then by coil number, this matrix has a banded structure [3]. Likewise, the unacquired-portion G_u has a similar banded structure. Using estimates of the coil sensitivity, one can construct the composite matrix $G_uG_a^{\dagger}$, an example of which is shown in Fig. 2. As seen in Fig. 2, each band is a shifted version of the previous band. This redundancy enables GRAPPA to construct high-quality images using a small number of reconstruction parameters, namely the coefficients found in Eq. 4, after some re-ordering. The dashed box in Fig. 2 shows the region corresponding to the GRAPPA parameterization.

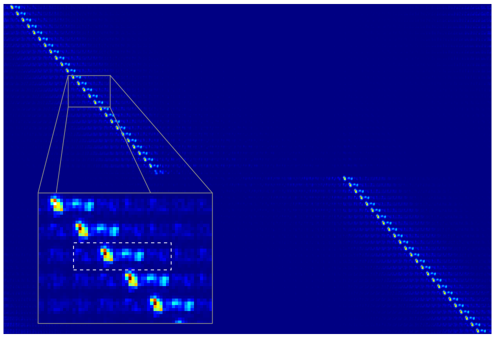

Fig. 2. Illustrated structure of the composite coil-by-coil encoding system, $G_u G_a^\dagger$. The dashed box shows the region estimated by the 4x1 GRAPPA kernel, which is repeated across multiple horizontal bands.

Our suggestion is to use estimates of the coil sensitivity to construct a regularized version of the composite matrix $G_u G_a^\dagger$. We then use this estimate in GRAPPA, taking data corresponding to the reconstruction kernel from one or more bands of $G_u G_a^\dagger$ to build a prior, n_0, in the GRAPPA parameter estimation equation. Specifically, Eq. 4 is augmented by including the prior estimate

$$\begin{bmatrix} s \\ n_0 \end{bmatrix} = \begin{bmatrix} J \\ \lambda^2 I \end{bmatrix} n \tag{5}$$

which is solved in a least-squares fashion. This biases the GRAPPA reconstruction parameters towards the structure given by the sensitivity estimates.

3. RESULTS

Here, we demonstrate a few results employing GRAPPA-guided-SENSE and SENSE-guided-GRAPPA.

3.1. GRAPPA-guided-SENSE Results

To illustrate the advantages of GRAPPA-guided-SENSE, Fig 3 shows two images reconstructed from subsampled FGRE data. The data was acquired using an 8 channel cardiac coil on a 1.5T MR Scanner (GE Medical Systems). The *k-space* sub-sampling scheme used 8 $(1\Delta k)$ lines in the low-frequency region. The $(2\Delta k)$ region spanned the phase index region from -23 to 23. Beyond the $(2\Delta k)$ region, the sampling density was exponentially weighted [3], growing from $(3\Delta k)$ in the mid-frequency range to $(9\Delta k)$ in the high-frequency range. A total of 74 lines, out of 256, were acquired giving an effective acceleration factor of almost 3.5x.

GRAPPA was employed to estimate missing lines in the $(2\Delta k)$ region. This allowed self-referenced coil sensitivity estimates, as per [8], to be derived from 46 (8 $(1\Delta k)$ + 19 odd $(2\Delta k)$ + 19 estimated even) central lines of k-space. These estimates were then used in a SPACE RIP reconstruction (a SENSE-like approach) with Tikhonov regularization [9]. Although we could have included the GRAPPA-estimated data in the SPACE RIP reconstruction, we chose not to do so here in order to emphasize the differences produced by the different coil sensitivity estimates.

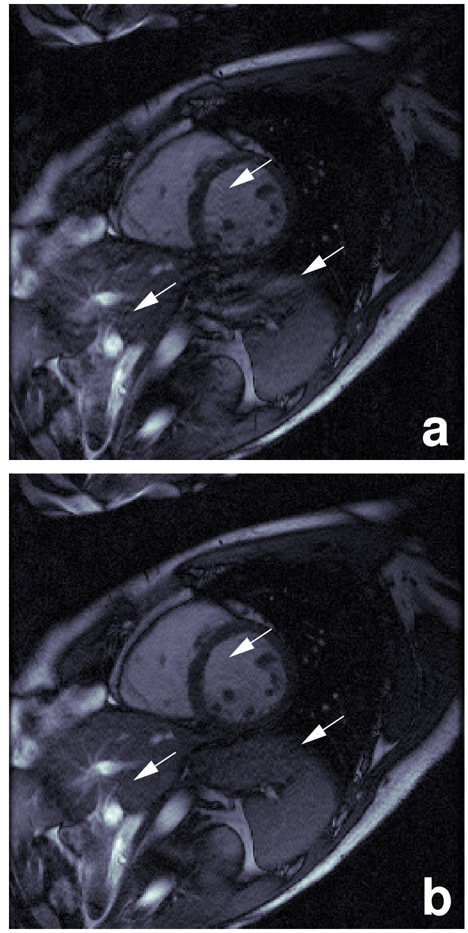

Fig. 3. SPACE RIP reconstructions of 3.5x accelerated data using (a) 8 and (b) 46 (27 raw + 19 GRAPPA estimated) autocalibration lines to estimate coil sensitivity. The raw data for both images is identical. The improved quality visible in (b) is attributed to the use of GRAPPA to improve the coil sensitivity estimates.

