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Relation Between Wavelet-Biopotential Likeness and Wavelet Compression of Biopotential Trains

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Abstract - The wavelet transform (WT) gains ground every day over other methods of biosignal processing. Yet there is still much ambiguity when deciding which particular wavelet to use. The aim of this work is to consider the relationship between (a) the likeness of a wavelet to a transitory signal (the Hodgkin-Huxley action potential), and (b) the performance of the WT compressing a train of said signal. In order to achieve this, a new method of quantifying likeness is developed. The method is insensitive to amplitude, DC level, frequency and phase of the signals it is comparing. The compression performance of the WT is evaluated for several number of thersholding schemes (soft, hard, global, level, etc.). **For all evaluations, the wavelet most alike the transitory signal yielded the best compression results**. This study opens the door and establishes a methodology for many possible future researches.

KEY WORDS: Wavelets, action potential, likeness quantification, compression merit figures.

I. INTRODUCTION

Muscle fibers and neurons are both excitable cells. These communicate by means of electrical impulses. This discrete impulse is called the action potential (AP). It can be measured by taking the potential difference between the inner and outer part of the cell membrane. The mathematical description developed by Hodgkin and Huxley (H-H) morphologically reproduces the potential and also explains the phenomenon [1].

The H-H equation system was originally intended for the squid AP, but the concept behind it constitutes the paradigm to explain any case of communication among the inner parts of beings (including humans), e.g., nerves, cardiac cells or muscular cells in general; i.e., the equations for other cells are obtained by slight modifications in the coefficients and/or adding some terms. It is even understood that the form of the signal will be more or less similar in any of the mentioned cases. Part of the value of the H-H study is this generality. The H-H circuit model and formulation is as follows:

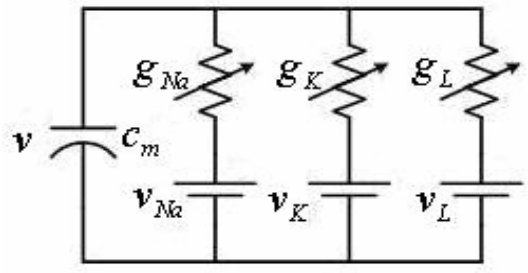

Fig. 1 H-H electrical circuit model.

$$c_m \frac{\partial v}{\partial t} = -g_{Na}(v - v_{Na}) - g_K(v - v_K) - g_L(v - v_L) \qquad \text{(Ec. 1a)}$$

$$\text{Where: } \begin{aligned} g_K(v,t) &= g_{K\infty} n^4(v,t) \\ g_{Na}(v,t) &= g_{Na\infty} m^3(v,t) h(v,t) \end{aligned} \qquad \text{(Ec. 1b)}$$

The voltage and time dependant functions n, m and h are defined in the form $dx/dt = -\alpha_x(1-x) - \beta_x x$ (Ec. 1c), for x = n, m, or h. All 6 values of α and β (2 for each subscript: n, m and h) have an exponential form and depend on voltage v and temperature [2], [3]; c_m and the different gs are the membrane capacitance and conductance, respectively.

The WT is a great tool for signal and image processing in biomedicine and bioengineering fields. That the WT consistently outperforms other methods is usually explained by the facts that biosignals are non-stationary and that the WT is best suited to deal with these types of signals [4], [5]. Yet the use of WT to stationary, time invariant biosignals (outperforming Fourier transform and short-time Fourier transform) makes this explication at least incomplete [6].

One possible reason for the better results of the WT in certain biosignal applications lies in the likeness between the potentials that form the biosignal and the wavelet, something previously suggested but not proven [6]. This seems justified physiologically by the accepted model for the formation of superficial biosignals [7], [8] and by the different results obtained in analyzing the same biosignal with WT using different wavelets [6].

Some other works seem to show a subjective, almost visual justification of the selection of the particular wavelet on the basis of likeness; and do not compare the performance between several wavelets [9], [10]. An analysis of the H-H AP with WT utilizing several wavelets is proposed. The results find application in physiological modeling, pathology diagnosis and biosignal analysis in general. The generalization of the method could have application in every wavelet signal processing area.

The proposal consists in first, evaluating the WT performance, using different wavelets, in the compression of AP trains; and second, to compare those results with the likeness quantification between the AP and the wavelets used. Therefore, a new, fair, likeness-quantifying algorithm was developed.

II. Methodology

The H-H system (Ec. 1) has been solved using a fixed-step method. The step is chosen small enough to assure stability and convergence. The Matlab™ technical computing language (version 6.5, release 13, running on a Pentium 4, 2.4 GHz, HP laptop) is used to obtain the approximations of the wavelet functions, it is also used for all other computation and programming. Symmetric-padding mode is used. The following wavelets were considered: Haar, Daubechies 2-10 (db2-10), Symlet 2-8 (sym2-8), Coiflet 1-5 (coif1-5), and discrete Meyer (dmey); which are some of the most used. The WT is performed at level 3.

A new algorithm that uses the correlation coefficient is developed in order to obtain a fair measure of likeness between the wavelets and the H-H AP. When two signals are compared, the algorithm yields an answer, between 1 (for equal signals) and -1 (for opposite signals), which is independent of amplitude, DC level, time of occurrence of the signal (phase), and frequency (scale). Fig. 2 shows the conceptual steps behind the algorithm.

Ec. 2 shows the correlation coefficient (C.C.) formula; as can be seen, it is insensitive to amplitude and DC level variations. Fig. 3 shows how the insensitiveness for the time of occurrence (phase) and for frequency (scale) of the signal is achieved. The C.C. is a measure of the extent to which one signal can be approximated by a linear function of another signal.

$$r = \frac{\sum_m (A_m - \bar{A})(B_m - \bar{B})}{\sqrt{\left(\sum_m (A_m - \bar{A})^2\right)\left(\sum_m (B_m - \bar{B})^2\right)}} \qquad \text{(Ec. 2)}$$

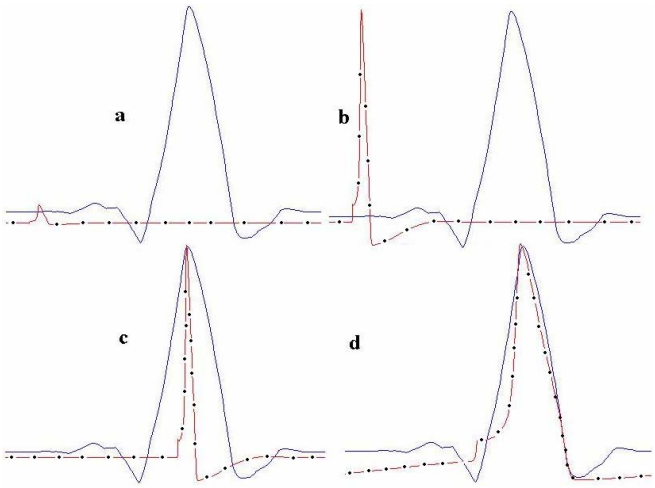

Fig. 2. Graphical illustration of the idea behind the likeness algorithm. The AP (dotted line) fits the symlet 4 scaling function (continuous line) through steps: a) no fitting; b) amplitude and DC fitting; c) time (phase) fitting; and d) frequency fitting.

The AP train is generated with values: $t_{START} = 0$; $t_{END} = 79$ ms; and dt = 10 μs; the activation current starts at 7.75 ms; and the cycle resets every 18 ms. Meaning the AP shots up at times: 7.75, (18 + 7.75), (36 + 7.75) and (54 + 7.75) ms. A train of 4 APs is thus formed. After generation, the AP is subsampled at 11.1 KHz, which is a theoretically and practically consistent value [11], [12]. In order to have a fair fitting of the frequency (scale) between the signals, the single AP for likeness-quantification has more than enough post-transitory values, Fig. 2a, (high t_{END}, 300-350 ms).

Compression was done by soft and hard thresholding and also by global and level dependant thresholding. The empirical method Balance Sparcity-Norm is used to determine the threshold value. With this option the percentages of retained energy and number of zeros are the same.