Fig. 3(a) shows the image reconstructed using coil sensitivity estimates from only 8 lines of k-space. Fig. 3(b) shows a similar reconstruction using the GRAPPA-guided sensitivity estimates. The arrows in both images show locations where the improved coil sensitivity estimates provided by GRAPPA produce fewer artifacts. Specifically, there appear bright spots from fat regions on the edge of the thoracic cage that are aliased into the central field of view, and lines of poor resolution through the left ventricle.

3.2. SENSE-guided-GRAPPA Results

Here, we compare images reconstructed by our SENSE-guided-GRAPPA (SgG) method with the recently proposed Spatially Variant GRAPPA (SV-GRAPPA) [6] approach. An accelerated acquisition was simulated by 4x subsampling an 8-coil low-resolution phantom data set, and using 32 lines for auto-calibration. Note that the matrix diagram of Fig. 2 corresponds to this sampling pattern. Panel (a) of Fig. 4 shows the SV-GRAPPA reconstruction. While predominantly artifact free, the reconstruction is hampered by high frequency noise which completely obscures the central radial pattern of low-contrast dots. In contrast, the SgG reconstruction accurately shows the low-contrast dots, but with significantly greater artifacts. For comparison, we show the comparable SPACE RIP image in panel (c), which was reconstructed from the same raw data using the regularization parameter setting as in [9]. It is notable that the SgG reconstruction approaches that in (c), but displays significantly more artifacts.

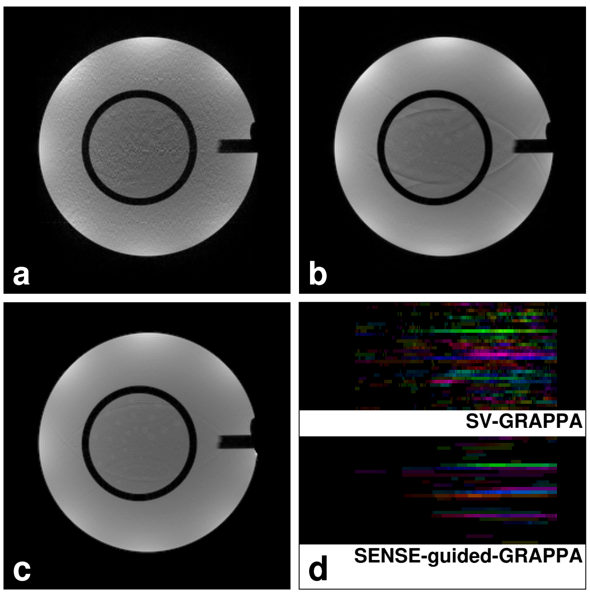

Fig. 4. Reconstructions of a low-contrast slice using (a) SV-GRAPPA with 4x1 kernel, (b) SgG, and (c) SPACE RIP with optimized regularization. Panel (d) illustrates the SV-GRAPPA and SgG reconstruction parameters for one coil across multiple columns.

A comparison between the SV-GRAPPA and SgG reconstruction parameters is shown in panel (d) of Fig. 4. The parameter values are shown with intensity mapped to magnitude and phased mapped to color. The top image shows the SV-GRAPPA parameter estimates, while the bottom row shows estimates regularized using SENSE-guided priors. Note that the two parameter estimates are quite visually similar, but the SgG-regularized parameters show significantly less variance horizontally as the parameters change from column to column.

4. DISCUSSION

We have demonstrated here two approaches to combining SENSE and GRAPPA in the reconstruction of subsampled parallel MRI data. In particular, we have demonstrated that GRAPPA can be used to improve the coil sensitivity estimates used in SENSE-like reconstruction algorithms. Alternatively, coil sensitivity estimates and the SENSE reconstruction equations can be used to guide the estimation of GRAPPA reconstruction parameters. There is little downside to using GRAPPA-guided-SENSE. With a small number of parameters to estimate, and a relatively small number of lines to reconstruct, adding GRAPPA to SENSE reconstructions incurs very little processing overhead. The same cannot be said about SENSE-guided-GRAPPA, however. This approach requires estimation of the coil sensitivities and an inversion of the SENSE system of equations, both of which add significant processing overhead to standard GRAPPA implementations. However, we anticipate that further benefits may be achieved by using the two reconstruction methods iteratively. This could potentially enable the reconstruction of highly accelerated data with a very low number, perhaps as low as 2, auto-calibration lines.

5. REFERENCES

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