The merit figures for compression are the L^2 recovery score, a global measure of retained energy (Ec. 3); and root mean square error, a local point-to-point measure (Ec. 4).

$$\%L^2 = 100\|S_P\|^2 / \|S_O\|^2 \qquad \text{(Ec. 3)}$$

$$\text{RMS} = \sqrt{(1/N)\sum^{N} (S_P - S_O)^2} \qquad \text{(Ec. 4)}$$

Where S_P is the processed signal and S_O is the original signal. A high L^2 score and low RMS error is desired.

The block corresponding to noise in the left side of Fig 3 was added to make the merit figure evaluation more real and robust. Three types of noises were added: 60Hz (0.0015 amplitude), 60Hz-Gaussian (0.0015 amplitude and standard deviation), and 60Hz-White (0.0015 and 0.002 amplitude and standard deviation, respectively). In total, 4 different AP trains were compressed: 3 with noise added and one with no noise added.

III. Results

Table 1 shows, in descending order, the absolute values of the correlation coefficients (C.C.) between: (a) the AP and the scaling function, and (b) the AP and the mother wavelet.

Table 1. a) C.C. for the AP with the scaling functions (S.F.). b) C.C. for the AP with the mother wavelets (M.W.).

S.F.	C.C.	M.W.	C.C.
sym4	0.96041	coif1	0.85887
coif1	0.95898	db2	0.81786
sym6	0.95155	sym2	0.81786
coif2	0.94947	coif2	0.80015
db4	0.9474	db3	0.79942
sym8	0.94538	sym3	0.79942
db3	0.94537	sym4	0.79599
sym3	0.94537	sym6	0.76948
coif3	0.94361	coif3	0.76577
coif4	0.93976	sym5	0.76498
coif5	0.93691	sym8	0.74765
db5	0.93478	coif4	0.74347
dmey	0.92395	db4	0.73098
db6	0.92269	coif5	0.72744
db7	0.91355	sym7	0.7009
sym7	0.90978	haar	0.67673
sym5	0.90721	dmey	0.6637
db8	0.90586	db5	0.6521
db9	0.89845	db6	0.6445
db2	0.89177	db7	0.64097
sym2	0.89177	db8	0.62324
db10	0.89128	db9	0.59583
haar	0.28045	db10	0.56498

a b

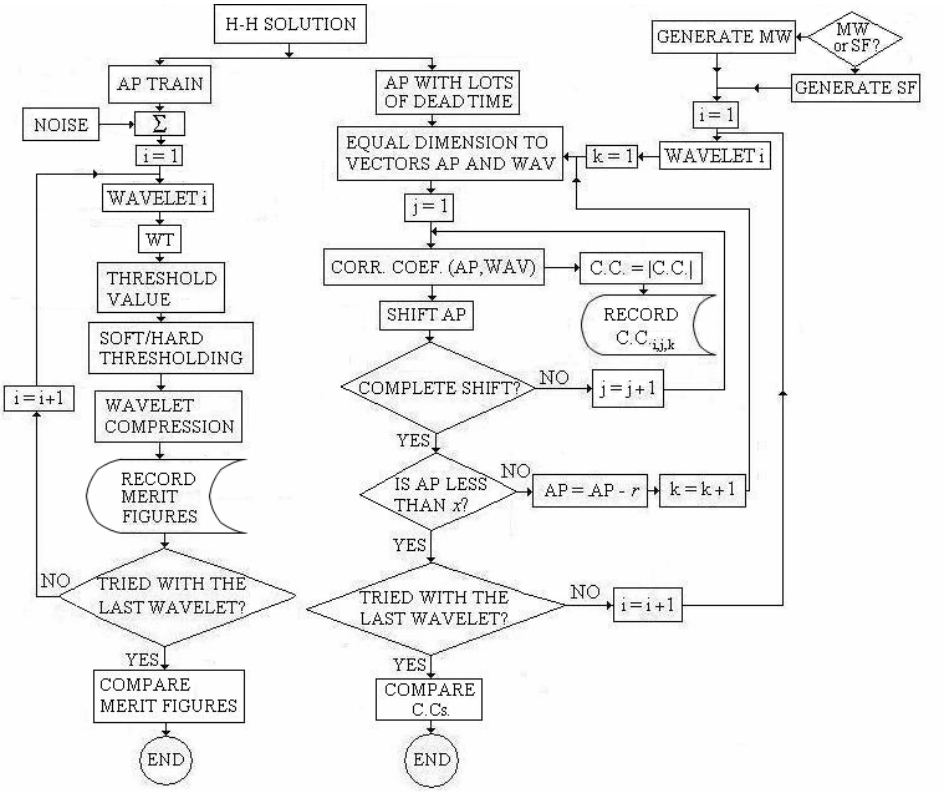

Fig. 3 Flow chart of the train-compression (left) and likeness-quantification (right) methods. Either 60Hz, 60Hz-Gaussian, 60Hz-White, or no noise is added to the AP train; the resulting signal is compressed. Threshold value is calculated for global and level options. Soft and hard thresholding is performed. This is repeated for all the considered wavelets. For the likeness algorithm, either the mother wavelet (M.W) or the scaling function (S.F.) is chosen; and refer to as WAV thereafter. The correlation coefficient (C.C.) between vectors AP and WAV for each shift, frequency (scale) and wavelet is calculated and its absolute value stored. When the AP dimension is less than x, the wavelet is changed; x is chosen so that the likeness evaluation is not done with a deceptively small and physiologically nonsensical part of the AP. The cut r is chosen small enough to assure a thorough and complete evaluation of all possible frequencies (scales) of the AP.

In all, 32 compression tables are evaluated: 3 noised and one pure AP train (4 different trains); soft and hard thresholding (2); global and level-dependent threshold calculations (2); and 2 merit figures for each (4 X 2 X 2 X 2). Table 2 shows the average of the merit figures for all 32 compression schemes. The highest and lowest values are highlighted. The merit figure values presented apply to soft and hard thresholding.

IV. Discussion

Fitting the AP presents difficulty due to some of its characteristics, e.g., it has non-zero mean, and it is completely asymmetrical, among others. Yet the likeness-quantifying algorithm is able to find an optimum fit for amplitude, DC level, phase, and frequency (scale).

Merit figures present the same values with either soft or hard thresholding (table 2). These same values suggest that the thresholding method yields relative small thresholds.

Because of the experimental nature of this work, it would not be prudent to assert definitive statements regarding the relationship between the AP-wavelet likeness, and the WT performance in compression of AP trains. However, several tendencies are noticeable: the Symlet 4 wavelet performs the best for every thresholding and noise. Other wavelets that perform well are: db5, db3, sym3, coif3, db7, coif5 and coif1. The worst performing wavelets are: sym8, db6, coif2, db4, Haar, db2, sym2; table 1.

Table 2. Averages of RMS and L^2 percentage values of all 32 compression evaluations. The values apply for soft and hard thresholding. For each column the highest value is highlighted with horizontal lines and the lowest value with diagonal lines.

WAVELETS	RMS (E-03)	STD (E-05)	%L²	STD (E-02)
haar	4.6277	5.6455	99.502	2.08
db2	5.1501	5.1988	99.393	2.2222
db3	2.4684	10.915	99.869	1.9233
db4	4.6216	5.7815	99.527	1.9509
db5	2.255	12.009	99.895	1.793
db6	4.2772	6.2356	99.61	1.8133
db7	2.6217	10.277	99.859	1.8876
db8	3.9462	6.7565	99.678	1.6799
db9	3.0292	8.8545	99.814	1.8282
db10	3.51	7.6141	99.755	1.6324
sym2	**5.1501**	5.1988	**99.393**	2.2222
sym3	2.4684	10.915	99.869	1.9233
sym4	**2.1306**	12.733	**99.904**	1.8813
sym5	3.7352	7.1759	99.697	2.078
sym6	3.2056	8.3395	99.785	1.6722
sym7	3.5321	7.5903	99.739	1.9185
sym8	4.1776	6.3923	99.638	1.6769
coif1	2.8085	9.5612	99.826	2.1352
coif2	4.3926	6.0712	99.589	1.648
coif3	2.5857	10.439	99.866	1.8726
coif4	4.0423	6.6044	99.68	1.5895
coif5	2.6586	10.152	99.87	1.7253
dmey	3.5794	6.9081	99.801	1.425

With respect to the scaling function-AP likeness sorting (Table 1a); among the 8 best performing wavelets, 6 are in the upper half (including the best one), 1 being in the middle; and among the 7 worst wavelets, 4 (including the worst 3) are in the bottom half. With respect to the mother wavelet-AP likeness sorting (Table 1b); among the 8 best performing wavelet, 5 are in the upper half; and among the worst, 3 are in the bottom half (2 of the worst 3 are not in the bottom half).

Likeness to the scaling function seems to be more related to performance, rather than likeness to the mother wavelet. This is consistent with filter bank representation of the WT. The Symlet 4 is the scaling function most alike the AP and the one that performs the best for in every single threshold option (Fig. 2d). Nevertheless, although some relationship is seen, it is not a direct one; i.e., more likeness does not necessarily imply better performance. Some other factors may also influence wavelet performance

V. Conclusion

Optimizing biosignal processing is of great theoretical and practical interest. Mathematically, WT is a state of the art tool; and it gives excellent results when applied to biological systems.

In order to evaluate the likeness between signals in a fair, unambiguous way, a new algorithm was developed. This algorithm yields a result between 1 and -1 which is invariant to amplitude, DC level, phase, and frequency.

For the compression of AP trains, it is found that the scaling function most alike the AP (Symlet 4) is the one that yields the largest L^2 recovery score and smallest root mean square error. The Symlet 4 performance is the same for soft and hard thresholding, global and level thresholding, and for the pure and noised AP trains. Though not a direct relationship, there seems to be some link between scaling function-AP likeness and AP train wavelet compression performance.

This research represents the beginning of other important works; developing also an adequate procedure for the development of these.

References

[1] Hodgkin, A. L., Huxley, A. F. A Quantitative Description Of Membrane Current And Its Application To Conduction And Excitation. Nerve. J. Physiol., 1952, London 117:500-44.

[2] Hille, B. Ion Channels of Excitable Membranes. Third Edition. Sinauer. 2001.

[3] Hobbie, R. K. Intermediate Physics for Medicine and Biology, Third Edition. AIP Press. 1997.

[4] Akay, M. Wavelet applications in medicine. IEEE Spectrum, Pp50-56. May 1997,

[5] M. Unser; A. Aldroubi A. A review of wavelets in biomedical applications. Proc. IEEE, Vol. 84, No. 4, April 1996, 626-638.

[6] Englehart, K., Hudgins, B., Parker, P. A. A Wavelet Based Continuous Classification Scheme for Multifunction Myoelectric Control. IEEE Trans Biomed Eng, No. 3, BME-48:302-311. March 2001

[7] De Luca, C. J. Physiology and Mathematics of Myoelectric Signals. IEEE Trans Biomed Eng, 1979, BME-26:313-325.

[8] R.M. Rangayyan. Biomedical Signal Analysis. A case-study approach. IEEE Press, 2001.

[9] Hasegawa, H. A Wavelet Analysis of Transient Spike Trains of Hodgkin-Huxley Neurons. http://arxiv.org/PS_cache/cond-mat/pdf/0109/0109444.pdf, 2001.

[10] J. Fang; C.G. Agarwal; B.T. Shahani. Decomposition of Multiunit Electromyographic Signals. IEEE Trans Biomed Eng, Vol.46, No.6, June, 1999, 685-697.

[11] J. D. Bronzino (Editor). The Biomedical Engineering Handbook. IEEE Press, 2000.

[12] J.G. Webster (Editor). The Measurement, Instrumentation, and Sensors Handbook. CRC Press IEEE Press, 1999.